E.M waves in metals

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial E}{\partial t} + \mu_0 \vec{J}$$

$$\begin{cases} \rho \to \rho_f \\ \vec{J} \to \vec{J}_f \end{cases} \qquad \varepsilon_0 \mu_0$$

$$\sigma \vec{E} = \vec{J} \qquad \text{Ohm's law}$$

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\sigma \vec{\nabla} \cdot \vec{E} = \frac{\sigma}{\varepsilon_0} \rho = -\frac{\partial \rho}{\partial t}$$

$$\rho(t) = \exp\left(-\frac{\sigma}{\varepsilon_0} t\right) \rho(0)$$

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\dot{\vec{B}} & \vec{\nabla} \times \vec{B} = \frac{1}{c^2} \dot{\vec{E}} + \mu_0 \sigma \vec{E} \end{cases}$$

$$\begin{cases} \Delta \vec{E} = \frac{1}{c^2} \ddot{\vec{E}} + \mu_0 \sigma \dot{\vec{E}} \\ \Delta \vec{B} = \frac{1}{c^2} \ddot{\vec{B}} + \mu_0 \sigma \dot{\vec{B}} \end{cases}$$

$$\begin{cases} \vec{E} = \vec{E}_0 \, e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \vec{B}_0 \, e^{i(\vec{k}\vec{r} - \omega t)} \end{cases}$$

$$k^2 = \frac{\omega^2}{c^2} + i \mu_0 \sigma \omega$$

$$k = \frac{\omega}{c} \sqrt{1 + i \frac{\sigma}{\varepsilon_0 \omega}}$$

$$n = \sqrt{1 + \frac{i\sigma}{\varepsilon_0 \omega}} = n_1 + i n_2$$

If $\quad \frac{\sigma}{\varepsilon_0} >> \omega \quad$ (good metal)

then

$$\begin{cases} n \cong \sqrt{i \frac{\sigma}{\varepsilon_0 \omega}} \\ n_1 \cong n_2 \cong \sqrt{\frac{\sigma}{2\varepsilon_0 \omega}} \end{cases}$$

$$k = \frac{\omega}{c} n = \frac{\omega}{c} n_1 + i \frac{\omega}{c} n_2$$

$$\vec{E} = \vec{E}_0\, e^{i(kz-\omega t)} =$$

$$= E_0 \left\{ e^{i\omega\left(\frac{n_1 z}{c} - t\right)} \right\} \underbrace{e^{-\frac{n_2 \omega z}{c}}}_{e^{-\frac{z}{d}}}$$

$$d^{-1} = \frac{n_2 \omega}{c} \qquad d = \frac{c}{\omega n_2}$$

for good metals

$$d \cong \frac{c}{\omega \sqrt{\frac{\sigma}{2\varepsilon_0 \omega}}} = c\sqrt{\frac{2\varepsilon_0}{\sigma\omega}}$$

$$d = c\sqrt{\frac{2\varepsilon_0}{\sigma\omega}}$$

Skin depth

$$\begin{cases} \sigma = 6 \cdot 10^{7} \ (\Omega \cdot m)^{-1} \\ \omega = 4 \cdot 10^{15} \ 1/s \end{cases} \qquad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^{9}$$

$$d \cong 5 \cdot 10^{-9} \ m = 5 \cdot 10^{-7} \ cm$$

$$\approx \underline{50 \ \mathring{A}}$$

$$\omega = \frac{2\pi c}{\lambda} = \frac{2\pi \cdot 3 \cdot 10^{8}}{0.5 \cdot 10^{-6}} \cong 4 \cdot 10^{15}$$

