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# ACOUSTIC IMPEDANCE SPECTRA OF CLASSICAL AND MODERN FLUTES

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Instruments in the flute family, unlike most wind instruments, are played with the input of the instrument open to the atmosphere. Consequently, they operate at minima in the spectrum of acoustic input impedance. Detailed examination of these minima requires measurements with large dynamic range, which is why the flute has not been hitherto investigated in detail. We report the application of a technique with high precision and large dynamic range to measurements of the impedance spectra of flutes. We compare the acoustical impedance spectra of two examples of the modern orchestral flute and an example of the classical flute. For each instrument, we measured several dozen of the most commonly used different acoustic configurations or fingerings. The results are used to explain features of the spectra of the sound produced, to explain performance features and difficulties of the instruments, and to explain the differences between the performances of the classical and modern instruments. Some hundreds of spectra and sound files are given in JSV+ to allow further examination.



## 1. INTRODUCTION

The flute is much older than history [1], and today is one of the most popular wind instruments. The sound is excited by a jet of air striking an edge and thus it belongs to the diverse air jet family, which includes the recorder, shakuhachi, syrinx, quena, ocarina and organ pipes. An excellent account of the history and construction of the flute is given in reference [2].

The air jet can be excited by a transverse acoustic flow, and it then amplifies sinuous waves travelling along it. In the presence of an acoustic resonator—the body of the instrument—alternate deflections of the jet into and out of the resonator can sustain oscillations in the resonator over a small range of suitable values of the jet velocity and length. Thus, the operating frequency is largely determined by the resonances of the bore of the instrument and the speed and length of the jet. The interaction between the two can be understood in terms of the acoustic impedance of the resonating bore $\mathbf{Z}(f)$ (the ratio of acoustic pressure to volume flow). The operation of various flute-family instruments is discussed in detail in reference [3] and a brief summary is given later in the present article.

Because it involves air jet excitation, the input aperture (embouchure hole) of flutes is necessarily open to the external air, whereas in most other wind instruments it is sealed by a reed and/or the player's lips. The flute therefore operates at minima in the $\mathbf{Z}(f)$ of the resonator. This is the reason why there are relatively few experimental studies of the linear

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acoustics of the flute: to determine $\mathbf{Z}(f)$ sufficiently well to study the minima precisely, one requires measurements with large dynamic range. Indeed, for most measurements of reed and lip–reed instruments, $\mathbf{Z}(f)$ is plotted on a linear scale, so that most of the curve lies on the $Z = 0$ axis, or is obscured by noise.

In this paper, we describe a technique that achieves the required dynamic range. It is an adaptation of a spectrometer originally designed to operate rapidly for measurements on the vocal tract during speech [4, 5]. We synthesize a signal comprising all of the desired frequency components. For the flute, we use the range from 200 Hz (lower than the lowest note on the instrument) to either 3000 or 4000 Hz. The impedance spectrum has little structure above about 3 kHz and the standard range of the instrument does not include notes with fundamentals above 3 kHz. This synthesized signal is converted to an acoustic current via an amplifier, loudspeaker, impedance matching horns and an attenuator with a high acoustic impedance. Because it is difficult to measure the volume flow with high precision over a large range, our technique, like most previous techniques [6–8], compares known and unknown impedances. We use this instrument to measure the acoustic impedance at the input or embouchure hole of flutes.

In the flute, as in other woodwind instruments, the resonances of the instrument are varied by opening and closing different combinations of keys. Each combination is called a fingering, and good players know at least several dozens of these. A much longer list is given by Dick [9]. Much of the interest in the acoustic performance of the instrument comes from comparing and contrasting the performance of different fingerings for the same or for different notes, so we report acoustic impedance spectra for several dozen different fingerings for the flutes studied. We also report sound spectra. Together, there are too many data for conventional publishing, so in this paper we choose a small number of curves to illustrate some points, while we publish the complete set (including sound files) electronically in JSV+ .

## 2. THE ANATOMY AND EVOLUTION OF THE FLUTE

### 2.1. TYPES OF FLUTES

We report studies on flutes that are variants on two basic geometries (Figure 1). The body of the modern or Boehm flute is nearly cylindrical, but the head joint is tapered towards the embouchure end. The flute of the 18th and early 19th centuries (the classical flute) is approximately conical over much of its length, with a cylindrical head joint. The constraints of manufacturing the varying tapers means that both flutes are usually made in sections. The head joint includes the embouchure hole and has no keys. In the modern flute, the body is a long joint that has most of the holes and/or keys. In the classical flute studied here, the "body" comprised two sections, one with the holes operated by the left hand, and the other with all or most of the holes operated by the right. Various different foot joints are used on different flutes. On the modern flute, the C foot has three tone holes (all operated by the right little finger) and permits a lowest note of C4. The longer B foot has an extra tone hole and allows B3 to be played. This change produces substantial acoustical differences, especially at the extremes of the range. The classical flute is and was much less standardized. It may have a short foot stopping at D4, or a longer foot that may descend to C4. The geometry of the foot may continue the cone of the body, or may flare out slightly at the end. The modern flutes studied here are production-line models, which are widely available at relatively modest price. This choice was made to facilitate comparisons by other researchers. There are no production line classical flutes. Further, the original instruments of that period, usually made of wood, have changed their geometry over time. The

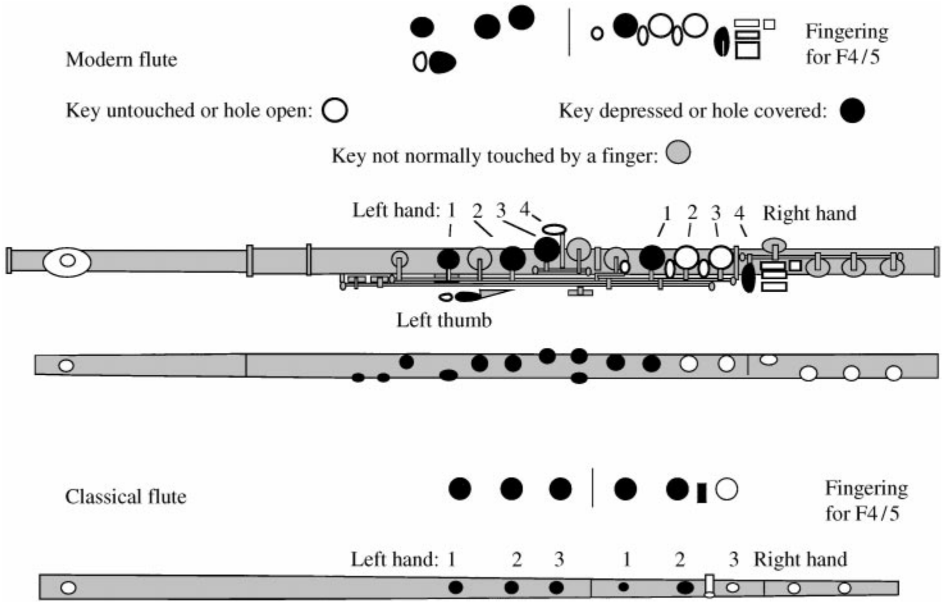


Figure 1. Fingering and acoustic schematic diagram for the modern and classical flutes. From the top are shown a fingering diagram for the modern flute, a sketch of the modern flute and an acoustic schematic of that instrument. Next is a fingering diagram and schematic for the classical flute. A similar legend is used for all the fingerings and notes in the database. The example chosen here is the fingering for the notes F4 and F5.

instrument we studied was made by a local maker. It is pitched at A4 = 440 Hz, as are the modern flutes, which facilitates comparison with them. It has three interchangeable feet, examples of the three mentioned above.

