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# Stress tensor

$$\overleftrightarrow{T}_{ij} = \epsilon_0 \left( E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) \quad (1)$$

$$+ \frac{1}{\mu_0} \left( B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$

$$\left( \overleftrightarrow{\nabla \cdot T} \right)_j = \frac{\partial}{\partial r_i} T_{ij} =$$

$$= \epsilon_0 \left( (\overrightarrow{\nabla} \cdot \vec{E}) E_j + (\vec{E} \cdot \overrightarrow{\nabla}) E_j - \frac{1}{2} \overrightarrow{\nabla}_j E^2 \right)$$

$$+ \frac{1}{\mu_0} \left( (\overrightarrow{\nabla} \cdot \vec{B}) B_j + (\vec{B} \cdot \overrightarrow{\nabla}) B_j - \frac{1}{2} \overrightarrow{\nabla}_j B^2 \right)$$

$$= \left( \overleftrightarrow{\nabla \cdot T} \right)_j \quad (2)$$

$$\vec{F} = \int (\vec{E} + \vec{v} \times \vec{B}) \rho d^3r =$$

$$= \int \underbrace{(\rho \vec{E} + \vec{J} \times \vec{B})}_{\vec{f}} d^3r \quad (3)$$

$$\vec{f} = \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \vec{E} + \frac{1}{\mu_0} \left( \vec{\nabla} \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \right) \times \vec{B} + \vec{E} \times (\vec{\nabla} \times \vec{E}) \quad (4)$$

$$\frac{\partial \vec{E}}{\partial t} \times \vec{B} = \frac{\partial}{\partial t} (\vec{E} \times \vec{B}) - \underbrace{\vec{E} \times \frac{\partial \vec{B}}{\partial t}}_{-\vec{\nabla} \times \vec{E}} \quad (5)$$

$$\vec{f}_{(4,5)} = \epsilon_0 \left[ (\vec{\nabla} \cdot \vec{E}) \vec{E} - \vec{E} \times (\vec{\nabla} \times \vec{E}) \right] + \frac{1}{\mu_0} \left[ \vec{B} \times (\vec{\nabla} \times \vec{B}) + (\vec{\nabla} \cdot \vec{B}) \vec{B} \right] - \frac{\epsilon_0}{c^2} \frac{\partial}{\partial t} (\vec{E} \times \vec{B}) \quad (6)$$

$$\vec{E} \times (\vec{\nabla} \times \vec{E}) = \frac{1}{2} \vec{\nabla} (\vec{E}^2) - (\vec{E} \cdot \vec{\nabla}) \vec{E} \quad (7)$$

$$\begin{aligned} \vec{P} &= \epsilon_0 \left[ (\vec{\nabla} \cdot \vec{E}) \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{E} \right] \\ &+ \frac{1}{\mu_0} \left[ (\vec{\nabla} \cdot \vec{B}) \vec{B} + (\vec{B} \cdot \vec{\nabla}) \vec{B} \right] \\ &- \vec{\nabla} \left( \frac{\epsilon_0 \vec{E}^2}{2} + \frac{\vec{B}^2}{2\mu_0} \right) \\ &- \epsilon_0 \frac{\partial}{\partial t} (\vec{E} \times \vec{B}) \end{aligned} \quad (8)$$

$$\vec{f} = \vec{\nabla} \cdot \vec{T} - \epsilon_0 \mu_0 \frac{\partial \vec{S}}{\partial t} \quad (9)$$

(8,2)

$$\vec{F} = \int \vec{T} \cdot d\vec{s} - \epsilon_0 \mu_0 \frac{d}{dt} \int \vec{S} d^3v$$