

PHYS1231 end of year test 2003

The following equations may be used without proof.

$$PV = NkT = nRT$$

$$P = \frac{1}{3} \rho \overline{v^2} \quad I = e\sigma T^4$$

$$x' = \gamma(x - vt) \quad t' = \gamma(t - vx/c^2) \quad u'_x = \frac{u_x - v}{1 - u_x v/c^2} \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad E^2 = p^2 c^2 + m^2 c^4$$

$$\lambda_{\max} T = 2898 \mu\text{m}\cdot\text{K} \quad \lambda - \lambda' = \frac{h}{m_e c} (1 - \cos \theta) \quad E_n = -\frac{13.6 \text{ eV}}{n^2} \quad p = h/\lambda$$

$$m_e = 9.1 \cdot 10^{-31} \text{ kg} \quad m_p = 1.67 \cdot 10^{-27} \text{ kg} \quad e = 1.6 \cdot 10^{-19} \text{ C} \quad h = 6.63 \cdot 10^{-34} \text{ Js}$$

$$k = 1.38 \cdot 10^{-23} \text{ JK}^{-1} \quad \sigma = 5.67 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \quad \mathbf{F}_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \quad \mathbf{F}_g = G \frac{Mm}{r^2} \hat{\mathbf{r}}$$

Question 1 (21 marks)

A tank is in the shape of a cube, each side of which has a length L_0 at temperature T_0 . The material of which the tank is made has a linear coefficient of thermal expansion α .

The tank contains air, which you may treat as an ideal gas with molar mass m . It is open to the outside atmosphere, so its pressure remains constant at P_A .

- i) Derive an expression for $V(T)$, the volume of the tank as a function of temperature T . Using the usual approximation that $\alpha(T-T_0) \ll 1$, make your expression a linear function of temperature. Express your answer as a fractional increase in volume, $\Delta V/V_0$, where V_0 is the volume at T_0 . (4 marks)
- ii) From the equation of state, derive an expression for $\rho(T)$, the density of air in the tank as a function of temperature. Express your answer as a fractional increase in volume, $\Delta\rho/\rho_0$, where ρ_0 is the volume at T_0 . Simplify the expression. (5 marks)
- iii) Take $\alpha = 2.0 \cdot 10^{-5} \text{ K}^{-1}$, $T_0 = 293 \text{ K}$ and $\Delta T = 20 \text{ K}$. Calculate the percentage change in the mass of gas in the tank due to (a) the change in volume of the tank alone (ie neglect density change) and (b) the change in density of the gas alone (ie neglect volume change). State the sign of the change in each case. (4 marks)

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Take the average temperature of the Earth's atmosphere to be 290 K.

- iv) Calculate the root mean square velocity v_{rms} for Nitrogen (molar mass 28 kg/kmol) and Hydrogen (2 kg/kmol) in the Earth's atmosphere. (5 marks)
- v) The escape velocity for the Earth is $11 \text{ km}\cdot\text{s}^{-1}$. Comment briefly on the significance of your results for part (iv) for the composition of the atmospheres of the Earth, the Earth's moon, and Jupiter. (Three four clear sentences should suffice.) (3 marks)

Question 1

$$i) \quad V = L^3 = L_0^3(1+\alpha(T-T_0))^3 = L_0^3(1+3\alpha(T-T_0)) \cong L_0^3(1+3\alpha(T-T_0))$$

$$\frac{\Delta V}{V} = \frac{L_0^3(1+3\alpha(T-T_0)) - L_0^3}{L_0^3} = 3\alpha(T-T_0)$$

$$ii) \quad \rho \equiv \frac{M}{V} = \frac{nm}{V} \quad \text{where } n \text{ is number of moles in tank. Substituting from } PV = nRT,$$

$$\rho = \frac{P_A m}{RT} \quad \text{so} \quad \frac{\Delta \rho}{\rho_0} = \frac{\frac{P_A m}{RT} - \frac{P_A m}{RT_0}}{\frac{P_A m}{RT_0}} = \frac{\frac{1}{T} - \frac{1}{T_0}}{\frac{1}{T_0}} = \frac{T_0}{T} - 1$$

$$iii) \quad a) \quad PV = nRT \text{ so } nm \propto V. \quad i) \rightarrow \frac{\Delta V}{V} = 3\alpha(T-T_0) = 0.12\%$$

$$b) \quad m \propto \rho \quad ii) \rightarrow \frac{\Delta \rho}{\rho_0} = \frac{T_0}{T} - 1 = -6.8\%$$

$$iv) \quad \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \quad \therefore \quad v_{r.m.s.} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{\text{mol wt}}}$$

