Radiation

$$a \ll \lambda \qquad (1)$$

$$\underset{(1)}{ka \ll 1} \qquad (2)$$

$$\underset{(1)\text{ or }(4)}{v \ll c} \qquad (3)$$

$$\left\{ \begin{array}{l} T \cong 2\pi \dfrac{a}{v} \\ \lambda = cT \cong 2\pi \dfrac{ca}{v} \\ \dfrac{v}{c} \ll 1 \iff \lambda \gg a \end{array} \right. \qquad (4)$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \int' \frac{\vec{J}(\vec{r},t')}{|\vec{r}-\vec{r}'|} \, d^3r' \qquad (5)$$

$$t' = t - \frac{1}{c}|r - r'|$$

$$|\vec{r}-\vec{r}'| \cong \vec{r} - \vec{r}_0 \qquad (6)$$

$$t' \cong t - \frac{1}{c}|\vec{r} - \vec{r}_0| \qquad (7)$$

(10)

$$\omega t' \underset{(5)}{=} \omega t - \frac{\omega}{c} |\vec{r} - \vec{r}\,'| =$$

$$= \omega t - \frac{\omega}{c} \left| \vec{r} - \vec{r}_0 + \vec{r}_0 - \vec{r}\,' \right|$$

$$\cong \omega t - \frac{\omega}{c} \left(|\vec{r} - \vec{r}_0| + \vec{n} \cdot (\vec{r}_0 - \vec{r}\,') \right) \quad (8)$$

$$\frac{\omega}{c} \left| \vec{r}_0 - \vec{r}\,' \right| \simeq \frac{\omega a}{c} \underset{(2)}{\ll} 1 \quad (9)$$

$$\omega t' \underset{(8,9)}{\simeq} \omega t - \frac{\omega}{c} |\vec{r} - \vec{r}_0| \quad (10)$$

$$\vec{A}(\vec{r}, t) \underset{(5,6,7)}{\cong} \frac{\mu_0}{4\pi R} \int \underbrace{\vec{J}(\vec{r}\,', t')}_{(7) \Rightarrow t' \text{ independent of } r'} d^3 r' =$$

$$= \frac{\mu_0}{4\pi R} \left(\sum_i q_i \vec{V}_i \right)_{t'} \quad (11)$$

$$\vec{A} = \frac{\mu_0}{4\pi R} \frac{d}{dt} \left(\sum_i q_i \vec{r}_i \right) \quad (12)$$

$$R = | \vec{r} - \vec{R}_0 | \quad (13)$$

$$\vec{d} = \sum_i q_i r_i \quad (14)$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi R} \dot{\vec{d}} =}$$

$$= \left(\frac{1}{4\pi \varepsilon_0} \right) \frac{\dot{\vec{d}}}{c^2 R} \quad (15)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad (16)$$

$$\vec{\nabla} \times \vec{A} \simeq \frac{1}{c}\, \dot{\vec{A}} \times \vec{n} \qquad (17)$$

$$\vec{n} = \frac{\vec{R}}{R}$$

$$\vec{\nabla} \times \vec{A} \underset{\substack{(15)\\(17)}}{=} \frac{\mu_0}{4\pi c} \frac{\ddot{\vec{d}} \times \vec{n}}{R} = \vec{B} \qquad (18)$$

$$\vec{E} = c\, \vec{B} \times \vec{n} = \frac{\mu_0}{4\pi} \frac{\left(\ddot{\vec{d}} \times \vec{n}\right) \times \vec{n}}{R} \qquad (19)$$

$$dI = c\, \underbrace{u_{em}}\, \underbrace{\vec{n} \cdot d\vec{A}}_{R^2\, d\Omega} \qquad (20)$$

$$\frac{\varepsilon_0 \vec{E}^2}{2} + \frac{\vec{B}^2}{2\mu_0} \underset{\text{EM wave}}{=} \cancel{2} \cdot \frac{\vec{B}^2}{\cancel{2}\mu_0} \qquad (21)$$

$$d\underline{I} \underset{(20,21)}{=} \frac{c\,\vec{B}^2}{\mu_0} R^2\, d\Omega$$

$$= \frac{c}{\mu_0} \cdot \left(\frac{\mu_0}{4\pi c}\right)^2 \frac{\left(\ddot{\vec{d}} \times \vec{n}\right)^2}{\cancel{R^2}} \cancel{R^2}\, d\Omega$$

$$= \frac{\mu_0}{4\pi c}\, \ddot{\vec{d}}^{\,2} \sin^2\theta \left(\frac{d\Omega}{4\pi}\right) \qquad (22)$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{1}{c^3}\, \ddot{\vec{d}}^{\,2} \sin^2\theta\, \frac{d\Omega}{4\pi}$$

$$\underline{I} \underset{\substack{(22)\\(24)}}{=} \left(\frac{1}{4\pi\varepsilon_0}\right) \cdot \frac{2}{3}\, \frac{\ddot{\vec{d}}^{\,2}}{c^3} \qquad (23)$$

$$\int \sin^2\theta\, \frac{d\Omega}{4\pi} \underset{(25)}{=} \frac{2}{3} \qquad (24)$$

$$\frac{2\pi}{4\pi} \int_0^{\pi} \underbrace{\sin^2\theta}_{1-x^2}\, \underbrace{\sin\theta\, d\theta}_{dx} \underset{x=\cos\theta}{=} \frac{1}{2} \int_{-1}^{1} (1-x^2)\, dx = \frac{2}{3} \qquad (25)$$

$$\vec{d} = \vec{d_0}\cos\omega t$$

$$\ddot{\vec{d}} = -\omega^2 \vec{d_0}\cos\omega t$$

$$I = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2}{3}\,\frac{\omega^4 \vec{d_0}^{\,2}}{c^3}\cos^2\omega t$$

$$\langle \cos^2\omega t\rangle = \frac{1}{2}$$

$$\vec{d_0}\cos\omega t = \underbrace{\frac{\vec{d_0}}{2}}_{\vec{d_\omega}} e^{-i\omega t} + \frac{\vec{d_0}}{2} e^{i\omega t}$$

$$\vec{d_\omega} = \frac{1}{2}\vec{d_0}$$

$\langle \cos^2\omega t\rangle$ $\qquad d_0$

$$\langle I\rangle = \frac{1}{4\pi\varepsilon_0}\underbrace{\frac{2}{3}\cdot\frac{1}{2}\cdot 4}_{4/3}\,\frac{\omega^4 \vec{d_\omega}^{\,2}}{c^2}$$

If $\quad \vec{d} = \vec{d}_\omega e^{-i\omega t} + \ldots$

then

$$\langle I \rangle = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{4}{3} \frac{\omega^4 |\vec{d}_\omega|^2}{c^3}$$

Magnetic dipole radiation

$$I = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{2}{3} \frac{|\ddot{\vec{m}}|^2}{c^5}$$

If $\vec{m} = m_\omega e^{-i\omega t}$

then

$$\langle I \rangle = \frac{1}{4\pi\varepsilon_0} \frac{4}{3} \frac{\omega^4 |\vec{m}_\omega|^2}{c^5}$$

$$[m] = [I \cdot a] = \left[\frac{q}{t} \cdot x^2\right]$$

$$= \left[\frac{qx}{v}\right] = \frac{[d]}{[v]}$$

$$[d] = [q\,x]$$

$$\vec{m} \Longleftrightarrow \frac{\vec{d}}{c}$$