

## II. Electrostatic

The Maxwell's equations read

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho \quad (2.1)$$

$$\nabla \times \mathbf{E} = 0 \quad (2.2)$$

Here  $\epsilon_0$  is a constant

$$\frac{1}{4\pi\epsilon_0} \cong 9 \cdot 10^9 \quad (2.3)$$

Consider the potential of the electric field  $V$  (called also the scalar potential)

$$\mathbf{E} = -\nabla V \quad (2.4)$$

Eqs.(2.1) gives

$$\Delta V = -\frac{1}{\epsilon_0} \rho \quad (2.5)$$

The **Gauss theorem**

$$\int_V \nabla \cdot \mathbf{W} d^3r = \oint_S \mathbf{W} \cdot d\mathbf{s} \quad (2.6)$$

Here  $V, S$  are a volume and a closed surface surrounding this volume. From Eqs.(2.1),(2.6) one finds

$$\int_V \nabla \cdot \mathbf{E} d^3r = \oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_V \rho d^3r = \frac{1}{\epsilon_0} Q \quad (2.7)$$

The Gauss theorem for the **flux of the electric field** reads

$$\boxed{\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} Q} \quad (2.8)$$

Here  $Q$  is the total charge inside the volume surrounded by the surface  $S$ .

Taking one point-like charge, and applying Eq.(2.8) to the sphere, which has the charge located at its center, one recovers the Coulomb law

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{n}}{r^2} \quad (2.9)$$

If the electric field is created by several charged, point-like particles, then the Coulomb law reads

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3} Q_i \quad (2.10)$$

Here  $\mathbf{r}_i, Q_i$  are the radius vectors indicating the location and charge of the particle  $i$ , the summation runs over all particles, which create the electric field.

From Eqs.(2.4),(2.10) one derives

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{Q_i}{|\mathbf{r} - \mathbf{r}_i|} \quad (2.11)$$

Assume that there is some density of charge  $\rho(\mathbf{r}')$  distributed within some volume  $V$ . Then Eq.(2.10) can be rewritten

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \rho(\mathbf{r}') d^3r' \quad (2.12)$$

Here  $\rho(\mathbf{r}')d^3r' = dQ$  describes the charge located within the volume  $d^3r'$ . Correspondingly, Eq.(2.11) reads

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \quad (2.13)$$

### Summary:

The Maxwell's equations read

$$\begin{cases} \nabla \times \mathbf{E} = 0 \\ \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho \end{cases} \quad (2.14)$$

They can be presented as **one** equation for the potential, which can be written either as

$$\Delta V(\mathbf{r}) = -\frac{1}{\epsilon_0} \rho(\mathbf{r}) \quad (2.15)$$

or alternatively as

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \quad (2.16)$$

### Comment.

Sometimes it is convenient to use a short-cut notation saying that if

$$\Delta f(\mathbf{r}) = g(\mathbf{r}) \quad (2.17)$$

then

$$f(\mathbf{r}) = \frac{1}{\Delta} g(\mathbf{r}) \quad (2.18)$$

Eqs.(2.15),(2.16) show that this short-cut means

$$f(\mathbf{r}) = \frac{1}{\Delta} g(\mathbf{r}) = -\frac{1}{4\pi} \int \frac{g(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \quad (2.19)$$