PHYS1231 end of year test 2004   Answers

Question 1 (16 marks)

i) Temperature is that quantity that is equal in two bodies in thermal equilibrium. (1)

ii) The heat capacity C of a body is the heat required to raise its temperature by unit temperature, or

Heat capacity $C = \dfrac{Q}{\Delta T}$ where Q is the heat added and ΔT is the temperature change resulting, or equivalent. (1)

iii) The specific heat c of a substance is the heat required to raise the temperature of unit mass (*or one mole*) of the the substance by unit temperature, or (1)

Specific heat $c = \dfrac{Q}{m\Delta T}$ where Q is the heat added, m is the mass (*or , for the molar heat capacity, the number of moles. Either to get a mark*) and ΔT is the temperature change resulting, or equivalent.

iv) The tank expands proportionally in all directions, so the increase in its volume is given by

$$\Delta V_{tank} = \beta_{steel} V_{20} = 3\alpha_{steel} V_{20} \Delta T.$$

The fuel expands by $\Delta V_{fuel} = \beta_{fuel} V_{20} \Delta T$.

Neglecting effects due to pressure increase (these will be small) the amount overflowing is

$$\Delta V = \Delta V_{fuel} - \Delta V_{tank} = \beta_{fuel} V_{20} \Delta T - 3\alpha_{steel} V_{20}$$

$$= (\beta_{fuel} - 3\alpha_{steel}) V_{20} \Delta T = (1.40\ 10^{-3}\ °C^{-1} - 3*2.3 \times 10^{-5}\ °C^{-1})(35\ l)25°C$$

$= 1.1$ litres. (6)

(If you neglect tank expansion, the answer is 1.2 litres. A steel tank gives an answer between these.)

v) Let the initial volume of gas contain n moles. $P_A V_0 = nRT_0$, where R is the gas constant.

At the new thermal equilibrium, the pressure P satisfies $PV = PAh_0 = nRT_0$.

Mechanical equilibrium requires $mg = (P - P_A)A$ so $P = P_A + mg/A$

Combining these equations and rearranging: $h_0 = \dfrac{nRT_0}{PA} = \dfrac{P_A V_0}{PA} = \dfrac{P_A V_0}{(P_A + mg/A)A}$

$h_0 = \dfrac{P_A V_0}{P_A A + mg} = \dfrac{V_0/A}{1 + mg/P_A A}$ or an equivalent expression. (5)

vi) The pneumatic 'spring' is stiffer for a fast deformation. If it is slow enough, the process will be isothermal, so PV^1 = constant. If it is rapid enough, the process will be adiabatic so PV^γ = constant. γ, the ratio of specific heats, is greater than 1 so, for an adiabatic process a given change in V produces a larger change in P and so a larger force. (2)

Question 2 (30 marks)

i) a) 20% efficient, so rate of heat production = 4 times rate of mechanical power.

$$H = 4\frac{d}{dt}mgh = 4mg\frac{dh}{dt} = 4*(80\text{ kg})*(9.8\text{ms}^{-2})(0.55\text{ m.s}^{-1}) = 1.7\text{ kW} \qquad (3)$$

b) Rate of heat lost from skin by radiation – rate of heat absorbed by skin from surroundings

$$H_{rad} = A\sigma\left(e_oT_o{}^4 - e_sT_s{}^4\right)$$

$$= (1.8\text{ m}^2)(5.67\ 10^{-8}\text{ Wm}^{-2}\text{K}^{-4})*0.8*\left((307\text{ K})^4 - (303\text{ K})^4\right)$$

$$= 37\text{ W} \qquad (3)$$

c) Energy to evaporate water $= L.m_{sweat}$ where m_{sweat} is the mass of sweat evaporated

Rate of heat lost by evaporation $H = L\dfrac{dm_{sweat}}{dt}$

Rate of sweating = H/L = $(1.7\text{ kW})/(2.5\text{ MJ.kg}^{-1}) = 7.8\ 10^{-4}\text{ kg.s}^{-1}$
= 2.8 kg/hr = 2.8 litres/hour. (4)

d) Under most conditions (except perhaps very dry conditions) some sweat is lost because it falls off or is wiped away, so the rate of water loss would be higher than calculated here.

