PLAYING FREQUENCY SHIFT DUE TO THE INTERACTION BETWEEN THE VOCAL TRACT OF THE MUSICIAN AND THE CLARINET

Claudia Fritz, Joe Wolfe

School of Physics, UNSW, Sydney
cfritz@phys.unsw.edu.au

Jean Kergomard, René Caussé

LMA, Marseille and IRCAM, Paris, France

ABSTRACT

Previous qualitative studies (Mooney [1], Benade [2], Hoekje [3]) confirm musicians' opinions that the vocal tract (VT) affects both timbre and pitch. Johnston, Troup and Clinch [4] modelled the tract as a one peak resonator, which, if tuned to the fundamental f_0 of a clarinet, gives a playing frequency of f_0. But this result depends upon their particular tract impedance Z, which is small and real at harmonics of f_0. In general, the playing frequency is shifted.

We relate the flow and the pressure difference at the reed by the usual non linear Bernoulli's equation in the time domain, simplified as a third order polynomial. In the frequency domain, the impedances of the clarinet and the vocal tract are in series. To obtain an analytical result for the frequency shift, we expand about the threshold oscillation. We compare this result with numerical results from the harmonic balance method. Finally, impedance spectra measured on an artificial VT with a discrete but variable area function are used in this theoretical study. We suggest that tuning the vocal tract to f_0 may be rare and unnecessary.

1. PHYSICAL MODELLING

The clarinet is modelled here as a self-sustained oscillator with a linear exciter (the reed) which is coupled nonlinearly to two linear resonators: the pipe and the vocal tract. In previous studies, the tract was often ignored. A sketch of the mouthpiece is shown in Figure 1, which includes the meaning of the physical quantities used. In contrast to most previous models, here the mouth pressure p_{mouth} is not constant but instead has time variation:

$$p_{mouth} = P_{m0} + p_m(t) \tag{1}$$

where P_{m0} is the DC component (used alone when the effects of the VT are omitted), which is assumed to be the source, and $p_m(t)$ is the acoustic component, assumed periodic.

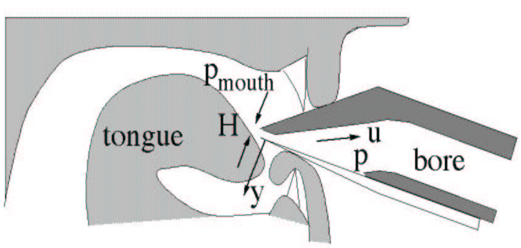

Figure 1: *Schematic view of the system with dimensionless physical quantities*

1.1. The reed

As the reed is not our main interest, we will restrict discussion to the case where the playing frequency is much lower than that of the reed resonance and describe the reed as a simple spring, of stiffness k_r so

$$k_r y(t) = -(p(t) - p_m(t) - P_{m0}) \equiv -(\Delta p - P_{m0}) \tag{2}$$

1.2. The pipe

The pipe is usually characterized by its input impedance, which describes its resonances. High frequencies are not important in our study so, for the modelling only, our 'clarinet' is a cylinder, whose length l is assumed to include end effects for radiation and the volume velocity due to reed displacement, and in which radiation losses are neglected compared to visco-thermal ones. It has neither tone holes nor bell (which are not important to the oscillation mechanism). Dispersion is as well neglected. We indicate the frequency domain by using capital letters and write the dimensionless equation

$$P(\omega) = Z_p(\omega)U(\omega), \tag{3}$$

The dimensionless input impedance is given by [5]:

$$Z_p(\omega) = i\tan(kl) \tag{4}$$

$$\text{where} \quad kl = \frac{f}{4f_p} - i\psi\eta\sqrt{\frac{f}{f_p}} \tag{5}$$

and where $\psi \simeq 1.3$ for common conditions in air, $f_p = c/4l$ is the first resonance frequency of the pipe, and η is a dimensionless loss parameter: $\eta = \sqrt{l_v l / r^2}$ as derived in ref. [6, Ch.6].