Figure 1 shows a schematic diagram of the modern and classical flutes, a simplified acoustical schematic of each, and fingering diagrams that would be immediately familiar to a flutist. The example chosen is the fingering used to play either F4 or F5 (the player adjusts the jet length and speed to play the two notes). The fingering diagram shows a player how to finger these notes: for the modern flute, three fingers and the thumb of the left hand depress their home keys, as do the first and fourth fingers of the right hand. This can readily be related to the sketch of the flute, indicating which keys are touched on the instrument, and also to a simplified acoustic schematic of the instrument. For the note F4, the first six holes most remote from the embouchure hole are open, because F4 is six semitones above the lowest note (B3) on this instrument. Eleven holes are closed: this is possible because one finger key may close more than one hole, and because some holes are opened by keys rather than closed. All tone holes on the instrument are approximately the same diameter, as shown in the acoustic schematic, except for the three holes closest to the embouchure hole. The two closest are the smallest holes, and are used as register holes for some of the highest notes, or for trills (rapid alternations of two notes) which would otherwise cross the "break" between the first register (fundamental) and second register (second harmonic). The third small hole is used as a register hole for several notes (D5, D6, D#5, A6) and also as a tone hole for the transition between C5 and C#5, or between C6 and C#6. This double use requires that it be further up the tube than the expected place for such a tone hole, and this in turn requires it to be small so as to have a large end effect. Most of the holes are approximately in line, those off-line are shown as ellipses. In one case there are two keys at the same distance from the embouchure hole: these are alternates: they are very rarely opened together (one fingering for C#7 is an exception), and exist mainly as an historical accident. The trill keys, the home key for the left fourth finger and the home key for the right fourth finger are closed when not touched. In the modern flute, the body is cylindrical and the head had a slight conical taper. An adjustable cork in the head makes the acoustic length rather shorter than the length of the instrument, as shown. In contrast, the classical flute (shown below) has a body with a slight conical taper. The fingering diagram (again for F4/5) shows that three fingers of the left hand cover the holes assigned to them. The first two fingers of the right hand close their holes. The third finger has two jobs: it can either cover a hole or depress the "F" key. Here it does the latter, as indicated by the black shaded key on the fingering diagram. On this flute, this is the only key, and it opens a hole when depressed, so the ellipse at the end of the "F" key is shown white for this fingering.

The radical change in the flute in the 19th century was much larger and more abrupt than that of the other woodwind instruments. This was due to the flutist and flute maker Theobold Boehm, who aimed to make the flute louder, its timbre more homogeneous from note to note, and its tuning more in accord with that of other instruments [10].

In the classical flute, successive opening of the finger holes produces a diatonic scale (D major). Some of the remaining notes are produced by opening an extra tone hole, which is normally closed by a key. Others are produced by "cross fingering", i.e., closing one or more holes downstream of the first open hole. This gives different timbre to adjacent notes.

Boehm's first innovation was the introduction of larger holes on the conical-bore flute, which made the instrument louder, and key rings and a coupling mechanism to avoid cross-fingering, which made the timbre more homogeneous. This system allowed the notes of the chromatic scale over most of the range to be played with all of the holes downstream of a particular point open. In search of a still bigger sound, he redesigned the bore: a cylindrical body and a tapered head profile, which he (inaccurately) called "parabolic". The cylindrical bore of the modern Boehm flute is larger (19 mm diameter) than that of the classical flute everywhere except near the embouchure hole, where it tapers to about 17 mm. This larger bore reduces energy losses near the walls. More importantly, the tone holes are considerably larger (about 13 mm diameter) and almost uniform in size, and so radiate more strongly. They also produce a brighter sound, as discussed later. Boehm developed a new system of coupled keys to cover these tone holes, now much too large for the unaided fingers. Thus, most of the ordinary notes of the instrument require no cross-fingering and this, together with the large tone holes, makes the downstream bore more nearly approximate a truncated pipe. This produces a more homogeneous timbre. The changes to the flute since Boehm have been relatively minor. Boehm's revolutionary changes to the flute influenced the design of the saxophone, and were also imitated (to a lesser extent) on the clarinet. (The oboe and bassoon remain much more like their ancestors, having small tone holes and, particularly on the bassoon, less rational key systems.)

## 2.2. SOUND PRODUCTION IN THE FLUTE

It is possible to link the findings of this empirical study with conclusions reached by theoretical means. Detailed discussion, with extensive references, is given in reference [3], but it is helpful to summarize the theoretical conclusions here.

### 2.2.1. *The air jet*

Only a simplified discussion of the sound production process in the flute is required for our purposes. First, the blowing pressure and jet length must be sensitively adjusted by the player to give the correct phase relation between the acoustic flow that is exciting the jet and the propagation of sinuous waves along the jet to vary its inflow into the embouchure hole. This is why the example fingering shown in Figure 1 is cited for two different notes: a faster (or shorter) jet can be used to excite a standing wave at the second, rather than at the first, impedance minimum. More importantly for our present concerns is the response of the instrument to this oscillating jet flow. If the jet has air velocity **V** and area $S_J$ and the flute tube has area $S_P$, then the acoustic wave sustained in the flute by the jet has volume flow

$$\mathbf{U}_P = \frac{(\mathbf{V} + \mathrm{j}\omega \Delta L)\rho\,\mathbf{V} S_J}{S_P \mathbf{Z}(f)}, \tag{1}$$

where $\Delta L$ is the end-correction at the embouchure, $\omega = 2\pi f$ is the angular frequency and $\rho$ is the density of air. (Bold characters represent complex quantities.) $\mathbf{Z}(f)$ is the input impedance of the flute as measured from outside the embouchure hole, in such a way that the radiation impedance of the embouchure hole is included. This equation indicates the importance of measuring this input impedance, and shows that the flute sounds at the minima in $\mathbf{Z}(f)$. The whole situation is, however, rather more complex than this analysis suggests, for the calculation must actually be extended to include harmonics of the fundamental tone. Sound generation is most efficient when the frequencies of the impedance minima closely approximate a harmonic series (1, 2, 3,...), for then all harmonics generated by the non-linear jet process are reinforced.