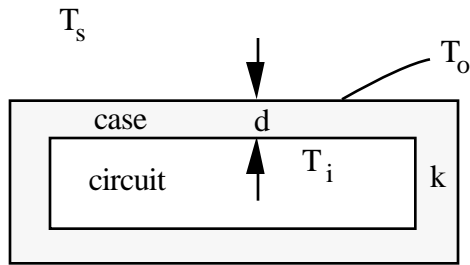
$$\text{for } N_2: \quad = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293 \times 6.02 \times 10^{23}}{0.028}} = 510 \text{ ms}^{-1}$$

$$\text{for } H_2 \quad = \quad = 1.9 \text{ kms}^{-1}$$

- v) Earth: for hydrogen, although the $v_{rms} < v_{escape}$, the random motion of molecules means that occasionally hydrogen molecules in the Earth's atmosphere acquire escape velocity and are lost. Consequently nearly all hydrogen in Earth's atmosphere has escaped. For nitrogen, $v_{rms} \ll v_{escape}$, so we have retained the nitrogen. Jupiter is colder and has a higher escape velocity: it has retained its hydrogen atmosphere. The moon has the lowest escape velocity and virtually no atmosphere.

Question 2 (16 marks)

a)



A circuit is operating in a vacuum, and you are worried about overheating, so you have installed an infrared camera to observe it.

You constructed the circuit inside a sealed, black steel case, with thickness $d = 1.2 \text{ mm}$, area $A = 0.24 \text{ m}^2$ and thermal conductivity $k = 14 \text{ W.K}^{-1}.\text{m}^{-1}$. The thickness d is much smaller than \sqrt{A} , so you may assume that the inside and outside areas of the case are the same. A fan inside the case circulates air and keeps the internal temperature uniform.

From the spectra of the infrared camera, you can tell that

- the outer surface of the case is at temperature $T_o = 45 \text{ }^\circ\text{C}$
- that the surroundings are at temperature $T_s = 28 \text{ }^\circ\text{C}$ and
- that these are not changing with time.

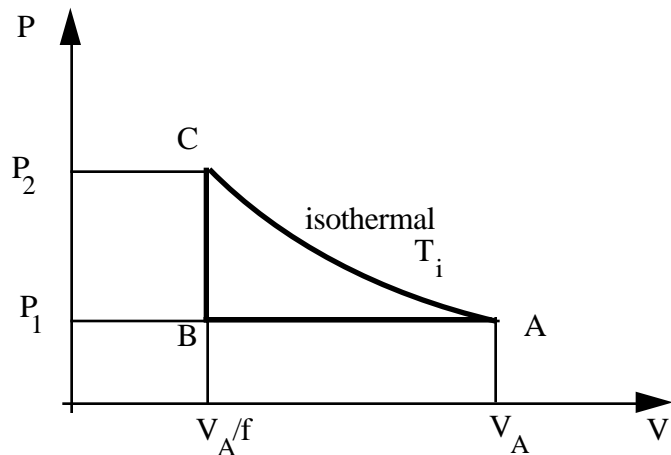
Both the surroundings and the case have emissivity of 1.0. Showing your working, determine the internal temperature T_i . (4 marks)

b)

A rather impractical closed cycle heat engine has been constructed solely for the purpose of an examination question. n moles of an ideal gas form the working substance that undergoes the cycle $A \rightarrow B \rightarrow C \rightarrow A$ as shown in the pressure-volume PV diagram at right, where the step CA is isothermal at temperature T_i . The volume of the isochoric step BC is a factor f smaller than the volume at A.

i) Derive expressions for the work done W in each step of the cycle (W_{AB} , W_{BC} , W_{CA}). **Important:** express your answers in terms of n , R , T_i and f only. (8 marks)

ii) For each of the steps, state whether heat is added to the gas or heat is lost from the gas. (3 marks)



Because the temperatures are not changing, the system is in steady state, so heat is radiated from the outer surface and conducted from the inner surface of the case at the same rate.