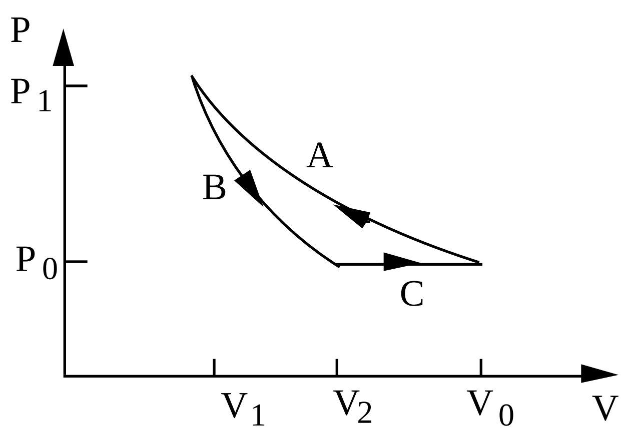
(If anyone says 'heat is lost by conduction as well, so this would be an overestimate, give them a mark too, even though it's not strictly an answer to the question.) (1)

e) Rate of heat produced – rate of heat lost $= H_{new} = H - 500\text{ W} = 1.2\text{ kW}$.

Definition of heat capacity: $Q \equiv mc\Delta T$ $\qquad Q = H_{new}t$

$$t = \frac{Q}{H_{new}} = \frac{mc\Delta T}{H_{new}} = \frac{(80\text{ kg})(4\text{ kJ.kg}^{-1}.°\text{C}^{-1})(2°\text{C})}{1.2\text{ kW}} = 500\text{ s} = 9\text{ minutes} \qquad (4)$$

ii)



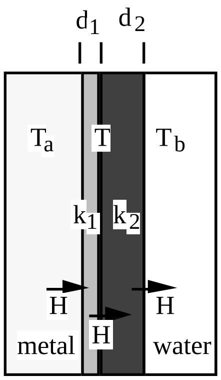
(4)

Step	Q	W	ΔU
A (isothermal compression)	-	-	0

B (adiabatic expansion)	0	+	-
C (isobaric)	+	+	+
Whole cycle (Σ)	-	-	0

(6)

iii)



In steady state, the temperature in the coatings is not changing. It follows that the heat flow H entering a given area of one side of either coating equals the heat flow H leaving the other side. Setting the heat flows through a given area A of the coatings to be equal and using the definition of the thermal conductivity k gives:

$$H \equiv k_1 A \frac{T_a - T}{d_1} = H \equiv k_2 A \frac{T - T_b}{d_2}$$

$$k_1 \frac{T_a - T}{d_1} = k_2 \frac{T - T_b}{d_2}$$

$$k_1 d_2 (T_a - T) = k_2 d_1 (T - T_b)$$

$$k_1 d_2 T_a + k_2 d_1 T_b = (k_2 d_1 + k_1 d_2) T$$

$$T = \frac{k_1 d_2 T_a + k_2 d_1 T_b}{k_2 d_1 + k_1 d_2} \qquad (5)$$

Question 3 (23 marks)

i) For the judges, using Pythagoras' theorem, the light beam travels $2\sqrt{(L/2)^2 + w_n^2}$. It travels at c, so the time taken by the car

$$t_{judge} = \frac{2\sqrt{(L/2)^2 + w_n^2}}{c} = \frac{\sqrt{L^2 + 4w_n^2}}{c}.$$ (simplification not required for marks) (3)

ii) For the judges, $v = L/t_{judge}$, so

$$v = \frac{cL}{\sqrt{L^2 + 4w_n^2}} = \frac{c}{\sqrt{1 + 4(w_n/L)^2}}$$ (2)

iii) $1 + 4(w_n/L)^2 = (c/v)^2$ so $L/w_n = \dfrac{2}{\sqrt{(c/v)^2 - 1}}$ (3)

iv) From Jane's point of view, the light beam travels at right angles to the car: it goes out to the right, strikes the reflector when the car is at the halfway point, and arrives at the detector at the finish. She is an inertial observer, so, according to the priniciple of special relativity, light travels at c with respect to her. Hence the time taken is

$$t_{Jane} = 2w_n/c$$ (2)

iv) Jane would conclude that, from her frame of reference, the course is actually shorter than L: it has suffered a relativistic length contraction due to its speed v, relative to her. (2)

v) The judges would conclude that Jane's clock is running slow. It has suffered a relativistic time factor due to its speed v, relative to them. (Note that the judges can measure the proper length L.) (2)

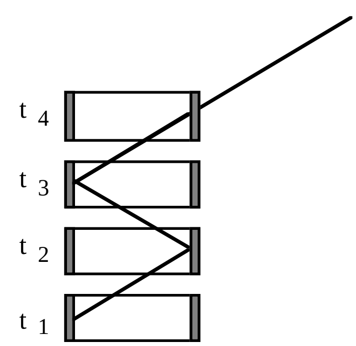
vi) $$t_{judge}/t_{Jane} = \frac{\sqrt{L^2 + 4w_n^2}}{2w_n} = \sqrt{(L/2w_n)^2 + 1} = \sqrt{\left(\frac{1}{\sqrt{(c/v)^2 - 1}}\right)^2 + 1} = \sqrt{\frac{1}{(c/v)^2 - 1} + 1}$$

$$= \sqrt{\frac{1 + ((c/v)^2 - 1)}{(c/v)^2 - 1}} = \sqrt{\frac{(c/v)^2}{(c/v)^2 - 1}} = \frac{1}{\sqrt{1 - (v/c)^2}}$$ $(= \gamma)$ (6)

As in (v), the judges observe Jane's clock to run slow: here we show that it runs slow by the factor γ, as predicted by special relativity.

vii) As described in (iv), from Jane's point of view, the laser beam travels at right angles to the car: so it should be pointed in this direction.

