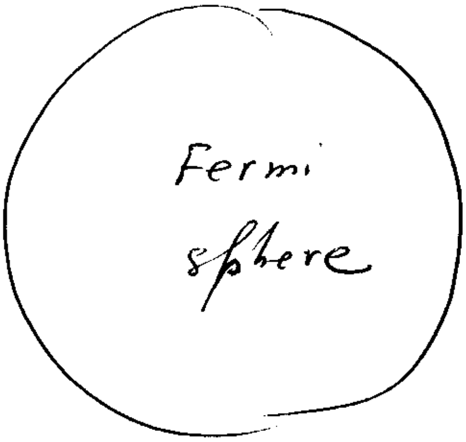
Free electron gas



Number of states

$$dN = \sum_S \frac{V\, d^3k}{(2\pi)^3} = 2 \cdot \frac{4\pi V}{8\pi^3} k^2 dk \qquad (1)$$

$$= \frac{V}{\pi^2} \frac{p^2 dp}{\hbar^3} = \frac{V p m}{\pi^2 \hbar^3} d\varepsilon$$

$$\varepsilon = \frac{p^2}{2m} \qquad (2)$$

$$dN = g(\varepsilon)\, d\varepsilon \qquad (3)$$

$$g(\varepsilon) = \frac{V p m}{\pi^2 \hbar^3} = \frac{V m \sqrt{2m\varepsilon}}{\pi^2 \hbar^3} \qquad (4)$$

Fermi energy

$$N = 2\int \frac{V\,d^3k}{(2\pi)^3} =$$

$$= \frac{V}{\pi^2}\int_0^{k_F} k^2 dk = \frac{V}{3\pi^2} k_F^3$$

$$\underline{k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3}} \qquad (5)$$

$$p_F = \hbar k_F \qquad (6)$$

$$\mathcal{E}_F = \frac{\hbar^2 k_F^2}{2m} = \frac{(3\pi^2)^{2/3}}{2}\frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3} \qquad (7)$$

$$E = \frac{V}{\pi^2\hbar^3}\int_0^{p_F} \frac{p^2}{2m} p^2 dp =$$

$$= \frac{V p_F^5}{10\pi^2 m\hbar^3} = \frac{3(3\pi^2)^{2/3}}{10}\frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3} N$$

$$T \ll \mathcal{E}_F$$

Heat capacity

$$N = \frac{V}{\pi^2}\int_0^\infty \frac{k^2 dk}{e^{\frac{\varepsilon_k - \mu}{kT}} + 1} \qquad (8)$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} \qquad (9)$$

$$\mu = \mu(T, N) \qquad (10)$$

$$E = \frac{V}{\pi^2}\int_0^\infty \frac{\varepsilon_k\, k^2 dk}{e^{\frac{\varepsilon_k - \mu}{kT}} + 1} \qquad (11)$$

$$E = E(T) \qquad (12)$$

$$C_V = \frac{dE}{dT} \qquad (13)$$

Estimate

$$E(T) \sim E(0) + g(\varepsilon_F)\,(kT)^2 \qquad (14)$$

$$C_V \sim k\, g(\varepsilon_F) \cdot kT \sim$$

$$\sim k \underbrace{g(\varepsilon_F)\cdot \varepsilon_F}_{\sim 3N} \cdot \frac{kT}{\varepsilon_F} \qquad (15)$$

$$C \sim 3Nk \cdot \frac{kT}{\varepsilon_F} \qquad (16)$$

Exact result

$$C = \left(\frac{\pi^2}{6}\right) \cdot 3Nk \left(\frac{kT}{\varepsilon_F}\right) \qquad (17)$$

Effective mass

$$m \to m^* \tag{18}$$

Conductivity

$$\frac{d\vec{p}}{dt} = \underbrace{(-e)\left(\vec{E} + \vec{v}\times\vec{B}\right)}_{\vec{F}_L} \qquad (1)$$

$e = |e| > 0$

Collisions

$$\frac{d\vec{p}}{dt} = \vec{F}_L - \frac{\vec{p}}{\tau} \qquad (2)$$

If $\quad \vec{B} = 0 \qquad (3)$

Then $$\frac{d\vec{p}}{dt} = (-e)\vec{E} - \frac{\vec{p}}{\tau} \qquad (4)$$

$$\vec{p} = (-e)\tau\vec{E} \qquad (5)$$

$$\vec{v} = \underbrace{-\frac{e\tau}{m}}_{\mu_e}\vec{E}$$

$$\mu_e = + \frac{e\tau}{m} \tag{6}$$

$$\vec{j} = (-e)\, n \cdot \vec{V} = \frac{n e^2 \tau}{m} \vec{E}$$

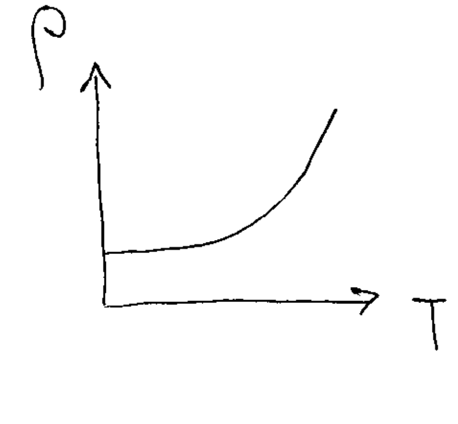
$$\vec{j} = \vec{\sigma}\, \vec{E}$$

$$\sigma = \frac{n e^2 \tau}{m} \tag{7}$$

$$\rho = \frac{1}{\sigma} = \frac{m}{n e^2} \frac{1}{\tau} \tag{8}$$

$$\rho = \frac{m}{n e^2} \left(\frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_0} \right) \tag{9}$$

Mattheisen's rule (10)



Thermal conductivity

$$W = \frac{1}{3} \bar{v} \ell \frac{dE}{dz} =$$

$$= \frac{1}{3} v \ell \frac{dE}{dT} \frac{dT}{dz} \quad (11)$$

$$W = k \frac{dT}{dz} \quad (12)$$

$$K = \frac{1}{3} v \ell \underbrace{\frac{dE}{dT}}_{C_V} \quad (13)$$

E - energy density

W - flux of energy

C_V - heat capacity per volume

$$\kappa = \frac{1}{3} C_V v_F^2 \tau \qquad (13)$$

$$= \frac{1}{3} \cdot \frac{\pi^2}{2} \frac{Nk}{V} \frac{T}{T_F} \frac{2\varepsilon_F}{m} \tau$$

$$= \frac{\pi^2 k^2}{3} \frac{n T \tau}{m} \qquad (14)$$

Weideman Franz law

$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \left(\frac{k}{e}\right)^2 \qquad (15)$$