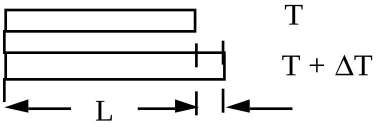
Definition of T:

T is equal in any 2 bodies at thermal equilibrium.

Thermal Expansion



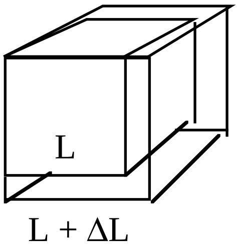
Define

$$\frac{\Delta L}{L} = \alpha \Delta T$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

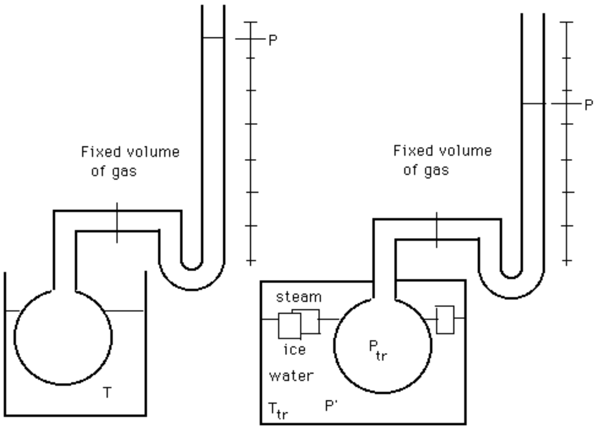
α is **coefficient of linear expansion**

β is **coefficient of volume expansion**



$$\Delta V = (L + \Delta L)^3 - L^3$$
$$= \;.....$$
$$\approx V.3\alpha\Delta T$$
$$\therefore \;\; \beta \approx 3\,\alpha$$

Formal definition of T (Ideal gas thermometer)



$$T = T_{tr}.\lim_{\rho_{tr}\to 0}\left(\frac{P}{P_{tr}}\right)_V \qquad \text{where } T_{tr} = 273.16\text{ K}$$

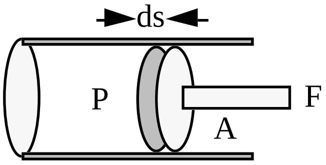
Heat Capacity: $C = \dfrac{\Delta Q}{\Delta T}$ *extensive quantity (for a body)*

Specific Heat: $c = \dfrac{\Delta Q}{M\Delta T}$ *intensive quantity (of a substance)*

Latent Heat: heat per unit mass for change of phase (at constant T).

Work done against pressure P

$$dW = \mathbf{F.ds}$$
$$= PA\,ds$$
$$= PdV$$

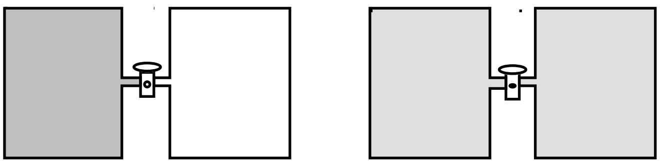


1st Law $dU = dQ - dW$ where U is a state function

Isobaric P const Isochoric V constant

Adiabatic Process: $\Delta Q = 0$ (fast or insulated)

Free expansion: open tap



No work done, Experimentally, find $\Delta Q = 0$

$$\therefore \;\; \Delta U = 0 \qquad \therefore \;\; U = U(T) \quad \text{only}$$

Calculating work for gases:

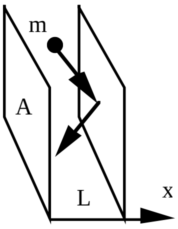
Isobaric: $W = \int P\,dV = P\Delta V$

Isothermal: $W = \int P\,dV = \int \dfrac{nRT}{V}dV = nRT\,ln\dfrac{V_f}{V_i}$

Adiabatic: $PV^\gamma = \text{constant}$ *revise derivation*

$$P = \frac{P_iV_i^\gamma}{V^\gamma} \qquad W = \int P\,dV = P_iV_i^\gamma\int\frac{dV}{V^\gamma}$$
$$= (\gamma - 1)P_iV_i^\gamma\left(\frac{1}{V_i^{\gamma-1}} - \frac{1}{V_f^{\gamma-1}}\right)$$