Viewed from the judges' frame of reference, both the light inside and outside the laser travel at the same angle to the car, but pointing the laser at right angles is still correct, as illustrated in the sketch at right.



(3)

Question 4 (13 marks)

i) (Iron is a very stable nucleus.) Because iron is stable, it takes energy to "pull its nucleus apart" into its components. Adding this energy (the binding energy E) to the components increases their mass. So the component nucleons are more massive by an amount $\delta m = E/c^2$ called the mass defect. (3)

ii) The proper time lifetime of the muons equals that measured at negligible speed, ie

$t_p = 2.2\ \mu s$. For the high speed muons, $t = 16\ \mu s$.

$$\frac{t'}{t} = \sqrt{1 - \frac{v^2}{c^2}} \qquad \left(\frac{t'}{t}\right)^2 = 1 - \frac{v^2}{c^2} \qquad 1 - \left(\frac{t'}{t}\right)^2 = \frac{v^2}{c^2}$$

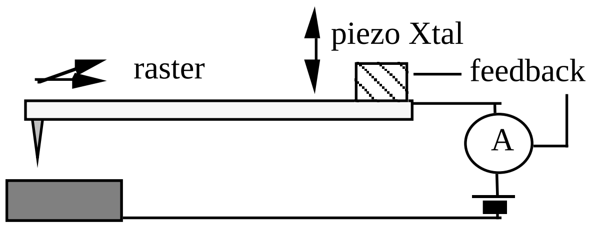
$$c\sqrt{1 - \left(\frac{t'}{t}\right)^2} = v = 0.99\ c$$ (5)

iii) Lots of examples are possible. Here is one. To a good approximation, the trajectory of a ball near the Earth's surface is described by its initial velocity and the forces (gravity and air resistance). For instance, a ball dropped falls vertically. Thus it agrees with **F** = m**a**. (2)

iv) Lots of examples are possible. Here is one. In an inertial frame, a pendulum swinging in a plane will always swing in the same plane, because its weight and the tension in the string are the only forces, and these forces are in the plane. However, on places at the Earth's surface (other than the equator), a pendulum is seen gradually to rotate. Thus Newton's laws don't apply to a frame in the Earth's surface, because it is rotating (and therefore accelerating) with respect to the distant stars. (3)

Question 5 (18 marks)

i) In the Scanning Tunnelling Electron Microscope (STEM), a very sharp point is held very close to the sample. The point scans over the sample (in a raster of x&y). An applied voltage allows electrons to tunnel across the gap (usually a vacuum)— a region in which they have negative kinetic energy (and so imaginary velociy). The electric current is measured, which is a measure of the rate of the tunnelling, and is a strong function of how close the tip is to the sample. (In the usual mode of operation, a feedback loop is applied in the z direction to keep the tunnelling current constant.) (*The level of detail expected is less than this: any comments about measuring the tunnelling current across the small gap to get the distance should score well. There may also be descriptions of a tunnel diode. Again, a reasonable explanation should do well.*)



(4 marks)

ii) a) A particle-antiparticle pair have opposite charge and spin, and the same mass, m. To create them from nothing requires an energy $E > 2mc^2$. Creating them for an indefinite time without this energy is impossible. However, for a time $t < \hbar /E$, conservation of energy is not violated because of Heisenberg's Uncertainty Principle. So a pair of virtual particle and antiparticle can spontaneously exist for such a time. (5 marks)

b) The virtual particles that mediate the strong force have finite mass and therefore limited lifetimes. They cannot travel further than c times their lifetime, so the range is finite. Virtual photons, which are massless, mediate the electrical interaction, so the range is potentially infinite. (4 marks)

iv) force $= \frac{\Delta p}{t} = \frac{np}{t}$ where n photons have momentum p each.

$$np = n\frac{h}{\lambda} = n\frac{hf}{c} = \frac{\text{total energy}}{ct} = \frac{P}{c} = \frac{10^3\ \text{W}}{3\ 10^8\ \text{ms}^{-1}} = 3\ \mu\text{N}$$ (5 marks)