### 2.2.2. *Bore and head joint*

The major acoustic losses in a wind instrument are created by viscous and thermal effects at the tube walls, rather than by acoustic radiation. Wave attenuation in the bore increases with increasing frequency as $\sqrt{f}$ and with decreasing tube radius as $r^{-1}$. There is thus more loss in the tapering tube of a classical flute than in a modern cylindrical Boehm flute, and all resonances become less pronounced at high frequencies. The taper of the head joint also has an important role in adjusting the flute intonation [11, 12] by decreasing the frequency of the low-octave resonances relative to those at higher pitch. This takes account of the changes in blowing pressure and in lip coverage of the embouchure hole in typical flute performance technique. The material from which the flute is made is expected to have little effect on the properties measured here, apart from its influence on surface finish (which should be very smooth) and the shape of finger holes (which should not have sharp edges). The detailed geometry of the lip plate and embouchure hole (shape, chimney height, undercut angle, edge sharpness, etc.) are all vitally important in determining the tone and responsiveness of the instrument.

### 2.2.3. *Finger holes*

Finger holes have important acoustic influences both when closed (when they contribute small extra volumes to the bore) and when open (when they provide an inertive shunt to the outside air). The internal standing wave in the flute bore always extends some distance past the first open hole (which is why "cross-fingerings" work to produce semitones) and, in the case of notes in the third octave, along the whole bore (which is why high-octave fingerings are complex) [13]. Benade [14] has examined the behaviour of a row of open finger holes and concluded that they act as a high-pass filter, the cut-off frequency being determined by the hole size and spacing in such a way that small holes give a low cut-off frequency, this frequency typically being in the range 1500–2000 Hz. Above the cut-off frequency, waves propagate rather freely along the instrument bore and are not reflected, hence eliminating high-frequency resonances and reducing the strength of higher harmonics in the tone. It is largely for this reason that classical flutes sound "mellow", with a cut-off of around 1500 Hz on most fingerings, while the modern flute is much "brighter" in tone quality, having a cut-off somewhat above 2000 Hz. This cut-off frequency also limits the pitch of the highest note playable on the instrument, about A6 on a classical flute and about F7 on a modern flute.

While the tone holes on a modern flute are all nearly the same size, the finger holes on a classical flute are much smaller and have different diameters, partly to bring their positions conveniently under the fingers, and partly to adjust the intonation on cross-fingered and high-octave notes. This means that the tone quality is not completely uniform from one note to another across the compass of the instrument, a feature regarded as a deficiency in modern music, but sometimes exploited to advantage in music of earlier centuries.

For notes in the third and fourth octaves (i.e., above C#6), it is not generally adequate to use simply a higher blowing pressure and the same fingering as for a lower octave. Part of the reason is that the lower-octave impedance minima are deeper than those at high frequencies, and are in addition supported by one or more upper resonances in closely harmonic relationship. The fingerings used for the uppermost octave are therefore designed to enhance the prominence of the desired upper-octave resonance while at the same time reducing the height or shifting the frequency of competing lower resonances. In hand-made classical wooden instruments, the maker may also introduce small variations in bore radius at appropriate places to enhance these effects.

## 3. MATERIALS AND METHODS

### 3.1. THE FLUTES

The modern flutes studied were production line instruments (Pearl PF-661, closed hole, C foot and B foot). On these mass produced instruments, both the open and closed hole variants use the same "scale" (i.e., positioning of tone holes). For standard measurements, the position of the cork in the head joint was set at 17·5 mm from the centre of the embouchure hole, and the tuning slide was set at 4 mm. These values are typical values used by flutists playing at standard pitch.

The classical flute was made by Terry McGee of Canberra, Australia. The dimensions of the instrument are based on those of a large-hole Rudall and Rose flute (R & R #655 from the Bate Collection in Oxford) but the scale has been adjusted to play at 440 Hz. It has a head joint, a joint with three tone holes closed by the fingers of the left hand, a further joint with three tone holes closed by the fingers of the right hand and one mechanical key, normally closed, whose opening makes the transition from E4 to F4 or from E5 to F5. It has three interchangeable feet: one is a C foot in the classical style, modelled on the Rudall and Rose instrument. Another is a D foot: it plays D4 with all holes closed. The third foot is also a C foot, but its bore is larger than that of the classical foot, and the bore increases slightly at the end. This foot was designed for use in Irish music, which often uses flutes of the classical design. This flute is made of gidgee wood (*Acacia cambagii*).

Most measurements were made at $T = 25 \pm 0{\cdot}5^{\circ}\mathrm{C}$ and relative humidity $58 \pm 2\%$. Both of these values are considerably lower than those in flutes being played, and thus the features of the measurements here occur at frequencies about 2% lower than they would be on a flute being played (about 30 cents flat). We judged that correcting for a different value of the speed of sound was easier than the inconvenience of operating at high temperature and humidity. Further, the temperature and humidity are presumably functions of position in the flute being played, and these conditions are difficult to reproduce in the spectrometer. (Cork and tuning slide position, temperature and humidity were also varied to determine their effect on relative and absolute tuning, and on harmonicity. These results will be discussed elsewhere.)

## 3.2. THE IMPEDANCE SPECTROMETER

The impedance spectrometer is shown in Figure 2(a). The signal is synthesized on a computer and output via a 16-bit AD card (National Instruments NBA-2100) to a power amplifier and pair of loudspeakers. An exponential horn (cut-off frequency 200 Hz) matches the speakers to the attenuator.

The attenuator comprises a truncated conical plug located inside, and coaxial with, a truncated conical hole. The two conical surfaces are spaced by three straight pieces of fine wire (diameter 120 $\mu$m) placed at 120° intervals in the annular space between them. The input side is shaped as shown (Figure 2(b)) to improve further the impedance matching. The choice of the output impedance of the attenuator is a compromise between a high value, which improves the accuracy for high measured impedances, and a low value, which allows a greater current, higher signal-to-noise ratios and therefore better precision for low-impedance measurements.

Small electret microphones are mounted at either end of the attenuator. Only the downstream microphone is used for calibration and measurement. The upstream microphone is used in a preliminary experiment to put an upper bound on the effect of reflections from the downstream end the attenuator. The output of the attenuator is a tube of 7·8 mm internal diameter, equal to that of our calibration reference and slightly smaller than the embouchure hole of each flute.

Our reference impedance for calibration is a "semi-infinite" cylindrical waveguide: a straight, stainless-steel pipe of 7·8 mm internal diameter. Its impedance is therefore purely resistive. Its length of 42 m was determined by the width of the Physics building. At 200 Hz, a tube of this diameter has an attenuation coefficient of $0{\cdot}11\ m^{-1} = -1{\cdot}0$ dB/m [3] so the echo returns with a loss of 80 dB or greater. (The dynamic range of the instrument is a little greater than 80 dB, but the precision is less than this. The echo returning during calibration coincides with a frequency component of the input which is 80 dB or more greater, and so can be ignored.) Its impedance is therefore resistive, with resistance equal to its characteristic impedance, $R_{ref} = 8{\cdot}5$ MPa s/m$^3$. Swagelock fittings are used to connect the

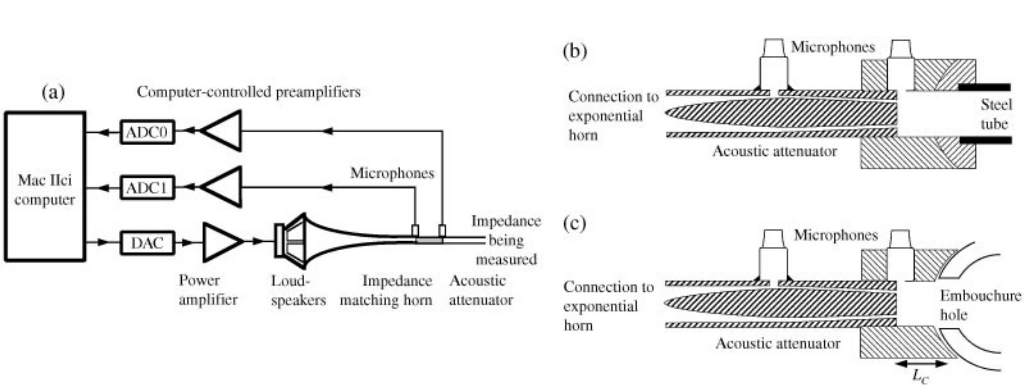


Figure 2. The impedance spectrometer (a) and the configuration for calibration (b) and for measurement (c). The figure is not so scale.

