Standard Model

Exam 2005 with answers

- *The answers to the exam questions are presented in italic and are marked by the dot-signs*
- *Probably the questions in Exam-2005 were not challenging enough; one might think of improving slightly in this department in 2006.*

Question 3 (20 marks)

i. Which gauge group governs the Standard Model ?

- $U(1)\times SU(2)\times SU(3)$, *of course.*

ii. Name all the forces, which this symmetry embraces

- *Electromagnetic (QED), weak (Weinberg-Salam theory), and strong (QCD) interactions*

iii. What is the scale of distances for each of this force ?

- $r=\infty$ *for QED*
- $1/m_W\ (=\hbar/m_W c)$ *for weak interactions*
- $1/\Lambda\,(=\hbar/\Lambda c)$ *for QCD, where* $\Lambda = 100-300\,Mev/c$ *is the radius of confinement*

iv. Which gauge bosons mediate each of these forces ?

- *Photon for QED,* $W^{\pm}, Z$ *for weak interactions, and gluons for QCD*

v. How many generations of leptons and quarks are known? Name all leptons and quarks in each generation.

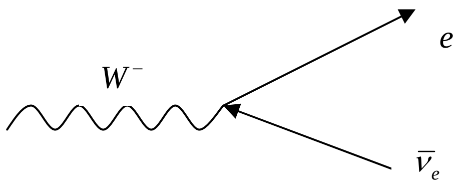
- *Three*
- $\begin{pmatrix}\nu\\ e\end{pmatrix}\quad\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\quad\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$
  $\begin{pmatrix}u\\ d\end{pmatrix}\quad\begin{pmatrix}c\\ s\end{pmatrix}\quad\begin{pmatrix}t\\ b\end{pmatrix}$

vi. Present all Feynman diagrams describing decays of the W^- boson into leptons
Hint: remember that there are several generations.

- *There are three channels for decay*

$$W^- \to e + \bar{\nu}_e$$
$$\to \mu + \bar{\nu}_\mu$$
$$\to \tau + \bar{\nu}_\tau$$

For example



and two more similar diagrams for other two channels

Question 4 (30 marks)

1. Consider a gauge transformation

$$\psi \to \psi' = U\psi$$
$$A_\mu \to A_\mu' = UA_\mu U^+ - \frac{i}{g} U\partial_\mu U^+ \qquad (4.1)$$

where ψ is a wave function, which is a n-dimension complex column, A_μ is a Hermitian traceless $n \times n$ matrix, and U is a $n \times n$ unitary unimodular matrix.

i. Verify that the gauge derivative $\nabla_\mu = \partial_\mu + i\,g\,A_\mu$ is transformed as

$$\nabla_\mu \psi \to U \nabla_\mu ' \psi' \qquad (4.2)$$

ii. Find from here the transformation law for the gauge field, which is defined as

$$F_{\mu\nu} = \frac{-i}{g}[\nabla_\mu, \nabla_\nu] \qquad (4.3)$$

(Eq.(4.3) results in $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g[A_\mu, A_\nu]$, but here it is more convenient to use (4.3) directly.)

Hint: argue that Eq.(4.2) can be rewritten as

$$\nabla_\mu \to \nabla_\mu' = U \nabla_\mu U^+ \qquad (4.4)$$

and then use definition (4.3).

iii. Using the transformation law for $F_{\mu\nu}$ verify that the Lagrangian of the gauge field

$$\mathcal{L} = -\frac{1}{4k} \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right) \qquad (4.5)$$

where k is a constant, is gauge invariant,

$$\mathcal{L} \to \mathcal{L} \qquad (4.6)$$

- *Questions related to Eqs.(4.2) - (4.6) are discussed in some detail in lecture notes 2006. (Probably notation is slightly different.)*

2. The Lagrangian, which describes the electro-weak interaction of fermions with the gauge field in the Standard model reads

$$\mathcal{L}_{\text{Weak-int}} = \frac{g_1}{2}\left\{ Y_L \left(\bar{L}\gamma^\mu L\right) + Y_R \left(\bar{e}_R \gamma^\mu e_R\right) \right\} B_\mu + \frac{g_2}{2}\left(\bar{L}\gamma_\mu \tau^i L\right) W_\mu^i \qquad (4.7)$$

where L is the isotopic doublet, and e_R is the isotopic singlet

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - \text{doublet}, \ e_R - \text{singlet}$$

The Pauli isotopic matrices are

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

i. Extract from the Lagrangian (4.7) the term responsible for the interaction of neutrino with B_μ and W_μ^3 i.e. the term which contains the neutrino current $\left(\bar{\nu}_L \gamma_\mu \nu_L\right)$.

ii. Show that the photon and *Z*-boson should be defined as linear combinations of the B_μ and W_μ^0. Write these combinations as

$$Z_\mu = \cos\theta_W W_\mu^0 - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W W_\mu^0 + \cos\theta_W B_\mu$$

Hint: remember that the neutrino interacts only with Z_μ. Therefore, the gauge potential, which multiplies the neutrino current in the Lagrangian necessarily represents the *Z*-bozon.

- *This question, in a slightly rephrased form, enters the Assignment 2006, lecture notes 2006 could also be recommended.*

Find expressions for the Weinberg angle θ_W and the electron charge *e* in terms of the coupling constants g_1 and g_2.
Hint. To save time keep in mind that $Y_L = -1, \ Y_R = -2$ (do not derive these identities).

- *Again, see the Assignment 2006 and lecture notes 2006.*