1.3. The vocal tract

The vocal tract is modelled as a (linear) impedance Z_m, as 'seen' by the reed (see the discussion in a companion paper [7]). Flows entering the mouth and the instrument are in opposite directions, so:

$$P_m(\omega) = -Z_m(\omega)U(\omega), \tag{6}$$

Following Jonhston et al [4], we approximate the vocal tract as a one peak resonator, an admittedly crude physical model that ignores anatomical details, which are as yet not really well known. We note that only the lowest tract resonance falls in the range in which the instrument has strong harmonics unless played loudly. Further, this allows us to examine Jonhston et al's proposition that coincidence or non coincidence of the VT resonance with one of the instrument's resonance may be important.

1.4. The non linear coupling

Assuming some hypotheses, in particular that the system changes sufficiently slowly for Bernoulli's law to be valid and including the volume flow due to the reed displacement in the bore impedance of the bore as described above, the air flow at the reed is related to the pressure difference Δp across it thus:

$$u(t) = \begin{cases} S(t)\sqrt{2|\Delta p - P_{m0}|/\rho}\,\mathrm{sign}(\Delta p - P_{m0}) & \text{for } y > -H \\ 0 & \text{for } y \leq -H \end{cases} \tag{7}$$

where $S(t) = (H + y(t))w$ is the cross-sectional area of the opening, determined by the reed width w and displacement y.

Denoting dimensionless pressure and flow with a tilde, nondimensionalising pressures by dividing by that which closes the reed, P_M, impedances by dividing by the characteristic impedance of the bore Z_0, and defining a dimensionless mouth pressure γ, we write:

$$\tilde{\Delta p} = \frac{\Delta p}{P_M} \qquad \text{and} \qquad \tilde{u} = \frac{uZ_0}{P_M} \tag{8}$$

$$\gamma = \frac{P_{m0}}{P_M} \tag{9}$$

A dimensionless "embouchure" parameter characterizes the mouthpiece and the mouth position:

$$\zeta = Z_0 wH\sqrt{\frac{2}{\rho p_M}} \tag{10}$$

Substituting dimensionless quantities in equation (7) gives:

$$\tilde{u}(\tilde{\Delta p}) = \zeta(1 + \tilde{\Delta p} - \gamma)\sqrt{|\gamma - \tilde{\Delta p}|}\,\mathrm{sign}(\gamma - \tilde{\Delta p}) \tag{11}$$

for $\tilde{\Delta p} > \gamma - 1$ and 0 otherwise.

Hereafter, all parameters are dimensionless and the tilde is omitted. For small oscillations, i.e. close to the oscillation threshold, the previous equation can be expanded as a third-order polynomial:

$$u(\Delta p) = u_{00} + A\Delta p + B\Delta p^2 + C\Delta p^3, \tag{12}$$

where

$$\begin{aligned} u_{00} &= \zeta(1-\gamma)\sqrt{\gamma}, & A &= \zeta\frac{3\gamma - 1}{2\sqrt{\gamma}}, \\ B &= -\zeta\frac{3\gamma+1}{8\gamma^{3/2}}, & C &= -\zeta\frac{\gamma+1}{16\gamma^{5/2}}. \end{aligned} \tag{13}$$

Solving the non linear system composed of equations (3),(6) and (12) is simplified by combining the first two thus:

$$\Delta P = (Z_p + Z_m)U = \mathbb{Z}U \tag{14}$$

The impedances of the pipe and of the VT are thus in series, and $\mathbb{Z} = Z_p + Z_m$ is the equivalent impedance.