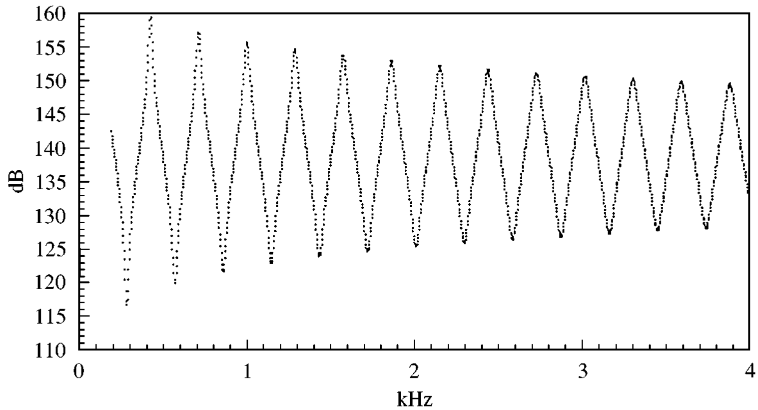


Figure 3. $\mathbf{Z}(f)$ for an open, stainless-steel cylinder of length 596·7 mm and diameter 7·8 mm. The impedance is expressed in dB with respect to one acoustic ohm, i.e. $20\log_{10}$ (Z/Pa.s.m$^{-3}$).

elements. The "semi-infinite" pipe is enclosed in a plastic pipe for sound insulation, and the plastic pipe is mounted from the ceiling via rubber bushes. The use of a frequency independent reference impedance improves the signal-to-noise ratio over the whole range, and obviates the need for more than one microphone in routine calibration or measurement.

Before calibration, the desired dependence upon frequency of the acoustic current $\mathbf{u}_{ref}(f)$ is chosen. If noise is distributed approximately equally across frequencies, it is advantageous to use a signal that is frequency independent to achieve uniform signal-to-noise ratio. For these experiments, in which low-frequency noise was not a major problem, the acoustic current was therefore chosen to be independent of frequency. A voltage waveform $\mathbf{V}_{ref}(f)$, with frequency independent harmonic components over the desired range, was synthesized and input to the power amplifier. The acoustic pressure is measured with the downstream microphone and sent via low noise preamplifiers, whose gain is computer controlled, to the AD card. The pressure spectrum $\mathbf{p}_{ref}(f)$ is calculated. $\mathbf{p}_{ref}(f)$ includes the frequency dependence of the AD card, amplifiers, speakers, horn, attenuator, other plumbing and microphone. A calibrated waveform is then synthesized with Fourier components $\mathbf{V}_{cal}(f)$ proportional to $\mathbf{V}_{ref}/\mathbf{p}_{ref}(f)$. This signal is then output, via the same elements, to the reference impedance. The microphone now returns a calibrated spectrum $\mathbf{p}_{cal}(f)$, in this case a "flat spectrum", i.e., a signal whose Fourier components are essentially independent of frequency. These $\mathbf{p}_{cal}(f)$ are recorded. The relative phases of the harmonic components of $\mathbf{V}_{ref}$ are adjusted to improve significantly the signal-to-noise ratios of our measurements [15].

Finally, the performance of the spectrometer was checked by measuring the impedance of a series of open, stainless-steel cylinders, for which the input impedance was calculated using the radiation impedance as the termination at the open end [3], and values of the speed of sound from [16] (Figure 3). The theoretical curve is not shown because it obscures almost all points. The discrepancies between theory and experiment occur when $\mathbf{Z}(f)$ approaches 100 MPa s/m$^3$, where the finite output impedance limits the precision, as discussed below.

### 3.3. CORRECTION FOR THE FINITE IMPEDANCE OF THE ATTENUATOR

For measurements of acoustic impedance very much less than that of the attenuator, the spectrometer source may be approximated as an ideal current source and so the acoustic current equals $\mathbf{u}_{ref}(f)$. The output of the attenuator is effectively in parallel with the system being measured, so the attenuator conductance was subtracted from both calibrations and measurements. From the dimensions of the attenuator we calculate a characteristic impedance for the attenuator $Z_a = 170$ MPa s/m$^3$. The actual impedance is likely to differ a little, because the spacing wires are not perfectly straight. We therefore determine the value of $Z_a$ by measuring known loads with high impedances, which in this case were the first maxima in the impedance spectra of open stainless-steel cylinders. This gives a value of 155 MPa s/m$^3$. The attenuation of the travelling wave in the narrow space between the cones is large, so the attenuator output is expected to behave like a pure resistance—another semi-infinite waveguide. This assumption requires that the wave travelling from the output of the attenuator, back to the input of the attenuator and then back to the output may be neglected. To check this, the upstream microphone was used to measure the pressure waveform at the input of the attenuator during calibrations (where the load was 8·5 MPa s/m$^3$, independent of frequency) and measurements of cylinders with strong resonances (where $\mathbf{Z}(f)$ varied from 20 kPa s/m$^3$ to 200 MPa s/m$^3$). No measurable difference was observed, so the attenuation in this wave is sufficient to permit its neglect in the measurements.

Thus, the subtraction of the attenuator conductance should allow precise measurements less than about 100 MPa s/m$^3$. This exceeds the maxima in $\mathbf{Z}(f)$ of the flute where, in any case, the minima are of much greater interest. (For measurements on other wind instrument families, where the maxima are of greater interest than the minima, the same technique may be used with a higher value output impedance.)

### 3.4. SIMULATING THE RADIATION IMPEDANCE AT THE EMBOUCHURE HOLE

As expressed in equation (1), the response of the flute to the driving force provided by an oscillating air jet is inversely proportional to an impedance $\mathbf{Z}(f)$ that is the sum of the input impedance of the flute tube as measured on a plane at the exterior entry to the embouchure hole plus the radiation impedance of the embouchure hole measured on this same plane [3]. Because it is relatively straightforward to estimate the value of this radiation impedance, our approach involves replacing it by the impedance of an appropriate short length of tube fixed to the exterior of the embouchure hole, and then measuring the input impedance of the flute at the outer end of this tube.

