Assignment 2009

Standard Model

Question 1. Phenomenology**.**

i. Fill in the following table:

Particle	Spin (0, 1/2 (indicate left or right), 1)	Mass	Colour (singlet, triplet, octet)	Weak isospin (0,1/2,1)	Charge
ν_{eL}					
e_L					
e_R					
u_L					
d_L					
u_R					
d_R					
$W^{\pm}$					
Z^0					
γ					
G					

ii. Draw all Feynman diagrams, which describe the parity-violating scattering of the electron by the neutron.

Question 2. Gauge transformations.

1. Consider a gauge transformation

$$\psi \to \psi' = U\psi$$
$$A_\mu \to A'_\mu = UA_\mu U^+ - \frac{i}{g} U \partial_\mu U^+ \qquad (2.1)$$

where ψ is a wave function, which is a n-th dimension complex column in the isotopic space, A_μ is a Hermitian traceless $n \times n$ matrix, and U is a $n \times n$ unitary unimodular matrix.

- Show that A'_μ in Eq.(2.1) is a Hermitian traceless matrix as well.
- Verify that the gauge derivative $\nabla_\mu = \partial_\mu + i\, g\, A_\mu$ of ψ is transformed as

$$\nabla_\mu \psi \to U \nabla_\mu ' \psi ' \qquad (2.2)$$

Question 3. The Higgs mechanism.

i. Consider the scalar field, which is transformed as a doublet under *SU(2)* gauge transformations

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{3.1}$$

Assume that the lower component of the doublet develops the nonzero vacuum expectation value $\langle \phi_2 \rangle = \mathrm{v}$, which spontaneously breaks the *SU(2)* group. Find the mass m of the triplet of the *SU(2)* gauge bosons.

Hint1. The mass term must give the contribution the following contribution to the density of energy of the gauge field

$$\left(\frac{dE}{dV}\right)_{mass,\,gauge} = \frac{1}{2} m^2 W^{am} W^{a\,m} \tag{3.2}$$

where $W^{a\mu}$, $a = 1,2,3$, $\mu = 0,\ldots 3$ is the triplet of the gauge bosons. Eq.(3.2) takes into account that the massive gauge field has the spin 1, therefore for zero momentum it has only three components $W^{am}, m = 1,2,3$, where m describes the projection of spin of the vector field

Hint 2. The necessary mass term originates from a part of the ensity of energy of the scalar field, which describes the scalar field

$$\left(\frac{dE}{dV}\right)_{scalar} = \frac{1}{2} \nabla^m \phi^+ \nabla^m \phi \tag{3.3}$$

where ∇^m, $m = 1,2,3$ is the spatial part of the covariant derivative $\nabla^\mu, \mu = 0,\ldots 3$, which is defined as follows

$$\begin{aligned} \nabla^\mu \phi &= \partial^\mu \phi + ig\, W^{a\,\mu} \frac{\tau^a}{2} \phi \\ \nabla^\mu \phi^+ &= \left(\nabla^\mu \phi\right)^+ = \partial^\mu \phi^+ - ig\, W^{a\,\mu}\, \phi^+ \frac{\tau^a}{2} \end{aligned} \tag{3.4}$$

It is handy also to remember the identity

$$\tau^a \tau^b = \delta_{ab} + i\varepsilon_{\mathrm{abc}} \tau^c \tag{3.5}$$