Higgs mechanism

Consider a superconductor

Meisner effect

Cooper's pairs $\Rightarrow$ n_c density of condensate (1)

$$\begin{cases} \Psi = |\Psi| e^{i\alpha} \\ |\Psi| = \sqrt{n_c} \end{cases} \quad (2)$$

$$\vec{j} = \frac{e_c}{2 i m_c}\left(\Psi^* \vec{\nabla}\Psi - cc\right) =$$

$$= e_c \frac{\vec{\nabla}\alpha}{m_c} \quad (3)$$

$e_c = 2e$

$m_c = 2m$

(3) $\Rightarrow$ $\begin{cases} \text{different phase in (2)} \rightarrow \\ \text{different current} \rightarrow \\ \text{different state of the superconductor} \end{cases}$ (4)

(4) ⇒ gauge invariance is lost

QED equations are gauge invariant

Wave function is <u>not</u> gauge invariant (5)

<u>Spontaneous symmetry breaking</u>

Current in the external field

$$\vec{\nabla} \Rightarrow \vec{\nabla} - ie\vec{A} \qquad (6)$$

comp. (4) "Gauge inv"

Signs are <u>correct</u>

$$\vec{j} = \frac{e_c}{2im_c}\left[\varphi^*\left(\vec{\nabla} - ie\vec{A}\right)\varphi - cc\right]$$

$$= e_c \frac{\vec{\nabla}\alpha}{m_c} - \frac{e_c^2 n_c}{m_c}\vec{A} \qquad (7)$$

$$\left(\frac{\partial^2}{\partial t^2} - \Delta\right)\vec{A} = 4\pi\vec{j} =$$

$$= \frac{\vec{\nabla}\alpha}{m_c} - \frac{4\pi n_c e_c^2}{m}\vec{A}$$

$$\left(\underbrace{\frac{\partial^2}{\partial t^2} - \Delta}\right)\vec{B} = -\frac{4\pi n e_c^2}{m}\vec{B}$$

$$\partial^2 = -\hat{p}^2 = -\hat{p}_\mu \hat{p}^\mu$$

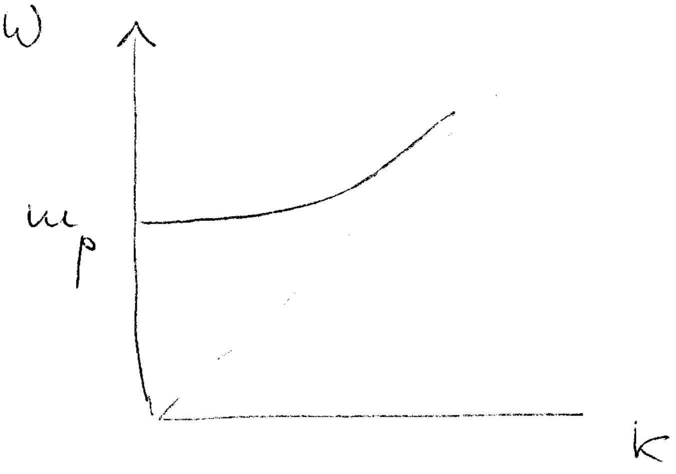
$$\boxed{-\partial^2 \vec{B} = m_{ph}^2 \vec{B}}$$

$$\boxed{m_{ph}^2 = \frac{4\pi n_c e_c^2}{m_c}}$$

Units

$$\frac{e^2}{\hbar c} = \frac{1}{137}$$

$$\omega^2 = k^2 + m_p^2$$



Meissner effect

$$\Delta \vec{B} = m_{ph}^2 \vec{B}$$

$$\vec{B} = \vec{B}_0 \exp(-m_p x)$$