The impedance head used here (Figure 2(c)) is designed to allow a measurement that, without any further corrections, approximates the input impedance of the flute under typical playing conditions. The dimensions are several millimetres and the frequencies of interest ($<3$ kHz) have wavelengths longer than 100 mm, so the plane wave approximation should be good at low frequencies and acceptable at high frequencies.

As discussed above, the impedance head includes a short length of tube designed to compensate for the effects at the embouchure hole under playing conditions. The radiation impedance of a baffled pipe is approximately equal to that of an ideal tube with a length about 0·85$a$, where $a$ is its radius. In the absence of the player's face, the radiation load at the embouchure would therefore be approximately that of a pipe with the same area and length 0·85$a$. In practice, the equivalent length is longer because the player's lower lip occludes part of the hole and his face acts as a baffle, reducing the solid angle available for radiation.

Let the player's lips leave open a fraction $g$ of the hole, and assume that the face leaves open a fraction $h$ of the solid angle available for radiation. The radiation impedance would then be

$$Z_{rad} = \frac{0{\cdot}85a\,\mathrm{j}\omega\rho}{\sqrt{g}\,hS_{emb}}, \tag{2}$$

where $S_{emb}$ is the area of the embouchure hole (about 90 mm$^2$). We use a pipe with smaller cross-section than the embouchure hole so that positioning of the impedance head is not critical. The impedance $Z_c$ of the coupling section of the pipe that is to simulate the radiation load, is

$$Z_c = \frac{\mathrm{j}\omega\rho L_c}{S_c} = Z_{rad} = \frac{0{\cdot}85\, a\, \mathrm{j}\omega\rho}{\sqrt{g}\, h S_{emb}}, \tag{3}$$

where $L_c$ is the length and $S_c$ the cross-sectional area. We use a value of 6 mm for $L_c$. This is chosen empirically so that, at the corresponding temperatures and humidities, the impedance minima correspond to the played notes. Therefore,

$$h\sqrt{g} = \frac{0{\cdot}85 a}{L_c}\frac{S_c}{S_{emb}} = 0{\cdot}3, \tag{4}$$

which approximately corresponds to $g = h = 0{\cdot}5$. The effect of changing $g$ and $h$ can therefore be simulated by changing $L_c$. Further, this can be done theoretically using the transfer matrix for a cylindrical pipe where, for this length, losses may reasonably be neglected [3].

### 3.5. MEASURING $Z(f)$

The impedance head, including the microphones and attenuator, were connected to the flute via the adaptor shown in Figure 2(c), which is acoustically equivalent to a cylinder. The adaptor has the same radius as the calibration cylinder and is 6·0 mm long. This load may be removed by calculation from the resultant measurement, but its dimensions were chosen so as to give an impedance equivalent to that of the radiation impedance of the embouchure hole under typical playing conditions (see the previous section and reference [17] for more details). A thin gasket made of flexible casting compound (Wacker Elastosil M4503, Barnes, Sydney) is used to seal the measurement head to the flute.

The reference signal is applied to the flute and the Fourier components $\mathbf{p}_{meas}(f)$ are then measured. The total admittance $\mathbf{p}_{cal}(f)/(\mathbf{p}_{meas}(f)\cdot R_{ref})$ is calculated, and the attenuator conductance is subtracted to give the flute admittance. When measurements are finished, the spectrometer is reconnected to the reference impedance and a measurement is taken to compare with the calibration.

Background noise in the laboratory, although rather quiet, could affect measurement of the weak pressure signals at the impedance minima. For this reason, the musical instrument, or other load to be measured, was placed inside a rigid box, which was lined with acoustically absorbent materials to minimize resonances of the box. The operator (who is also a flutist) passed his arms through rigid "sleeves" on the box to finger the instrument and triggered measurements with a foot control.

### 3.6. SOUND RECORDINGS

Because sound spectra depend very strongly on parameters determined by the player, by the radiation pattern of the flute, and by recording conditions, sound spectra recorded for human players can only be regarded as examples. The obtaining of examples typical of playing by a flutist under realistic, musically comfortable conditions are not consistent with obtaining exactly repeatable spectra. Sound spectra were measured at a distance of about

1 m directly in front of the player in a room that was moderately quiet and with little reverberation. The player, Geoffrey Collins, is one of Australia's foremost flutists. The notes were written out in traditional musical notation, with dynamic markings *ff*, *mf* and *p*. Where the fingerings were not standard, they were indicated using a fingering diagram of the sort shown in Figure 1. For the classical flute, he used the McGee instrument described above. For some of the notes on the modern flutes, he used the Pearl flutes discussed above. For the rest of the notes and multiphonics on the modern flute he used one of his own flutes, which is a hand-made Brannen–Cooper flute with a Nagahara head.

## 4. RESULTS AND DISCUSSION

Before discussing the impedances of flutes with their complicated geometry, we present a measurement of the acoustic impedance measured on a simple cylindrical tube made of stainless steel (Figure 3). The error bars in frequency are smaller than the points. Where the slope is least, the points appear to make a continuous line and so the error bars in $Z$, typically a few tenths of a dB, are omitted for clarity. For convenience, these impedances are plotted on a logarithmic scale analogous to the decibel scale for pressure, using the relation

$$Z(\text{dB}) = 20 \log_{10} \frac{Z}{Z_0}, \quad \text{where } Z_0 = 1\,\Omega\,(\text{SI}) = 1 \text{ Pa s/m}^3. \tag{5}$$

As for the flute, the measurements are made only above 200 Hz, which is the cut-off frequency of our horn. The mean value of the impedance is determined by the characteristic impedance (here 8·5 MPa s/m$^3$). The minima and maxima in $Z(f)$ occur approximately at frequencies of

$$f_{min} = n \frac{c}{2L} \quad \text{and} \quad f_{max} = \left(n - \frac{1}{2}\right) \frac{c}{2L}, \tag{6}$$

where $n$ is a positive integer, $c$ is the speed of sound and $L$ is the length of the pipe. (More precisely, $L$ includes a small end-correction with a weak dependence on frequency.) The size of the maxima and minima gradually decreases with frequency because wall losses become successively more important. The expression for $\mathbf{Z}(f)$ is given by reference [3].

Figure 4 shows $\mathbf{Z}(f)$, the acoustic impedance of a modern flute with a C4 foot for the fingering for the note C4. The magnitude ($Z$) and the phase $\phi$ are shown separately. To zeroth order, a flute is a cylindrical pipe and so, with all its keys closed, its impedance spectrum resembles qualitatively that of the simple pipe. Because the input is open to the atmosphere, the air jet excites modes with large air flow and modest variations from atmospheric pressure. Thus, the operating frequencies are the minima of $Z(\equiv |\mathbf{p}/\mathbf{u}|)$. For the configuration with all tone holes closed, these minima are in the harmonic series (equation (6)). For the flute, the maxima are irrelevant. One could also say that, to zeroth order, a clarinet with its bell removed is also a cylindrical pipe, and so its input impedance should also resemble that of a simple pipe; and so it does (data not shown). The difference is that, because its input is closed by the reed, mouthpiece and the player's mouth, it operates near the maxima in $\mathbf{Z}(f)$. Its sound spectrum has predominantly the odd harmonics (the second equation (6)) and its fundamental is approximately an octave lower than that of the flute. (The first maximum is not shown in Figure 3 because it lies below the frequency range of the measurements, but the reader may readily extrapolate it.) The length of pipe used for Figure 3 is comparable with that of the air column of a flute or a clarinet.

