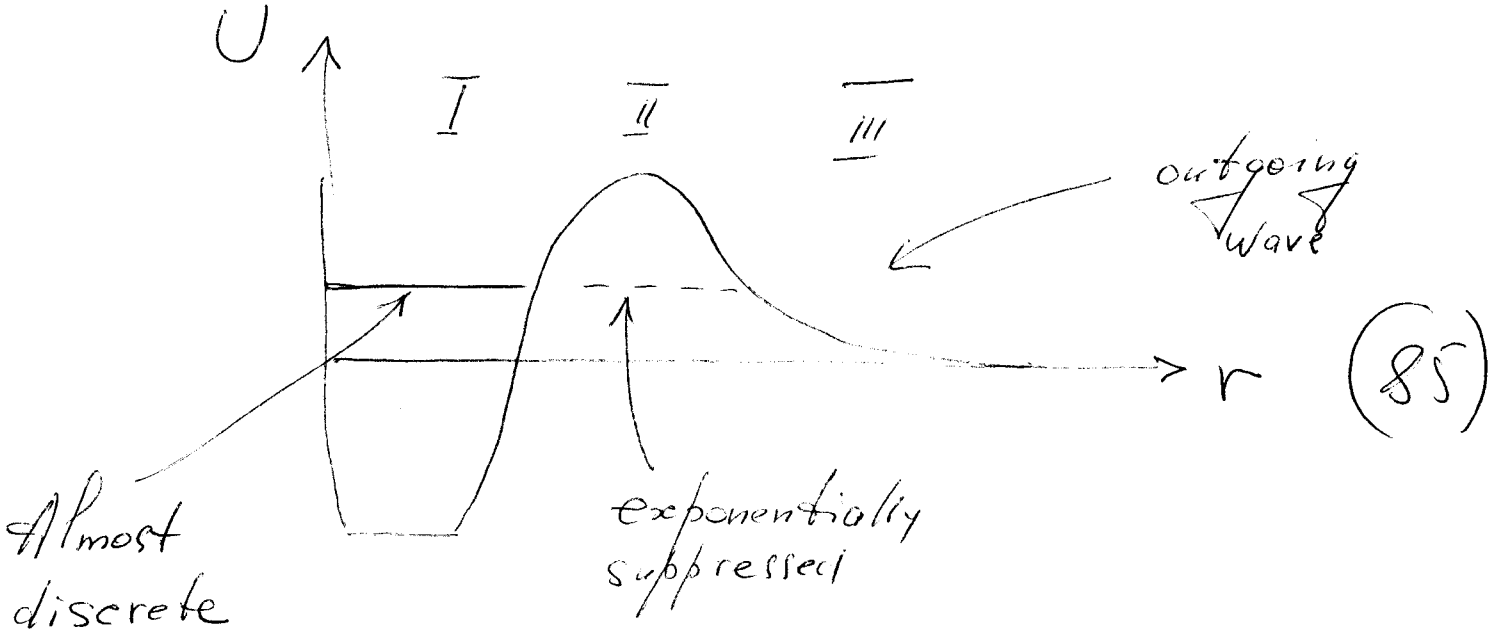
Resonance scattering



(85)

$$\psi(r,t) = e^{-i\varepsilon_0 t - \frac{1}{2}\frac{t}{\tau}} \psi(r) \quad (86)$$

$$\Gamma = \frac{t_u}{\tau} \quad (87)$$

$$\exp -i\left(\varepsilon_0 - i\frac{\Gamma}{2}\right)t$$

$$E \underset{(86,87)}{\Rightarrow} \varepsilon_0 - i\frac{\Gamma}{2} \qquad (88)$$

$$\psi(r) \underset{r\to\infty}{\simeq} \frac{2}{r} e^{ikr} \qquad (89)$$

↑ Region III in (85)

$\boxed{\ell = 0}$ $\longleftarrow$ $-2q-$ for simplicity

$$R_{k0} \simeq R_{k\ell}\Big|_{\ell=0} \underset{(69)}{=} \frac{2}{r} \sin\left(kr + \delta_0\right)$$

$$= \frac{1}{ir}\left[e^{i\delta_0} e^{ikr} - e^{-i\delta_\ell} e^{-ikr} \right] \quad (90)$$

$$S_\ell = e^{2i\delta_\ell} \quad (91)$$

$$R_{k0} \simeq \frac{1}{r}\left[a\, e^{ikr} + b\, e^{-ikr} \right] \quad (92)$$

$a = a(E)$
$b = b(E)$

$$\begin{cases} a \underset{(90)}{=} b^* & (93) \\ S^\ell \underset{(91)}{=} -\frac{a}{b} & (94) \end{cases}$$

$$\begin{cases} b(E) \underset{(89)}{=} B\left(E-\varepsilon_0+i\frac{\Gamma}{2}\right) \\ a(E) = B^*\left(E-\varepsilon_0-i\frac{\Gamma}{2}\right) \end{cases} \tag{95}$$

$$S \underset{(94,95)}{=} \overbrace{e^{2i\delta(0)}}^{E-\varepsilon_0+i\frac{\Gamma}{2}-i\Gamma} - \frac{B^*}{B}\,\frac{E-\varepsilon_0-i\,\Gamma/2}{E-\varepsilon_0+i\,\Gamma/2} \tag{96}$$