Kinetic Theory



Δ momentum = $2mv_x$
collide every $2L/v_x$

$$\left|\overline{F}\right| = \left|\frac{\Delta p}{\Delta t}\right| = \frac{2mv_x}{2L/v_x}$$
$$= \frac{mv_x^2}{L}$$

$$F_{all} = PA = \frac{Nm\,\overline{v_x^2}}{L}$$

$v^2 = v_x^2 + v_y^2 + v_z^2$; random motion ⇒

$$\overline{v_x^2} = \frac{1}{3}\,\overline{v^2}\text{, so:}$$

$$PAL = PV = \frac{N}{3}m\,\overline{v^2} \qquad P = \frac{1}{3}\rho\,\overline{v^2}$$

$$\frac{1}{2}m\,\overline{v^2} = \overline{\varepsilon} \;\text{ and }\; PV = NkT$$

$$\therefore \quad \overline{\varepsilon} = \frac{1}{2}m\,\overline{v^2} = \frac{3}{2}\frac{PV}{N} = \frac{3}{2}kT$$

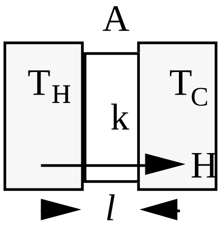
i.e. $\frac{1}{2}kT$ per **degree of freedom**

$$\frac{1}{2}m\,\overline{v^2} = \frac{3}{2}kT$$

$$v_{r.m.s.} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

density in field → Boltzmann Dist^n $\quad c = c_o\,e^{-E/kT}$

Heat conduction



$$H \equiv kA\frac{T_H - T_C}{l}$$

Planck's radiation law:

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\,\frac{1}{e^{hc/\lambda kT} - 1}$$

$$\frac{\partial S}{\partial \lambda} = 0 \rightarrow \qquad \lambda_{max} = \frac{(2898\ \mu m.K)}{T}$$

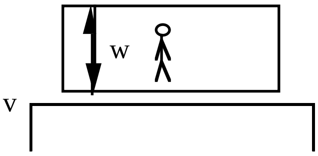
$\int d\lambda \rightarrow \infty$ **Black body radiation**

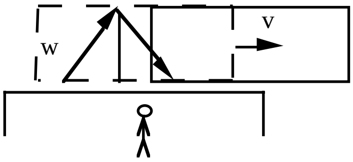
$$H_{nett} = e\sigma A(T^4 - T_{rad}^4)$$

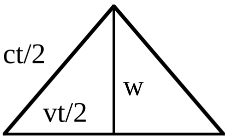
For both: in **steady state** $\left(\frac{d}{dt} = 0\right)$, $H_{in} = H_{out}$

Principle of special relativity:

Mechanics ***and Electromagnetism*** are the same in inertial frames.







$$t' = 2w/c \qquad t = 2\,\frac{\sqrt{w^2 + v^2(t/2)^2}}{c}$$

$$c^2t^2 = 4\left(\frac{c^2t'^2}{4} + \frac{v^2t^2}{4}\right)$$

$$\therefore \quad t' = t.\sqrt{1 - \frac{v^2}{c^2}}$$

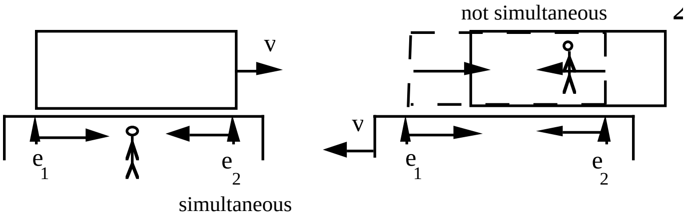
$t' = t/\gamma$ (*but from the vehicle...*)

Proper time t_o in the rest frame.

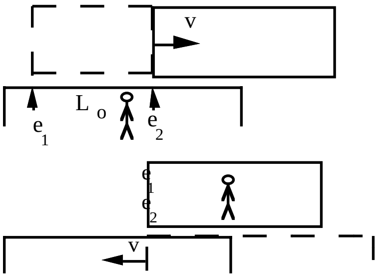
$$\frac{\Delta t}{\Delta t_o} = \gamma \geq 1$$

in all other frames t is faster than proper time

Relativity of simultaneity and time-order



Proper length L_o is length in the rest frame



$L_o = v\Delta t$ $\qquad L' = v\Delta t' = v\Delta t_o$

$$\frac{L'}{L_o} = \frac{v\Delta t_o}{v\Delta t} = \frac{1}{\gamma} \leq 1$$

$x' = Ax + Bt$
$y' = y \qquad z' = z$
$t' = Ct + Dx$

At $x' = 0$, $x = vt$. $\qquad \therefore A = -B/v$

At $t = 0$, $x' = \gamma x = Ax$. $\qquad \therefore x' = \gamma(x - vt)$

At $x = 0$, $t' = \gamma t$ $\qquad \therefore C = \gamma$

At $x' = 0$, $t = \gamma t'$ & $x = vt$

$t/\gamma = \gamma t - Dvt \qquad \therefore 1 = \gamma^2 - D\gamma v$

$$\therefore D = \frac{\gamma^2 - 1}{\gamma v} \quad \therefore$$

Lorentz transformations $v << c \Rightarrow$ Galileo

$x' = \gamma(x - vt)$
$y' = y \qquad z' = z$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$$u_x' = \frac{dx'}{dt'} = ... = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

not additive

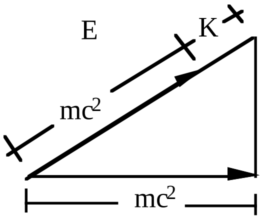
Relativistic Mechanics

Define $p \equiv \gamma mv$. Conserved in both frames.