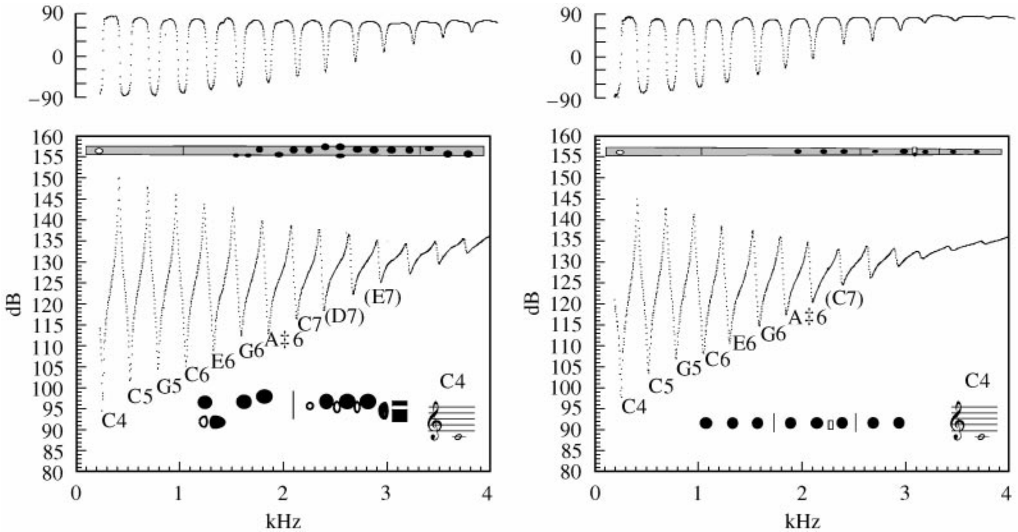


Figure 4. $\mathbf{Z}(f)$ for a modern flute (left) and a classical flute (right) for the note C4.

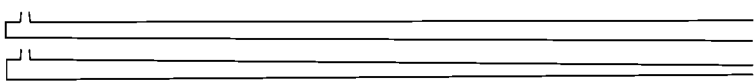

Figure 5. A simplified acoustic model of the flutes in the configuration to play their lowest notes.

The most important geometrical difference between a simple cylinder and the modern flute configured for C4 is that the flute is excited, and therefore here measured, at the embouchure hole, whose centre is 17·5 mm from the closed end. This hole is connected to the bore via a transverse, tapered hole. This embouchure hole has a considerably smaller cross-section than does the instrument. Further, the closed tone holes each have a little volume added to the side of the bore. Finally, the head joint of the modern flute is not cylindrical but tapered to a smaller diameter near the embouchure hole (see Figures 1 and 5). We shall begin by discussing the qualitative shape of $\mathbf{Z}(f)$.

The shapes of $\mathbf{Z}(f)$ for the flutes have many similarities to that of a plot of $\mathbf{Z}(f)$ for a simple cylindrical pipe. The main differences are (1) the maxima do not fall half-way between the minima, as they do for the simple cylinder; (2) the envelope of $\mathbf{Z}(f)$ rises gradually at frequencies above about 2 kHz; and (3) the maxima and minima become smaller more rapidly with frequency than they do for a cylinder of the same size. All of these are explained by the geometry shown in Figure 5. The "chimney"—the relatively narrow hole at the embouchure—has a larger characteristic impedance does than the flute tube itself. This has greatest effect on the impedance minima, where there is largest flow in the chimney. It has the effect of making the minima occur at lower frequencies than they otherwise would (which makes them asymmetrically distributed with respect to the maxima) and reduces the depth of the minima at higher frequencies. Further, the chimney tube plus the closed end of the flute tube together constitute a Helmholtz resonator, this is in parallel with the rest of the instrument. Its impedance rises with frequency, and it has a resonance at several kHz, which then dominates the $\mathbf{Z}(f)$ curve. It also decreases the frequencies at which the minima occur.

With the fingering shown in Figure 4, the flute can be made to play a series of notes at frequencies near to the first several minima in this spectrum, by adjusting the speed and geometry of the jet. As the minima become shallower, and as the required jet speed increases, the notes in this series become harder to play. The minima are almost exactly harmonic and so the playable series is C4, C5, G5, C6, E6, G6, A‡6, C7, where ‡ means a half sharp: the seventh note in the series is between A6 and A#6, slightly closer to the latter. Normally, this fingering is used only for C4, and other fingerings are used for the higher notes.

The vibration of the jet is periodic but not sinusoidal. Hence, when C4 is played with this fingering, several harmonics are present in the sound, and these have spectral components at frequencies corresponding to the minima, as shown in the figures in the database in JSV+ [18]. The harmonicity of the series of minima is important in determining the stability of the vibration régime, as discussed above.

The displacement of the embouchure hole from the end of the flute has important effects on tuning. As for as input impedance is concerned, the flute as a whole may be considered as a short (17·5 mm) closed tube (the closed part of the bore) in parallel with the rest of the bore. These two relatively large diameter tubes are in series with the relatively narrow embouchure hole and chimney (Figure 5). This is responsible for the gradual rise of $\mathbf{Z}(f)$ at high frequencies.

Figure 6 shows a configuration that illustrates several features relating the fingering pattern to the note produced. The first fingering shown (i) is the standard fingering for the notes G4 and G5. The eight lowest tone holes are open and the effective length of the flute is approximately half the wavelength required for G4 (frequency $f_0 = 392$ Hz). The flute readily plays G4 in this configuration. By using a higher blowing pressure and changing the embouchure, it will also play $2f_0$ (G5), $3f_0$ (D6), $4f_0$ (G6) and $5f_0$ (B6). The pressure standing waves for the first three of these are shown in simplified form. In practice, a single tone hole is not an acoustic "short circuit" and the standing wave extends a little way beyond the first open hole. It is difficult to play D6 softly in the configuration shown in (i). The standard fingering for D6 opens a hole (indicated by the arrow) at about two-thirds of the effective length, as shown in (ii). This favours a pressure node and facilitates playing D6, and makes it impossible to play G5 or G4. The fingering (ii) does, however, allow the playing of the note