2. THEORETICAL INFLUENCE OF THE VT ON THE PLAYING FREQUENCY

Kergomard et al. [5] showed that in the case of the clarinet, the variable truncation method is a good approximation to calculate the spectral envelope easily and analytically. In the first iteration, even harmonics are neglected. In the second, they are calculated from the odd harmonics. Higher harmonics are presumed not to influence lower harmonics so equations can be truncated at the order of the harmonic to be calculated, ie the equations are truncated at the first order when calculating the first harmonic. This solution is substituted into the equations truncated at the third order to calculate the third harmonic and so on.

Here we use the same analytical technique, but with these modifications to the physical situation: the pressure difference across the reed replaces the pressure in the mouth, and the series impedance of clarinet and VT replaces that of the clarinet. We do not expect that the addition of the VT impedance in series will strongly affect the amplitude of odd or even harmonics of pressure in the mouthpiece, unless a vocal tract resonance lies close to a harmonic. However, the VT impedance can have a larger effect on the pressure in the mouth, especially the even harmonics of the played note, and therefore on the pressure difference across the reed. This in turn can affect the playing frequency. Hence we do not neglect even harmonics and the reason for that is now explained.

The variable truncation method is in particular used to obtain easily the playing frequency. Equation (12) truncated to the first order and equation (14) give indeed for the first harmonic:

$$\mathbb{Y}_1\Delta P_1 = A\Delta P_1 + 3C\Delta P_1|\Delta P_1|^2 \tag{15}$$

where $\mathbb{Y} = 1/\mathbb{Z}$ is the admittance of the series combination. This gives

$$|\Delta P_1|^2 = \frac{\mathbb{Y}_1 - A}{3C} \tag{16}$$

This implies that $\Im(\mathbb{Y}_1) = 0$ because all other terms of this equation are real, and thus gives the playing frequency, called ω_0. The threshold pressure is as given by $A = \mathbb{Y}_1(\omega_0) = A_{10}$. For the most simplified system, a harmonic pipe for the clarinet (no dispersion) with no VT influence, this implies of course that the playing frequency is equal to the first resonance of the pipe. But if we now add a vocal tract tuned to the first resonance of the pipe, we would expect to obtain the same playing frequency. However, this result is not repeated for numerical solutions of less simplified versions of the problem. These solutions were calculated by the program *Harmbal* developed by Farner et al. [8] which uses the harmonic balance method [9] and obtains the solution for N harmonics from the solution for $N-1$ harmonics.

To see how introduction of the second harmonic produces a frequency shift, we will truncate equation (12) to the second order. The choice of zero in the time domain is arbitrary, so for convenience we choose ΔP_1 to be real. The following equations are then to be solved simultaneously:

$$\begin{aligned} \mathbb{Y}_1\Delta P_1 &= A\Delta P_1 + 2B\Delta P_1\Delta P_2 + 3C\Delta P_1^3 + 6C\Delta P_1|\Delta P_2|^2 \\ \mathbb{Y}_2\Delta P_2 &= A\Delta P_2 + B\Delta P_1^2 + 6C\Delta P_1^2\Delta P_2 + 6C\Delta P_2|\Delta P_2|^2 \end{aligned}$$

For simplicity we have chosen to use the Small Oscillations Approximation, used by Worman[10] and Grand et al. [11]. Near the threshold, ΔP_2 is a second order of ΔP_1 (Worman [10]) so we can neglect quadratic and higher terms in ΔP_1 to obtain:

$$\mathbb{Y}_1 - A = 2B\Delta P_2 + 3C\Delta P_1^2 \tag{17}$$

$$(\mathbb{Y}_2 - A)\Delta P_2 = B\Delta P_1^2 \tag{18}$$

So

$$\Delta P_2 = \frac{B\Delta P_1^2}{\mathbb{Y}_2 - A} \tag{19}$$

$$\Delta P_1^2 = \frac{(\mathbb{Y}_1 - A)(\mathbb{Y}_2 - A)}{2B^2 + 3C(\mathbb{Y}_2 - A)} \tag{20}$$

