EM waves in matter

Insulators

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = 0 & \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\dot{\vec{B}} & \nabla \times \vec{H} = \frac{1}{c^2} \dot{\vec{D}} \end{cases} \qquad (1)$$

$$\begin{cases} \vec{B} = \mu \vec{H} \\ \vec{D} = \varepsilon \vec{E} \end{cases} \qquad (2)$$

$$(1,2) \Rightarrow \begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & \vec{\nabla} \times \vec{B} = 0 \\ \bar{\nabla} \times \bar{E} = -\dot{\vec{B}} & \vec{\nabla} \times \vec{B} = \frac{1}{v^2} \dot{\vec{E}} \end{cases} \qquad (3)$$

$$\frac{1}{v^2} = \frac{\varepsilon \mu}{c^2} \qquad \begin{cases} v = \frac{c}{\sqrt{\varepsilon \mu}} \\ v = c/n \\ n = \sqrt{\varepsilon \mu} \; (> 1) \end{cases} \qquad (4)$$

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \Delta \right) \vec{E} = 0 \qquad \text{vacuum}$$

$$\left(-\frac{1}{v^2} \frac{\partial^2}{\partial t^2} + \Delta \right) \vec{E} = 0 \qquad \text{matter}$$

Dispersion

$$\begin{cases} \varepsilon \Rightarrow \varepsilon(\omega) \\ \mu \Rightarrow \mu(\omega) \end{cases} \quad (1) \qquad \begin{cases} n = n(\omega) = \dfrac{1}{\sqrt{\varepsilon(\omega)\mu(\omega)}} \\ v = v(\omega) = \dfrac{c}{n(\omega)} \end{cases} \quad (2)$$

$$\vec{E}, \vec{B}, \vec{D}, \vec{H} \sim e^{-i\omega t} \qquad (3)$$

$$\frac{\partial}{\partial t} \Rightarrow -i\omega \qquad (4)$$

$$\begin{cases} \vec{\nabla}\cdot\vec{E} = 0 & \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{E} = i\vec{B} & \vec{\nabla}\times\vec{B} = -\dfrac{i\omega}{v^2}\vec{E} \end{cases} \qquad (5)$$

$$\left(\frac{\omega^2}{v^2} + \Delta\right)\vec{E} = 0 \qquad (6)$$

$$\left(k^2 + \Delta\right)\vec{E} = 0 \qquad (7)$$

$$\frac{\omega}{v} = k = \frac{\omega}{c} n \qquad (8)$$