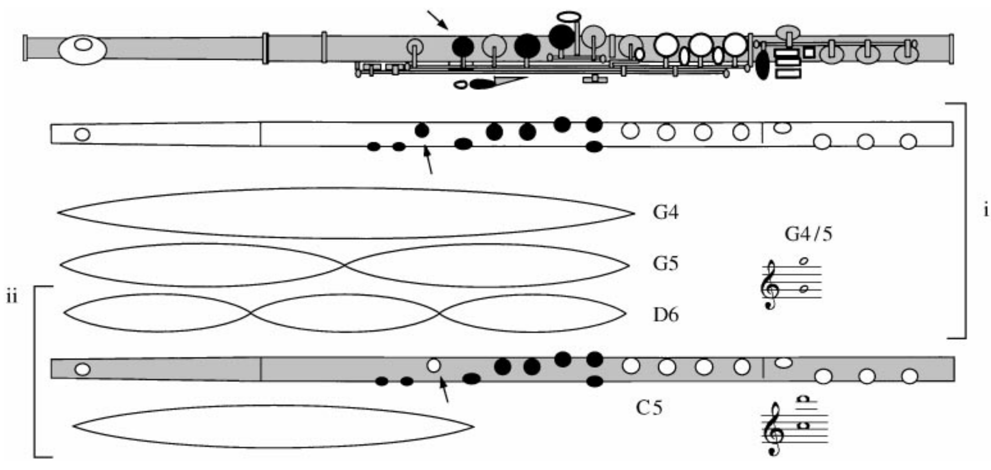


Figure 6. The fingering and acoustic schematics for (i) G4, G5 or D6 and (ii) C5 or D6 or the multiphonic C5&D6.

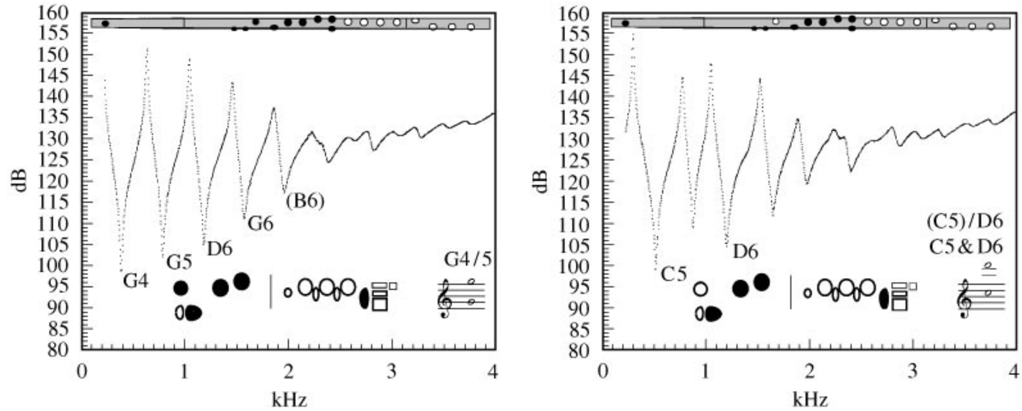


Figure 7. The input impedance $\mathbf{Z}(f)$ for the cases described in Figure 6.

C5, with a half wavelength that is 3/4 the half wavelength of G4, as shown in the sketch. (ii) is also a multiphonic fingering, because it is possible to play C5 and D6 at the same time using this fingering. With this fingering, one can blow softly and produce C5, gradually increase the blowing pressure until the multiphonic sounds, then continue increasing the pressure until the C5 disappears leaving a pure-sounding D6 (whose spectrum may nevertheless show a very weak signal at C5). Fingering (i) is not a multiphonic fingering: if one played G4 and G5 at the same time one would be playing G4, because the harmonics of G5 are a subset of those of G4. Multiphonics are discussed in more detail below.

Figure 7 shows the acoustic impedance of the two configurations shown in Figure 6. Note that the impedance curve for the fingering for G4/5 (1) has minima at approximately $f_0$ ($= 390$ Hz), $2f_0$, $3f_0$, $4f_0$, $5f_0$ and $6f_0$ for the note G4. The even members of this series are of course the first three harmonics of G5. The opening of the extra hole at about two-thirds of the effective length (2) displaces the first two minima so that the minima are no longer in a simple harmonic series. The first minimum is at the frequency of C5, the third is (still) at the frequency for D6. With this fingering, low blowing pressure and an appropriate embouchure, the flute will play C5 and D6 at the same time. Pressure spectra and sound files are in the JSV+ database [18].

## 4.1. MULTIPHONICS

Some of the unusual fingerings listed in reference [9] produce sounds termed multiphonics, in which a chord-like effect is heard. In these cases, the impedance curve has two or more deep minima, the frequencies $f_1$ and $f_2$ of which are not close to being in simple small-integer ratio. If the player uses a wide air jet, then it is possible to excite both resonances simultaneously, giving two notes, but the effect does not stop there. The two basic tones react back on the air jet which, because its interaction with the flute tube at the embouchure lip is non-linear, generates excitations at frequencies that are multiple sum or difference frequencies $nf_1 \pm mf_2$ where $n$ and $m$ are small integers. This can happen only if the two basic frequencies are not very close to being in an harmonic (small integer) relationship [19]. A simple multiphonic is shown in Figure 7, at the right. Further examples, including sound spectra and sound files, are given in the JSV+ database [18].

### 4.2. GENERAL OBSERVATIONS ABOUT IMPEDANCE SPECTRA

In all the flute + foot combinations we studied, these generalizations hold:

(1) With all of the holes closed, the impedance spectrum has about 10 sharp minima. Their frequencies are close to harmonic ratios (see Figure 4). These minima correspond to standing waves or resonances in the flute when played.

The depth and sharpness of the minima in $\mathbf{Z}(f)$ decreases more-or-less smoothly with frequency. The resonances corresponding to these minima in $\mathbf{Z}(f)$ can be played by overblowing, but this becomes more difficult as the minima in $\mathbf{Z}(f)$ become shallower and wider. For instance, on the Boehm flute, the eighth resonance is substantially shallower than the seventh; and one notices that the eighth harmonic is substantially harder to play than the first 7.

Further, harmonic minima in the $\mathbf{Z}(f)$ spectrum are well aligned in frequency with harmonic peaks in the sound spectrum.

(2) As holes are opened successively from the foot, the number of sharp, nearly harmonic resonances decreases (data in JSV+ [18]). At frequencies above about 2 or 3 kHz, $\mathbf{Z}(f)$ shows several very shallow minima, which are not in general harmonics of the low-frequency minima. This is explained due to the filtering effect of the tone-hole lattice downstream from the lowest closed hole, as discussed above. Further, some of the minima in the low-frequency end become slightly displaced from harmonic values.

The effect of this on the playing of these fingerings is that (1) it is harder to overblow the higher members of a harmonic series, and (2) the low notes played with these fingerings have less power in the high harmonics, even when the player attempts to produce a uniform sound (compare the sound and impedance spectra in JSV+). All else being equal, deeper minima in $\mathbf{Z}(f)$, which are harmonics of the fundamental, correspond to stronger harmonics in the sound spectrum of the fundamental note. This is most clearly shown when alternative fingerings give different relative depths for different harmonics. For a striking example, see the sound spectra for different fingerings for A4 on the Boehm flute. For a more subtle example, see the variant fingerings for A#4/5. These spectra, and the sound files, are in the database in JSV+ [18].

### 4.3. COMPARING THE BOEHM FLUTE WITH THE CLASSICAL FLUTE

(1) Even with all tone holes closed, the classical flutes have $\mathbf{Z}(f)$ spectra in which, while the low resonances are roughly as deep and as sharp as those on the Boehm flute, the higher resonances are significantly less deep and sharp (Figure 4).

