Quantum Field Theory

Program for exam 2009

I. Electromagnetic field in the vacuum. Casimir effect.

i. Contribution of each photon mode to the energy of the vacuum.

ii. Total energy of the photon vacuum:
- Why is it infinite ?
- Which effect this infinity bears onto quantum gravity ?
- How to make the energy finite, cut off with the help of the Plank mass

iii. The idea of the renormalization:
- Calculate those quantities, which are *directly* measured in the experiment
- Verify that all *infinities* are cancelled out in these calculations

iv. The energy of the vacuum of the electromagnetic field between two metal plates.

v. Casimir effect
- What the Casimir effect is
- Calculations, which lead to the Casimir effect (definitely the starting point)

II. Self-interaction of charged particles; Lamb shift

i. What is the Lamb shift.

ii. Physical ideas, which qualitatively explain the Lamb shit:
- Self-interaction of the electron
- Vacuum polarization

iii. Calculation of the Lamb shift in the logarithmic approximation; Bethe approach
- The second order of the perturbation theory
- The dipole matrix element responsible for absorption and radiation of dipole quanta, the normalization of the photon wave function
- Expression of the Lamb-shift via the matrix element of ΔU

iv. Final result for the Lamb shift of the ns level in the Hydrogen type ions, dependence on main quantum number n, on the charge of the ion Z, and on the fine-structure constant alpha. Comparison with the atomic energies and radiative widths.

v. Imaginary part of the Lamb shift, its relation with the probability of the radiative decay

vi. Derivation of the Lamb shift from the probability of the radiative decay.

III. Dirac equation
- Dirac equation for free electrons
- Dirac sea
- Contribution of electrons into the vacuum energy, energy in the Dirac see
    - a. Negative sign

b. Infinite value
c. How to make the energy finite, cut off with the help of the Plank mass

- Dirac equation in the external electromagnetic field
- Perturbation theory for the Dirac equation

IV. Vacuum polarization, Uehling potential, running coupling constant, Landau pole, asymptotic freedom.

i. Physical idea, which explain the vacuum polarization by external electromagnetic field, perturbation of electron wave functions in the Dirac sea

ii. Vacuum polarization

- How to organize the calculations:
  a. Calculate the correction to the wave-function of electrons in the Dirac sea.
  b. From the correction to the wave function find correction to the charge denity
  c. And finally from the charge density find polarization potential
- Renormalization

iii. Physical consequencies of the vacuum polarization:

- Increase of the charge at small distances (large momenta) in the lowest order of the perturbation theory over α
- The role of higher orders of the α expansion; running coupling constant; Landau pole.
- The role of the electron charge and spin in phenomena leading to the Landau pole
- The role of the spin and statistics
- Vacuum polarization and Landau pole for scalar particles
- Running coupling constant
- Vacuum polarization and asymptotic freedom for vector particles

iv. Uehling potential and related phenomena:

- Behaviour of the Uehling potential at large and small distances.
- Its contribution to the Lamb shift
- Relation to the Landau pole
- Contribution to the wave function of the electron at small distances
- Relation between the Uehling potential and the polarization operator (from the assignment)
- The vaciuum polarization for an arbitrary electromagnetic field (from the assignment)

v. The role of the spin and statistics in the polarization

- Polarization of the vacuum of scalar particles.
- Polarization of the vacuum of vector particles, asymptotic freedom.

V. Strong homogeneous static magnetic field, Heizenerg-Euler problem

i. Landau levels of electrons in in homogeneous static magnetic field, the role of spin and statistics
   - Scalar field
   - Spinor field
   - Vector particles

ii. QED (electron-positron) vacuum in homogeneous static magnetic field.
   - Physical idea of the vacuum as the Dirac see, i.e. electrons in the lower continuum influenced by the magnetic field.
   - Calcualtion of the energy density for the vacuum (certainly the main ideas of this calculation), succeptibility of the vacuum, critical magnetic field for the QED vacuum.
   - Ultra-strong magnetic field, log approximation
   - Ultra-strong magnetic field for the vacuum of scalars, spinors and vector particles
     a. Diamagnetism and paramagnetis
     b. Role od spin and statistics
     c. Comparison with the phenomena of the Landau pole and asymptotic freedom

VI. Schwinger phenomenon, e^+e^- production in homogeneous, static electric field

i. Density of energy of the electron-positron vacuum in the electric field
   - derivation from the density of energy of the vacuum in the magnetic field
   - real and imaginary parts of the energy, relation of the imaginary part with the pair production
   - probability of the pair production