Equations of motion of a charge

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

Derivation

Consider some moment of time t. Taka a coordinate system in which the velocity at this moment is $V=0$. Then nonrelativistic equation reads

$$m\vec{a} = q\vec{E}$$

Rewright it

$$ds^2 = -c^2 dt^2 + d\vec{r}^{\,2} =$$

$$= -c^2 dt^2 \left(1 - \frac{v^2}{c^2}\right)$$

$$ds^2 = dx^\mu dx_\mu \quad \text{inv}$$

$$ds = \sqrt{-ds^2} = \sqrt{1 - \frac{v^2}{c^2}}\, c\,dt = $$

$$\underset{v=0}{=} c\,dt$$

$$m\vec{a} \underset{v=0}{=} c\,\frac{d\vec{p}}{ds}$$

$$\vec{E}$$

$$ds^2 = dx^\mu dx_\mu = -c^2dt^2 + d\vec{r}^{\,2} =$$

$$= -c^2dt^2\left(1 - \frac{1}{c^2}\left(\frac{d\vec{r}}{dt}\right)^2\right)$$

$$\Rightarrow -c^2dt^2\left(1 - \frac{v^2}{c^2}\right)$$

$$dS = \sqrt{-ds^2} = \sqrt{1-\frac{v^2}{c^2}}\, c\,dt \Rightarrow$$

$$\underset{v=0}{\Rightarrow} c\,dt$$

$$u^\mu = \frac{dx^\mu}{dS} = \left(\frac{1}{\sqrt{1-v^2/c^2}}, \frac{\vec{v}}{\sqrt{1-v^2/c^2}}\right)$$

$$\underset{v=0}{\Rightarrow} (1, \vec{0})$$

$$\vec{a}^{\,i} = \frac{d\vec{v}^{\,i}}{dt} \underset{v=0}{=} c\frac{d}{dS}\left(\frac{\vec{v}^{\,i}}{\sqrt{1-v^2/c^2}}\right) = c\frac{d}{dS}u^i$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{0i} = \partial^0 A^i - \partial^i A^0 = -\dot{A}^i - (\nabla V)^i$$

$$= E^i$$

$$E^i = -F^{i0} \underset{\nu=0}{=} +F^{i\mu} u_\mu$$

$$mc \frac{d}{ds} u^i = q F^{i\mu} u_\mu$$

$$mc \frac{du^\mu}{ds} = q F^{\mu\nu} u_\nu$$

$$\underline{m \frac{du^\mu}{ds} = \frac{q}{c} F^{\mu\nu} u_\nu}$$