Resonant scattering at low energy

Low-energy (in scattering) $\qquad ka \ll 1 \qquad (45)$

Assume there is a binding state with low binding energy in S state i.e. $l = 0$ (46)

$$\psi_b \underset{r \geq a}{=} C \frac{e^{-\varkappa r}}{r} \tag{47}$$

(46)

$$E_b = -\frac{\varkappa^2}{2m} \tag{48}$$

$$\varkappa a \ll 1 \tag{49}$$

(46)

$$(47) \Rightarrow \left. \frac{(r\psi)'}{r\psi} \right|_{r=a} = -\varkappa \tag{50}$$

Consider scattering problem for low energy.
Condition (50) ~~do~~ is determined by the potential. Energy does not manifest itself there (51)
Therefore (50) should be valid for scattering as well

$$\psi_s \underset{(45)}{=} \underbrace{e^{i\vec{k}\vec{r}}}_{\approx 1} + \underbrace{f}_{\theta\text{-independent}} \cdot \frac{e^{i\overset{\text{keep } k}{\overbrace{k}}r}}{r} \quad (52)$$

$$\frac{(r\psi_s)'}{r\psi_s} = \left.\frac{(r + f e^{ikr})'}{r + f e^{ikr}}\right|_{r=a} \simeq$$

$$\simeq \frac{1 + ikf}{f} \qquad (53)$$

$(50, 53) \Rightarrow$

$$\frac{1}{f} + ik = -\varkappa$$

$$\boxed{f = -\frac{1}{\varkappa + ik}} \qquad (54)$$

$$\sigma \underset{(54)}{=} \frac{4\pi}{\varkappa^2 + k^2} = \frac{2\pi\hbar^2}{m} \frac{1}{E + |E_B|} \qquad (55)$$

$$a \underset{\substack{(54)\\(41)}}{=} \frac{1}{\varkappa} \qquad (56)$$

$$\begin{cases} a \gg a \\ \sigma \gg a^2 \end{cases} \qquad (57)$$

Virtual levels

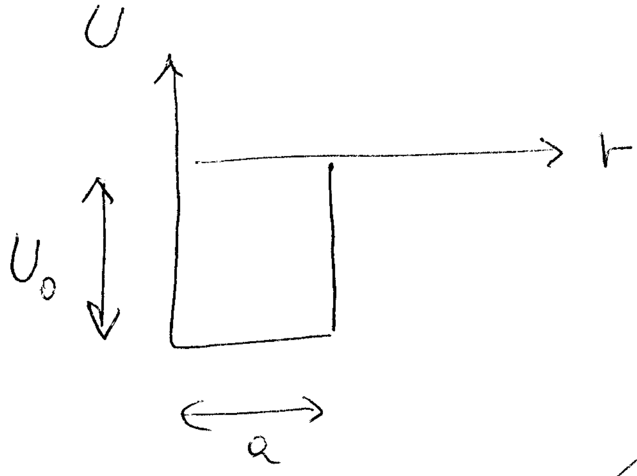
Eq. 50 results in the resonance cross section because $\varkappa$ is small.

The resonance nature of Eq. (55) remains valid even if

$$\varkappa < 0 \qquad (58)$$

If (58) is valid then there is no <u>bound state</u>, but the resonance in scattering is present.
The situation is called the virtual level

Example



(59)

Compare
Partial waves
analysed
below

$$r\psi = \sin pr \qquad r < a \tag{60}$$

$$p = \sqrt{2mU_0}$$

cotan

$$\left.\frac{(r\psi)'}{r\psi}\right|_{r=a} = p\,\mathrm{ctg}\,pa = -\varkappa \tag{61}$$

Rezonant scattering $\quad |\varkappa| a \ll 1$ (62)

(61) shows that tuning U_0, a one can make $|\varkappa|$ small and $\varkappa$ be ~~of~~ either positive, or negative