Nett rate of heat radiation loss = rate of heat conduction

$$A\sigma(e_o T_o^4 - e_s T_s^4) = \frac{kA}{d}(T_i - T_o)$$

$$\begin{aligned} T_i &= \frac{d\sigma}{k}(T_o^4 - T_s^4) + T_o \\ &= \frac{(1.2 \cdot 10^{-3} \text{ m}) \times (5.67 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4})}{14 \text{ W.K}^{-1}.\text{m}^{-1}} ((318 \text{ K})^4 - (301 \text{ K})^4) + 45^\circ\text{C} \\ &= 0.01 + 45^\circ\text{C} \cong 45^\circ\text{C} \end{aligned}$$

(Note that 45°C is the correct answer: data in the problem are given to 2 significant figures.)

$$\text{b) } W_{AB} = \int_A^B PdV = P_1 \int_A^B dV = P_1(V_B - V_A) = P_1 V_A \left(\frac{1}{f} - 1 \right) = nRT_i \left(\frac{1}{f} - 1 \right)$$

$$W_{BC} = \int_B^C PdV = 0$$

$$\begin{aligned} W_{CA} &= \int_C^A PdV = \int_C^A \frac{nRT_i}{V} dV = nRT_i \int_C^A \frac{dV}{V} = nRT_i [\ln V]_C^A \\ &= nRT_i (\ln V_A - \ln V_C) = nRT_i \ln \frac{V_A}{V_C} = nRT_i \ln f \end{aligned}$$

ii) A → B Heat lost (compressing a gas will raise its temperature unless Q is lost)

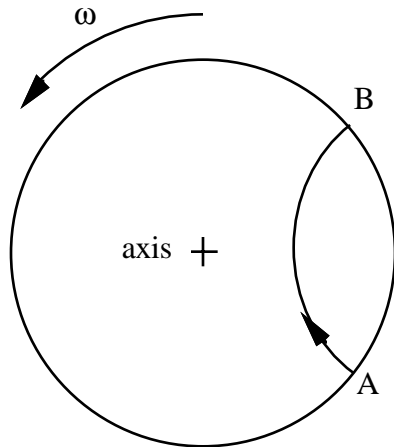
explanations not required

B → C Heat added (to raise the pressure at constant volume, T must rise so Q must be added)

C → A Heat added (T const so U const, work done so heat added)

Question 3 (16 marks)

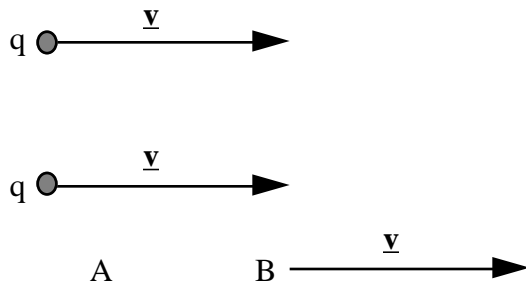
a)



A carnival ride comprises a closed cylinder that rotates about its axis, which is vertical. (The diagram is a view from above.) Riders stand inside the cylinder, against the vertical wall. One of these riders throws a ball from point A towards the axis. The thrower of the ball sees it follow the trajectory AB whose projection on the horizontal plane is shown in the diagram.

- Explain how the ball thrower's observations seem to be at variance with Newton's laws, as applied in this frame of reference. (One or two clear sentences should suffice.) (2 marks)
- Explain how the observed motion can be accounted for using Newton's laws. (A diagram and a couple of clear sentences should suffice.) (2 marks)
- What can the thrower of the ball deduce about her frame of reference from the observed trajectory? (One or two clear sentences should suffice.) (1 mark)

b)

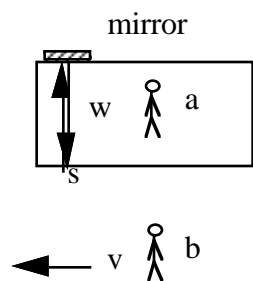


Two equal positive charges q travel to the right at speed v with respect to observer A, as shown. A calculates the force between the charges.

Observer B moves to the right with speed v with respect to observer A. B calculates the force between the charges.

- Do their calculations for the force give the same value? If not, whose value is greater? (1 mark)
- Explain your answer to part (i). (One clear sentence should suffice.) (2 marks)
- How is the theory of special relativity involved? (One or two clear sentences should suffice.) (1 mark)

c) (7 marks)



An observer a sees a beam of light travel from source s to a mirror and return to the source, as shown. The mirror and the source are at rest with respect to observer a, and are separated by a distance w.

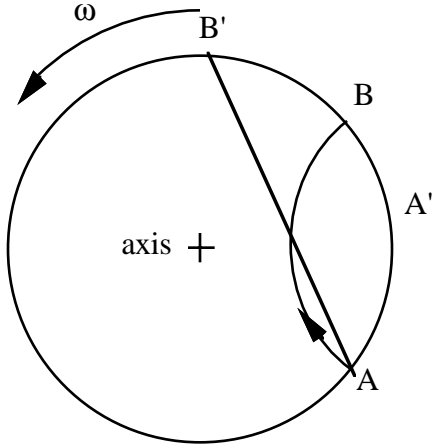
- i) State the time t_a light takes to travel from source to mirror and back again, as measured by observer a.