Following the development of Grand et al [11] for conditions close to the threshold, we can write $A = A_{10} + \delta A$. Further supposing that the playing frequency remains close to ω_0 we write $\omega = \omega_0 + \delta\omega$ and $\mathbb{Y}_1(\omega) = A_{10} + \delta A_1(\omega)$. If $\delta\omega \ll \omega_0$, this gives $\delta A_1(\omega) = A_{10}\left(2iQ\frac{\delta\omega}{\omega_0}\right)$, where Q is the dimensionless quality factor of the impedance peak near which the oscillation occurs. $\mathbb{Y}_2(\omega_0)$ is a priori complex so we write $\mathbb{Y}_2(\omega)$ in the following form:

$$\mathbb{Y}_2(\omega) = \Re(A_{20}) + i\Im(A_{20}) + \delta A_2(\omega) \qquad (21)$$

with $\delta A_2(\omega) = O(\delta(\omega))$, which we can neglect in first approximation.

As the left hand side of equation (20) is real, the imaginary part of the right hand side is zero, which gives:

$$\frac{\delta\omega}{\omega_0} = \frac{\delta A}{2A_{10}Q}\frac{2B^2\mathbf{I}}{2B^2\mathbf{R} + 3C(\mathbf{R}^2 + \mathbf{I}^2)} \qquad (22)$$

with $\mathbf{R} = \Re(A_{20}) - A_{10}$ and $\mathbf{I} = \Im(A_{20})$.

3. MEASUREMENTS

Measurements of the VT impedance were done by using the acoustic impedance spectrometer developed by Smith et al [12]. What is new is the coupling of the VT to the spectrometer and the use of an impedance head mounted inside a clarinet mouthpiece (Figure 2), that can be then coupled to the tract of a human player or to an artificial vocal tract (VT) composed of discrete elements. The artificial tract is modelled on the MRI data of Story and Titze [13] (Figure 3).

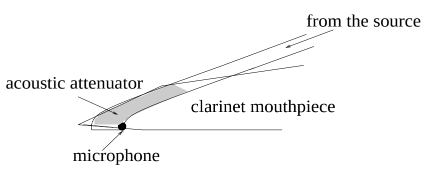

Figure 2: *Scheme of the measurement head*

As the impedance head has a cross section approximately equivalent to the part of the reed which is inside the player's mouth, we measure the impedance of the VT as "seen" by the reed.

The VT is coupled to a semi-infinite pipe which models the lungs, assumed non-reflective. The glottis can be chosen open or closed (20 or 6 mm diameter) as this varies among musicians, especially between beginners and professionals respectively [14].

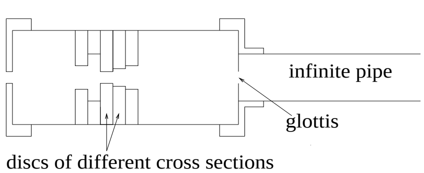

Figure 3: *Scheme of the artificial VT*

The discrete element tract gives weaker impedance peaks than does a continuous model, but they fall at nearly the same frequencies.

4. RESULTS

For experienced players who use their vocal tract, the glottis is nearly closed [14], which increases reflection at the glottis and so enhances VT resonances. We therefore consider only this case, and use the MRI data of Story and Titze [13] which were obtained in speech, where the glottis is almost closed. We present here, in Figure 4, the results for two vowels commonly cited by most of the players: /æ/ as in "had" for the low register and /i/ as in "heed" for the high register. This of course is only an approximation, because the embouchure constrains the player's jaw position.

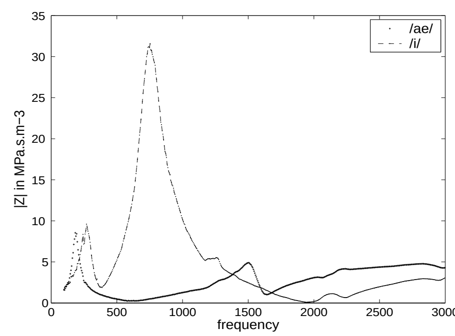

Figure 4: *Amplitude of the VT impedance for the vowels /æ/ and /i/.*

These results (amplitude and frequency of the resonances) are consistent with previous numerically simulated and experimental results (Hoekje [3], Sommerfeldt and Strong [15]).

