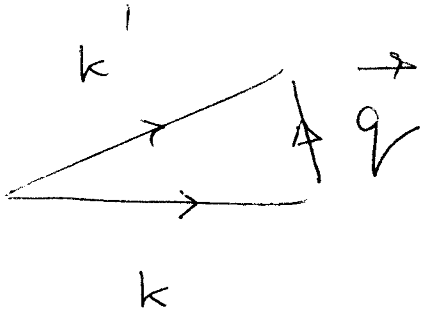
Born approximation

$$\psi_{\vec{k}}(\vec{r}) \simeq e^{i\vec{k}\cdot\vec{r}} \qquad (24)$$

$$f(\theta) \simeq f_B(\theta) = -\frac{m}{2\pi\hbar^2}\int e^{-i\vec{q}\vec{r}}\, U(r)\, d^3r \qquad (25)$$

$$\Rightarrow$$



$$\vec{q} = \vec{k}' - \vec{k}$$

$$\Rightarrow -\frac{m}{2\pi\hbar^2}\, U(\vec{q})$$

$$q = 2k\sin\frac{\theta}{2}$$

Applicability

1 $\quad ka \ll 1$

a – radius of the potential

$$\boxed{\frac{m|U|a^2}{\hbar^2} \ll 1} \qquad (26)$$

2. $\quad ka \gg 1$

$$\frac{m|U|a^2}{\hbar^2} \ll ka \qquad (27)$$

Born approximation + Low energy

$$E \to 0 \tag{28}$$

$$f_B \simeq \underbrace{-\frac{m}{2\pi\hbar^2}\int U(r)\,d^3r}_{\text{const}} \tag{29}$$

$$f \xrightarrow[E \to 0]{} -a \tag{30}$$

$$a \underset{\substack{\text{Born}\\ \text{approx}}}{\simeq} +\frac{m}{2\pi\hbar^2}\int U(r)\,d^3r \tag{31}$$

<u>Example</u> Coulomb potential

$$U = -\frac{Ze^2}{r} \tag{32}$$

$(27, 32) \Rightarrow$

$$\frac{m\,Ze^2 \cdot r^2}{r\hbar^2} \ll kr$$

$$\delta > \frac{Ze^2}{\hbar} = \frac{Ze^2}{\hbar c} \cdot c \tag{33}$$

$$= Z\alpha c$$

$$\frac{\delta}{c} > Z\alpha \tag{34}$$

$$f_B \underset{(25,32)}{=} \frac{m Z e^2}{2\pi \hbar^2} \underbrace{\int \frac{e^{-iqr}}{r} d^3r}_{\frac{4\pi}{q^2}} =$$

$$= \frac{2m Z e^2}{\hbar^2 q^2} = \frac{2m Z e^2}{\left(2\hbar k \sin\frac{\theta}{2}\right)^2} =$$

$$= \frac{Ze^2}{2\frac{p^2}{m}\sin^2\frac{\theta}{2}} = \frac{Ze^2}{4E}\frac{1}{\sin^2\theta/2} \tag{35}$$

$$E = \frac{p^2}{2m}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_B \underset{(35,13)}{=} \frac{1}{16}\left(\frac{Ze^2}{E}\right)^2 \frac{1}{\sin^4\theta/2} \tag{36}$$

Compare $\left\{\begin{array}{l}\text{exact result} \\ \text{eikonal approx} \\ \text{classical approx}\end{array}\right.$