$$= -\frac{B^*}{B}\left(1-\frac{i\Gamma}{\varepsilon-\varepsilon_0+i\frac{\Gamma}{2}}\right)$$

$$\approx e^{2i\delta_0}\left(1-\frac{i\,\Gamma}{E-\varepsilon_0+i\,\Gamma/2}\right) \tag{97}$$

$$\approx -e^{2i\delta_0}\,\frac{i\,\Gamma}{E-\varepsilon_0+i\,\Gamma/2} \tag{98}$$

$$f_0 = \frac{S_\ell - 1}{2ik} \simeq$$

$$\simeq \frac{1}{k} e^{2i\delta_0} \frac{\Gamma/2}{E - \varepsilon_0 + i\,\Gamma/2} \qquad (98) \qquad (99)$$

$$\sigma \simeq 4\pi |f_0|^2 =$$

$$= \frac{4\pi}{k^2} \frac{\Gamma^2/4}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2/4} \qquad (100)$$

(99)

For ℓ arbitrary

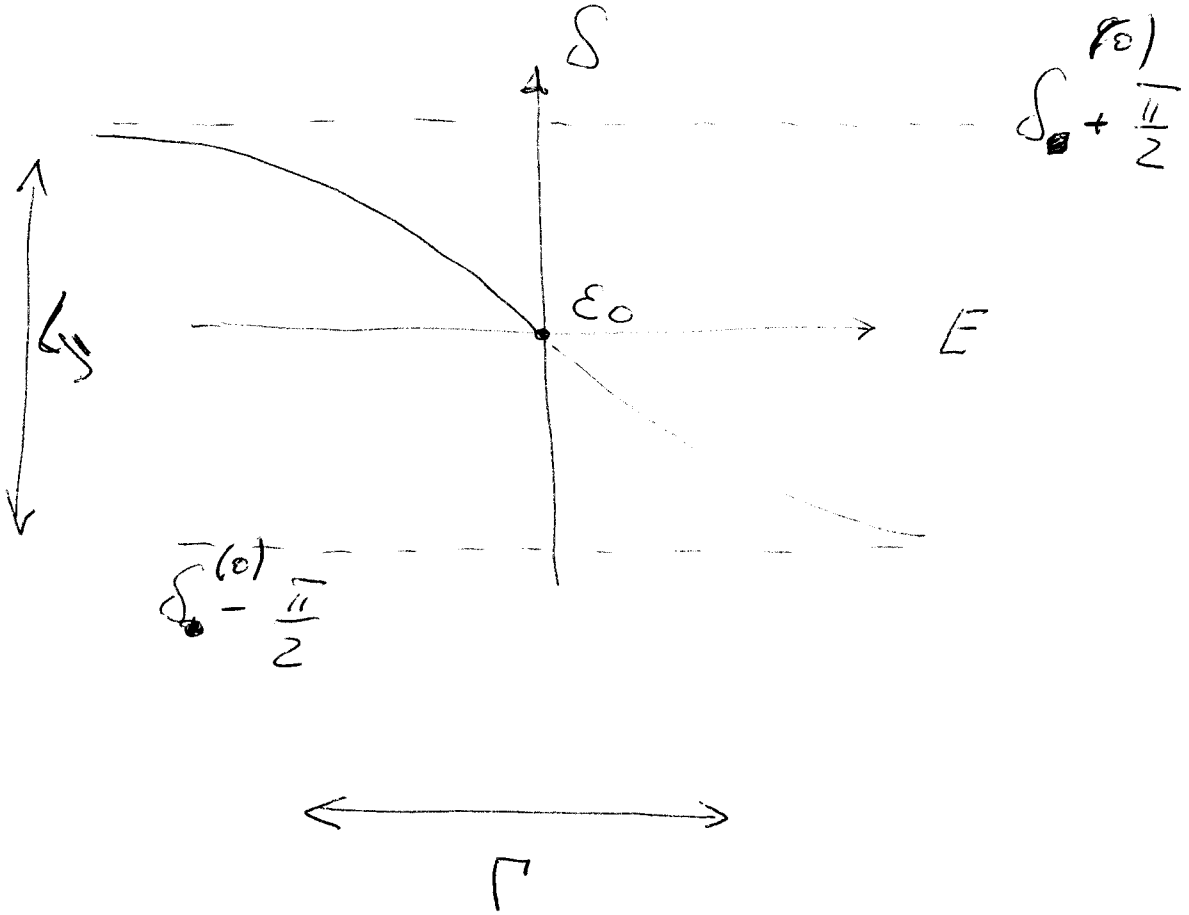
$$\sigma = \frac{4\pi}{k^2} \cdot (2\ell+1) \frac{\Gamma^2/4}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2/4}$$

$$e^{2i\delta} \underset{(96)}{=} e^{2i\delta_\blacksquare^{(0)}} \frac{E-\varepsilon_0 - i\Gamma/2}{E-\varepsilon_0 + i\Gamma/2} \quad (101)$$

$$\Downarrow$$

$$\delta = \delta_\blacksquare^{(0)} - \underbrace{\operatorname{arctan}}_{\tan^{-1}} \left(\frac{\Gamma/2}{E-\varepsilon_0} \right) \quad (102)$$

$$\Downarrow$$



(103)