$$\lambda = 0.5 \mu$$

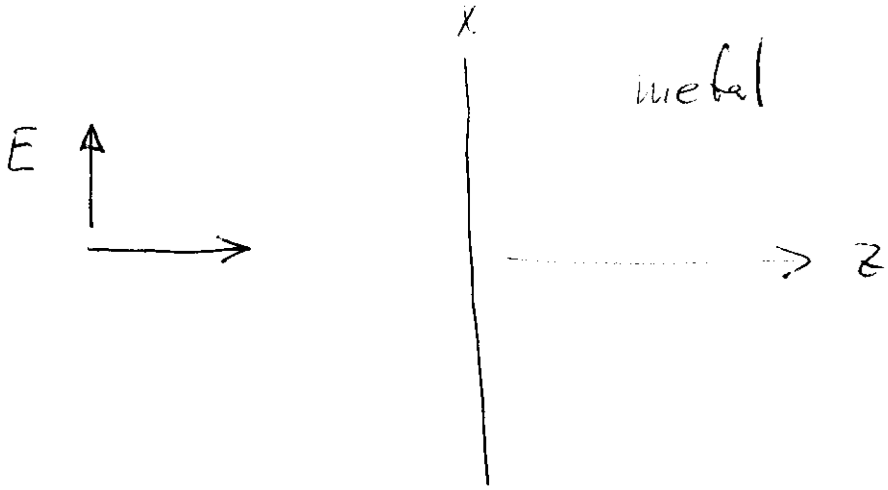
$$n = \sqrt{\varepsilon\mu} = \sqrt{1 + i\frac{\sigma}{\varepsilon_0\omega}}$$

$$\mu = 1$$

$$\boxed{\varepsilon = 1 + i\frac{\sigma}{\varepsilon_0\omega}}$$

Reflection off metal

Normal incidence



$$E_{0R} = \frac{\alpha - \beta}{\alpha + \beta} E_{0I}$$

$$E_{0T} = \frac{2}{\alpha + \beta} E_{0I}$$

$$\alpha = \frac{\cos\theta_R}{\cos\theta_I} = 1$$

$$\beta = \frac{\varepsilon_2}{n_2} \frac{n_1}{\varepsilon_1} = \sqrt{\frac{\varepsilon_2}{\mu_2} \frac{\mu_1}{\varepsilon_1}}$$

$$\alpha = 1$$

$$\beta = \frac{1}{\sqrt{\varepsilon_1}} \sqrt{1 + i \frac{\sigma}{\varepsilon_0 \omega}} = \frac{n_2}{n_1}$$

$$R = \left| \frac{1-\beta}{1+\beta} \right|^2$$

$$n_1 = \sqrt{\varepsilon_1} \simeq 1$$

$$R = \frac{(1 - \mathrm{Re}\, n_2)^2 + (\mathrm{Im}\, n_2)^2}{(1 + \mathrm{Re}\, n_2)^2 + \mathrm{Im}\, n_2} =$$

$$\boxed{R = \frac{(n'-1)^2 + n''^2}{(n'+1)^2 + n''^2}}$$

$$n = n' + i\, n'' = \sqrt{1 + i\,\frac{\sigma}{\varepsilon_0 \omega}}$$

$$R = \frac{|n|^2 - 2n' + 1}{|n|^2 + 2n' + 1} \approx$$

$$\approx 1 - 4\,\frac{n'}{|n|^2} \underset{n' = n''}{\approx}$$

$$\simeq 1 - \frac{4n'}{2n'^2} = 1 - \frac{2}{n'}$$

$$\cong 1 - 2\sqrt{\frac{\varepsilon_0 \omega 2}{\sigma}} =$$

$$\boxed{R = 1 - 2\sqrt{2\,\frac{\varepsilon_0 \omega}{\sigma}}}$$

$$n' \simeq n'' \cong \sqrt{\frac{\sigma}{2\varepsilon_0 \omega}}$$

Ag $T \simeq 300^\circ K$

$$\sigma = 6 \cdot 10^7 \ (\Omega \cdot m)^{-1}$$

$$\frac{\hbar \sigma}{\varepsilon_0} = 4.4 \text{ keV}$$

$$\lambda = 0.5 \mu$$

$$\omega = 4 \cdot 10^{15} \ 1/s$$

$$\hbar \omega = 2 \text{ eV}$$

$$R = 1 - 2 \sqrt{2 \frac{2}{4.4 \cdot 10^3}}$$

$$= 1 - \underbrace{0.069}_{\approx 7\% \text{ lost}}$$