(2) The difference between "all holes closed" and "*n* holes open" is greater on the classical flutes than on the Boehm flute. One might informally say that, because the Boehm flute has more and bigger tone holes, opening up the holes on a Boehm flute is closer to "sawing the end off", whereas the smaller and more widely spaced holes on the conical flutes have a greater effect when they are open downstream. This gives rise to the more mellow and less homogeneous timbre of the classical flute, as discussed above.

### 4.4. COMPARING THE CLASSICAL FLUTES WITH D FOOT AND WITH THE C FEET

For the C foot flute with all tone holes closed, $\mathbf{Z}(f)$ has 9 or 10 sharp, roughly harmonic minima starting with C4 (Figure 4). With the D foot in place there are also 9 or 10 sharp, roughly harmonic minima in $\mathbf{Z}(f)$, but they are the harmonics of D4 rather than C4. When the two flutes are compared on fingerings for the same note, then the longer flute, which has more tone holes open, show to a greater extent the effects (discussed above) of open tone holes. The effects of the more open C foot are rather subtle on notes from D4 and above. See the examples in JSV+ [18].

## 4.5. TECHNICAL OBSERVATIONS REGARDING THE PERFORMANCE OF FLUTES

The design of a flute is a compromise. For each note over a range of three and a half octaves, the instrument should produce a $\mathbf{Z}(f)$ with a narrow, deep minimum at the fundamental frequency of that note. Further, for the notes in the lower part of the range, the harmonics of the note should also coincide with minima in $\mathbf{Z}(f)$. Additionally, there should be no other deep minima near that of the desired fundamental. For the highest notes, this is difficult to achieve because of the shallowness of the minima at high frequency (as shown in the $\mathbf{Z}(f)$ for the notes in the fourth octave in the JSV+ database). As a result of this and of high speed of the air jet required, the highest notes are difficult to play quietly. Yet another complication is that the instrument should enable the player to make a smooth transition between a note and almost any other. An example of how this may be difficult, and how it is overcome, is shown in Figure 8.

The note E6 can in principle be played using the third impedance minimum in the configuration used for A4 and A5 (data in JSV+ [18]), but the third minimum is not very deep and it is easy for the player to misjudge the blowing pressure (particularly when playing softly) and to sound A5 instead. So instead, a cross-fingering is normally used. The same fingering produces one of two different acoustic configurations according to the design of the flute. A "split E mechanism" has a clutch which closes one extra tone hole in this fingering, and such a mechanism is available as an option on many flutes, where it facilitates playing E6 softly and slurs (uninterrupted transitions) between A5 and E6. The E6 fingering can be thought of as a variant of the fingering for A4, but with three or four downstream holes closed (see Figure 8). The flute does indeed play A4 and a note a little lower in pitch than A5 (called A5 $-\delta$ on the graph) in this configuration. It may also be considered as the fingering for E4, but with one or two register holes open, about one-quarter of the way along, to facilitate the fourth harmonic. Compare the $\mathbf{Z}(f)$ for E6 without and with this mechanism. Without the split E, the third labelled minimum is considerably less deep than the second, and hardly deeper than that of the fourth. Thus, skill

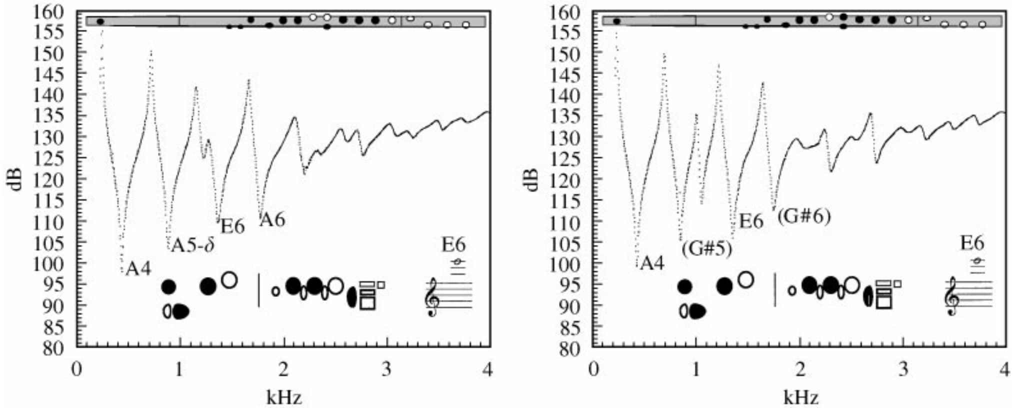


Figure 8. $\mathbf{Z}(f)$ for the note E6, without (left) and with a "split E mechanism" (right).

and reliable judgement of blowing pressure is required to sound the E6 rather than A6 or A5 $-\ \delta$, particularly in soft playing or in fast, slurred, leaping passages. With the split E mechanism, the third minimum (E6) is deeper and the fourth (G#6) less deep. Further, the second minimum (G#6) is both less deep and further out of tune, so it no longer has a nearly harmonic relation to the fourth minimum. It is therefore much less likely to sound A5, so the E6 is more stable in soft playing. There are further complications when slurs from E6 to A6 are required, and these can be overcome with use of another downstream key (data not shown).

### 4.6. SOUND FILES

When a variant fingering gives a deeper minimum in $\mathbf{Z}(f)$, the sound spectrum of a note involving that minimum usually has a larger component at that frequency. However, it should be noted that, while the impedance spectrum depends almost entirely on the flute, the sound spectra depend on many things. Most importantly, they depend on the player and on various parameters under his/her control. They depend on the way in which the note is played (loudness and timbre), as well as on the response of the room and on the relative position of the microphone and the instrument. They also depend on parameters used in calculating the spectrum such as the sample window used and the relative phase of the sound vibration and the sampling. There is therefore no "standard" sound spectrum for any note. It is also worth noting that the sound spectrum is only one of the things that determine the timbre of the note, so that one cannot easily get an idea of the sound of the note from just the spectrum (particularly one also needs to know the starting and finishing transients and the way the spectrum is affected by the vibrato). The sound files and sound spectra are recordings of an eminent professional flutist, using the flutes described in the Materials and Methods section.

### 4.7. FURTHER DATA ANALYSIS AND DISCUSSION

Particularly in higher registers, there are many technical problems such as those discussed in relation to Figure 8, and in many cases they can be explained in terms of the measured $\mathbf{Z}(f)$, or by comparing $\mathbf{Z}(f)$ with the sound spectrum $p(f)$ for the note. This is perhaps of greater interest to flute players and makers than to acousticians. When combinations of pairs of notes are considered, and when different fingerings and flute designs are possible for each note, the number of possibilities is very large. For this reason, we maintain an interactive web page with a diagrammatic fingering chart that allows flutists and makers to discuss such problems and to suggest further investigations beyond the several hundred graphs available in JSV+ [18]. The web site (www.phys.unsw.edu.au/music/flute) is used for extensions and additions to the database held by JSV+ .

## ACKNOWLEDGMENTS

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