The vowel /æ/ has relatively weak resonances so it should not affect much the sound of the clarinet. But the vowel /i/ presents a high peak, at 753 Hz, so we can expect an influence on the pitch around this frequency. The impedance is much higher because the tongue is in a high position which reduces the aperture inside the mouth.

We consider several cases. In one case, the pipe frequency was varied over the range 720-750 Hz: ie on the low frequency side of the strong peak in /i/. The playing frequency varies negligibly for /æ/. For /i/, the playing frequency is less than that for /æ/ by an amount that increases with frequency over this range: ie it increases as the VT impedance increases. Another case used a pipe frequency of 228 Hz, which lies in the middle between the impedance peaks for the first resonances in the two vowels chosen (Figure 4). At this frequency, the vocal tract is inertive for the vowel /i/ and compliant for /æ/. In both cases, the frequency shift is of order 0.2%, and the direction changes with mouth pressure, for reasons that we do not yet understand. We also consider the case where the first resonance frequency of the clarinet is 753 Hz (approximately the note G5), ie when the pipe frequency coincides with the peak in /i/. This case is of interest because of the proposal of formant tuning [4], and it is the one whose results are shown in Figure 5.

As the impedances are dimensionless in our calculation, we have to divide the impedance of the VT by the impedance of the clarinet. We can model it by the general formula of the impedance of one peak resonator:

$$Z_m(f) = iZ_d\frac{k}{k_1^2 - k^2 + i\frac{kk_1}{Q_1}} \qquad (23)$$

where Z_d is the dimensionless characteristic impedance, $k = 2\pi/f$ and $k_1 = 2\pi F_1/c$ and Q_1 are respectively the frequency and the quality factor of the resonance. Fitting the model to our measurement gives $F_1 = 753$ Hz, $Z_d = 35$ and $Q_1 = 6$. We can then calculate the values of the different parameters ($Q = 14$, $\mathbf{R} = 0.056$ and $\mathbf{I} = 0.567$) and obtain the shift in the playing frequency from equation (22) (see Figure 5).

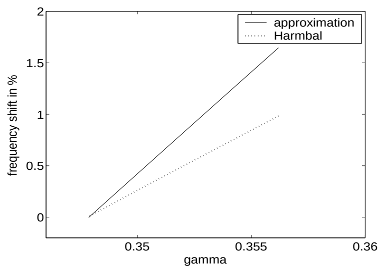

Figure 5: *Frequency shift in percent in function of the dimensionless mouth pressure, given by the approximation (22) and by the program* Harmbal

The results are compared to those of *Harmbal*. The difference is quite acceptable, so our approximations give a good idea of how the playing frequency evolves for small mouth pressures.

As well as analysing the effects of vocal tract impedance, the *Harmbal* technique can also be used to analyse the effect of changing reed parameters. This is interesting because, to change the pitch, clarinetists move both the tongue and the lower jaw. Lowering the jaw increases the effective length of the reed, decreasing its chacteristic frequency. The stiffness and the value of H, as functions of the force applied to close the reed (the 'jaw force'), were measured directly on a clarinet and reed. Using measured values for modest changes of this jaw force, and inputting the dependent dimensionless parameters to the model described above give changes in playing frequency of the same order as those given in Figure 5 for the effect of the vocal tract. The relative importance of these effects in playing remains, of course, a question to be settled experimentally.

5. CONCLUSION

The playing frequency shift obtained in the case of a VT tuned to the first resonance frequency of the clarinet is interesting. If players did tune their tract resonance to coincide with an instrument resonance the playing frequency would shift. However, we doubt that this is what players do. More importantly, this simple case shows a mechanism whereby the playing frequency can be changed by changes to vocal tract geometry. This is important because musicians claim to do it, and model experiments on other instruments show it [7].