$$K = \int_{v=0} dW = ... = mc^2(\gamma - 1)$$

$\gamma mc^2 = mc^2 + K \qquad = $ proper energy + K

Energy and momentum



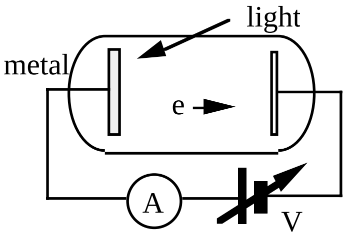
$p = \gamma m v$

$\therefore \quad p^2c^2 + (mc^2)^2$

$= \ \ldots\ldots \ = \ E^2$

$E = \sqrt{(pc)^2 + (mc^2)^2}$

The photoelectric effect



Experiment:
$eV_o = hf - hf_o$

$eV_{stop} = K$ $\quad hf = hf_o + K \quad$ hf_o = work function

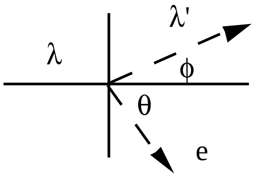
Photon energy $\quad E = hf$

$p = \frac{E}{c}$ $\quad$ *classical EM result*

for photon $\quad \therefore \quad p = \frac{hf}{c} = \frac{h}{\lambda}$

Compton scattering

X-ray photon from e



$$hf = hf' + mc^2(\gamma - 1)$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1)$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + mv\gamma\cos\theta$$

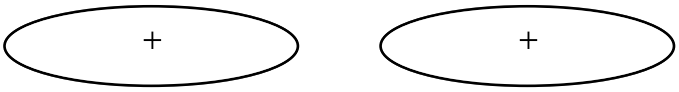
$$0 = \frac{h}{\lambda'}\sin\phi - mv\gamma\sin\theta$$

eliminate v,θ → $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$

de Broglie

$p = h/\lambda$ for massive particles

Bohr - de Broglie atom



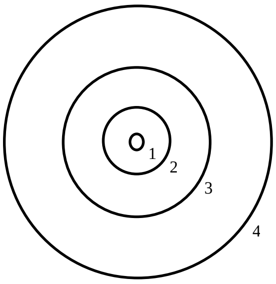
$2\pi r = n\lambda$
constructive interference

$2\pi r \neq n\lambda$
destructive interference

Constructive interference, $F_{elec} = ma = mv^2/r \rightarrow$

$$E = -\left(\frac{me^4}{8\varepsilon_o^2 h^2}\right)\frac{1}{n^2}$$

Hydrogen spectrum



Quantum numbers n.

e from n_2 to n_i →

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = - R.\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

R Rydberg's const

Photon: $\begin{pmatrix}\text{probability of}\\ \text{absorption}\end{pmatrix} \propto \begin{pmatrix}\text{amplitude}\\ \text{of EM field}\end{pmatrix}^2$

Massive particles $\begin{pmatrix}\text{probability of}\\ \text{'finding' it}\end{pmatrix} \propto \Psi\Psi^*$

Heisenberg's Uncertainty Principle

(Finite) Fourier transform: finite length sample → limited precision in f. $\delta f \geq 1/2\pi T$

$E = hf. \quad \therefore \quad \delta E = h\delta f \gtrsim h/2\pi\delta t$

$\delta E.\delta t \geq h/2\pi$

Similarly: $\delta(1/\lambda) \gtrsim 1/2\pi\delta x \rightarrow \delta p.\delta x \geq h/2\pi$

Examples: Heisenberg's microscope,
'Orbit' of the H atom (in ground state),
Virtual particles, especially Yukawa's pion,
Diffraction through a slit,
Young's experiment with localisation

Quantum tunnelling

$$\frac{h^2}{8\pi^2 m}\frac{\partial^2\Psi}{\partial x^2} + (E - V)\Psi = 0$$

if $K > 0$, Ψ has wave solutions (cos or sin x)

if $K < 0$ Ψ has exponential decay

Examples α decay, tunnel diode,
Scanning Tunnelling Electron Microscope