Observer a observes that observer b is travelling at speed v to the left, and at right angles to the path of the light, as shown. Both observers agree that the distance between the source and the mirror, in the frame of a, is w, and that the magnitude of their relative speed is v .

- ii) Draw a sketch of the path of the light ray, as observed by b. Using this sketch, determine the time t_b that light takes to travel from source to mirror and back again, as measured by observer b.
- iii) Write an equation for t_a/t_b , as determined by observer b.
- iv) What observation can b make about the rate of physical processes in a's frame of reference?

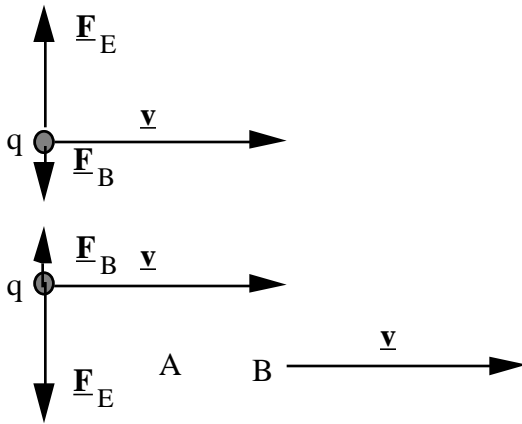
Question 3

a)



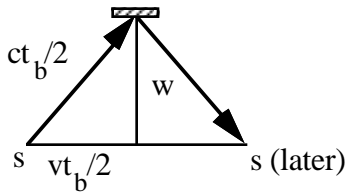
- i) There are no horizontal forces acting, so the horizontal velocity should be constant, so the projection of the trajectory in the horizontal plane should be a straight line.
- ii) The projection of the trajectory in the horizontal plane is a straight line for an observer in an inertial frame. An observer who is not rotating with the ride sees the ball travel along the trajectory AB', while the points A and B move to A' and B'.
- iii) The frame of reference in the ride is not an inertial frame. (It is accelerating (rotational motion) with respect to an inertial frame.)

b)



- i) B calculates a force that is repulsive, and that has a larger value than that calculated by A.
- ii) The electrostatic force is repulsive, the magnetic force is attractive (and weaker) so the net repulsion is less.
- iii) Forces measured or calculated in different frames of reference may be different. *or* Magnetism is the name of the relativistic correction applied to electric forces when the charges are moving with respect to the observer.

c)



(4 for the diagram)

- i) $t_a = 2w/c$, where c is the speed of light.
- ii) (From Einstein's principle of Special Relativity, both observers agree that the speed of light is c in all inertial frames.)

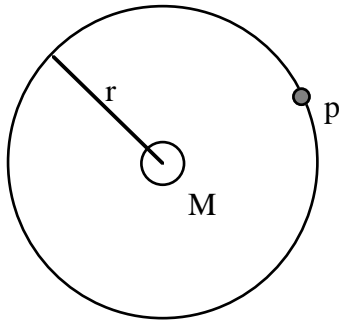
Pythagoras: $w^2 + (vt_b/2)^2 = (ct_b/2)^2$

$$w^2 = t_b^2((c/2)^2 - (vt_b/2)^2)$$

$$t_b = \frac{2w}{\sqrt{c^2 - v^2}}$$

iii) $\frac{t_b}{t_a} = \frac{1}{\sqrt{1 - v^2/c^2}}$

iv) b observes that processes (as exemplified by the round trip of light) occur more slowly than in b's own frame (by a factor $\frac{1}{\sqrt{1 - v^2/c^2}}$).

Question 4 (14 marks)

UNSW astronomers have discovered a cloud of interstellar gas. The gas has the exactly the same absorption spectrum as normal atomic hydrogen. However, from the motion of the gas in known magnetic fields, the astronomers determine that the atoms are positively charged. An early theory, which proposes that the gas is made of He^+ ions, is discarded. First, the spectrum has the wrong wavelengths. Second, the charge/mass ratio of the atoms is much, *much* too small for He^+ .

Eccentric scientist Josef Lupus proposes that each atom is in fact a small, uncharged, black hole of mass M , about which a normal proton (proton mass $m_p \ll M$) follows a circular orbit. Lupus' model resembles the Bohr-Sommerfeld model for hydrogen, but, because the 'nucleus' is uncharged, the attraction in this new 'atom' is purely gravitational, instead of electrostatic.,

Your job is to calculate the mass M of the black hole and the radius r of the orbit necessary for this 'atom' to have the same absorption spectrum as normal atomic hydrogen.

