Optical theorem

$$\mathrm{Im}\, f_\ell \underset{(77)}{=} \frac{1}{k} \sin^2 \delta_\ell =$$

$$= k\, |f_\ell|^2 . \qquad (82)$$

$$\mathrm{Im}\, f(0) \underset{\substack{(82)\\(78)}}{=} \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1) \sin^2 \delta_\ell \qquad (83)$$

$$\boxed{\frac{k}{4\pi}\, \sigma = \mathrm{Im}\, f(0)} \qquad (84)$$

Levinson's Theorem

Usually σ decreases at high energy faster than $\frac{1}{E}$ (81.1)

(80) $\Downarrow$

$$\sin \delta_\ell \underset{E \to \infty}{\longrightarrow} 0 \qquad (81.2)$$

It is convenient to presume that

$$\delta_\ell (E = \infty) = 0 \qquad (81.3)$$

$$k \to 0 \qquad \sigma \to \text{Const} \qquad (81.4)$$

usually

$(80,\ 81.4) \Rightarrow$

$$\sin \delta_\ell \underset{E=0}{=} n\pi \qquad (81.5)$$

Theorem

n = < number of discrete levels in the ℓ-th partial wave >

(81.6)

Low energy resonance

$$f \underset{(54)}{=} - \frac{1}{\varkappa + ik} \qquad (81.7)$$

$$ka \ll 1 \Rightarrow S\text{- wave} \qquad (81.8)$$

$$f = f_0 = \frac{e^{i\delta} \sin\delta}{k} = - \frac{1}{\varkappa + ik}$$

(81.9)

$$e^{i\delta} = \frac{1}{\cos\delta - i\sin\delta} \qquad (81.10)$$

$$f_0 \underset{\substack{(81.9\\81.10)}}{=} \frac{1}{k} \frac{\sin\delta}{\cos\delta - i\sin\delta} = \frac{1}{k\cot\delta - ik}$$

$$\underset{(81.9)}{=} - \frac{1}{\varkappa + ik} \qquad (81.11)$$

$$\boxed{\operatorname{ctg}\delta = -\frac{\varkappa}{k}} \qquad (81.12)$$

(81.9)