On a theoretical point of view, further study will be done in order to improve formula (22) by using an extension of the VTM to fit better to the numerical results of *Harmbal*.

In an other hand, the model reported here neglects several important physical features. Measurements of the impedance of the VT of musicians miming their playing embouchure will be carried out to try to analyse in more detail the role of the tongue and the glottis. The discrete element VT will now be used in a blowing machine to investigate this shift experimentally.

6. REFERENCES

[1] J. Mooney, "The effect of the oral cavity on the tone quality of the clarinet," Ph.D. dissertation, Brigham Young University, 1968.

[2] A. Benade, "Air column, reed and player's windway interaction in musical instruments," in *Vocal Fold Physiology*, I. Titze and R. Scherer, Eds. The Denver Center for the Performing Arts, 1983.

[3] P. Hoekje, "Intercomponent energy exchange and upstream/downstream symmetry in nonlinear self-sustained oscillations of reed instruments," Ph.D. dissertation, CaseWestern Reserve University, Cleveland, Ohio, 1986.

[4] R. Johnston, P. Clinch, and G. Troup, "The role of the vocal tract resonance in woodwind instrument playing," unpublished.

[5] J. Kergomard, S. Ollivier, and J. Gilbert, "Calculation of the spectrum of self-sustained oscillations using a variable truncation method : application to cylindrical reed instruments," *Acust.- Acta Acust.*, vol. 86, pp. 685–703, 2000.

[6] A. Hirschberg, J. Kergomard, and G. Weinreich, Eds., *Mechanics of Musical Instruments, Lecture notes CISM*. Springer-Verlag, 1995.

[7] J. Wolfe, A. Tarnopolsky, N. Fletcher, L. Hollenberg, and J. Smith, "Some effects of the player's vocal tract and tongue on wind instrument sound," in *Proc. SMAC*, Stockholm, Sweden, 2003.

[8] S. Farner, C. Vergez, and J. Kergomard, "Harmbal: A program for calculating steady-state solutions to nonlinear dynamical autonomous systems by the harmonic balance method. applications to a physical model of the clarinet," *To be submitted to Acustica-Acta Acustica*.

[9] J. Gilbert, J. Kergomard, and E. Ngoya, "Calculation of the steady-state oscillations of a clarinet using the harmonic balance technique," *J. Acoust. Soc. Amer.*, vol. 86, no. 1, pp. 35–41, 1989.

[10] W. Worman, "Self-sustained nonlinear oscillations of medium amplitude in clarinet-like systems," Ph.D. dissertation, Case Western Reserve University, 1971.

[11] N. Grand, J. Gilbert, and F. Laloe, "Oscillation threshold of woodwind instruments," *Acustica*, vol. 82, pp. 137–151, 1996.

[12] J. Smith, C. Fritz, and J. Wolfe, "A new technique for the rapid measurement of the acoustic impedance of wind instruments," in *Proc. of the Seventh International Congress on Sound and Vibration*, G. Guidati, H. Hunt, H. Heller, and A. Heiss, Eds., vol. III, Garmisch-Partenkirchen, Germany, 4-7 July 2000, pp. 1833 – 1840.

[13] B. Story, I. Titze, and E. Hoffman, "Vocal tract area functions from magnetic resonance imaging," *J. Acoust. Soc. Amer.*, vol. 100, no. 1, 1996.

[14] M. S. Mukai, "Laryngeal movement while playing wind instruments," in *Proc. International Symposium on Musical Acoustics*, Tokyo, Japan, 1992, pp. 239–242.

[15] S. Sommerfeldt and W. Strong, "Simulation of a player-clarinet system," *J. Acoust. Soc. Amer.*, vol. 83, no. 5, pp. 1908–1918, 1988.