- Derive a relation between the radius r of the orbit and the speed v of the proton (mass m_p) in gravitational orbit around the mass M . (You may assume that both special and general relativistic effects are negligible.) (3 marks)
- Use the de Broglie wavelength to derive an expression for the values of the radius of the orbit at which the de Broglie's waves for the proton give constructive interference. (5 marks)
- Using the above results, or otherwise, determine the value required for M such that the spectrum of this 'atom' has the same wavelength lines as normal atomic hydrogen. The ground state of normal hydrogen is -13.6 eV. You may use without proof the expression for the gravitational potential energy of two masses M and m at separation r : $U_g = -GMm/r$. (6 marks)

$$[m_p = 1.67 \cdot 10^{-27} \text{ kg} \quad h = 6.63 \cdot 10^{-34} \text{ Js} \quad e = 1.6 \cdot 10^{-19} \text{ C} \quad G = 6.67 \cdot 10^{-11} \text{ m}^3\text{s}^{-2}\text{kg}^{-1}]$$

Question 4

$$\text{i) } \frac{mv^2}{R} = F_{\text{centrip}} = G \frac{Mm}{R^2}$$

$$v = \sqrt{\frac{GM}{R}} \quad (3)$$

$$\text{ii) de Broglie: } \rightarrow p = h/\lambda \quad \text{constructive interference} \quad \rightarrow 2\pi R = n\lambda$$

$$2\pi r = n \frac{h}{m_p v} \quad \text{but substitution from (i) gives}$$

$$2\pi r = n \frac{h}{m_p} \sqrt{\frac{r}{GM}}$$

$$\sqrt{r} = n \frac{h}{2\pi m_p \sqrt{GM}}$$

$$r = n^2 \frac{h^2}{4\pi^2 G m_p^2 M} \quad (5)$$

$$\text{iii) } U_g = -\frac{GMm_p}{r_{\text{D}}} = -\frac{4\pi^2 G^2 M^2 m_p^3}{h^2 \text{D}} = U_H = -13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J.}$$

$$M = \sqrt{\frac{U_H h^2}{4\pi^2 G^2 m_p^3}} = \sqrt{\frac{13.6 \times 1.6 \times 10^{-19} \text{ J} \times (6.63 \times 10^{-34} \text{ Js})^2}{4\pi^2 (6.67 \times 10^{-11} \text{ m}^3 \text{s}^{-2} \text{kg}^{-1})^2 (1.67 \times 10^{-27})^3}} = 34,000 \text{ tonnes} \quad (6)$$

Question 5 (14 marks)

- a) i) State the Heisenberg Uncertainty Principle as it applies to energy. If your statement is an equation, then define all terms in the equation.
- ii) What are virtual particles? (Your explanation should refer to an expression from the Special Theory of Relativity and to Heisenberg's Uncertainty Principle. Four or five clear sentences.)
- iii) Why is the range of the strong nuclear force finite? (Hint: you may refer to your answer to part ii)
- b) de Broglie proposed that electrons could have a wavelength. Explain briefly the phenomenon that he explained using this wavelength. (Your explanation could be several sentences, *or* it could be in point form. In either case, a diagram may be useful.)

Question 5

- a) i) $\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$ where ΔE is the uncertainty in the measurement of energy, Δt is the uncertainty in the time of the energy measurement, and h is Planck's constant. (4 marks)
- ii) A particle-antiparticle pair have opposite charge and spin, and the same mass, m . To create them from nothing requires an energy $E > 2mc^2$. Creating them for an indefinite time without this energy is impossible. However, for a time $t < h/E$, conservation of energy is not violated because of Heisenberg's Uncertainty Principle. So a pair of virtual particle and antiparticle can spontaneously exist for such a time. (4 marks)
- iii) The virtual particles that mediate the strong force have finite mass and therefore limited lifetimes. They cannot travel further than c times their lifetime, so the range is finite. (2 marks)
- b) de Broglie introduced the idea to explain the quantisation of the energy of electron orbitals. In the Bohr model of the atom (small nucleus with classical circular electron orbits), the observed discrete emission spectrum can be explained if only certain orbits are possible. If the electrons have wavelengths, then they are stable only if the electron in an orbit interferes constructively with itself. (This condition, the Maxwell momentum expression, Newtonian mechanics and electrostatics, gives a model in good agreement with much of the observed spectrum of hydrogen.) (4 marks)