PHYS1231 end of year test 2003

The following equations may be used without proof.

$PV = NkT = nRT$ $P = \frac{1}{3}\rho\overline{v^2}$ $I = e\sigma T^4$

$x' = \gamma(x - vt)$ $t' = \gamma(t - vx/c^2)$ $u'_x = \frac{u_x - v}{1 - u_x v/c^2}$ $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $E^2 = p^2c^2 + m^2c^4$

$\lambda_{max}T = 2898\ \mu m.K$ $\lambda - \lambda' = \frac{h}{m_e c}(1 - \cos\theta)$ $E_n = -\frac{13.6\ eV}{n^2}$ $p = h/\lambda$

$m_e = 9.1\ 10^{-31}$ kg $m_p = 1.67\ 10^{-27}$ kg $e = 1.6\ 10^{-19}$ C $h = 6.63\ 10^{-34}$ Js

$k = 1.38\ 10^{-23}\ JK^{-1}$ $\sigma = 5.67\ 10^{-8}\ Wm^{-2}K^{-4}$ $\mathbf{F}_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}$ $\mathbf{F}_g = G\frac{Mm}{r^2}\hat{\mathbf{r}}$

Question 1 (21 marks)

A tank is in the shape of a cube, each side of which has a length L_o at temperature T_o. The material of which the tank is made has a linear coefficient of thermal expansion α.

The tank contains air, which you may treat as an ideal gas with molar mass m. It is open to the outside atmosphere, so its pressure remains constant at P_A.

i) Derive an expression for V(T), the volume of the tank as a function of temperature T. Using the usual approximation that $\alpha(T-T_o) << 1$, make your expression a linear function of temperature. Express your answer as a fractional increase in volume, $\Delta V/V_o$, where V_o is the volume at T_o. (4 marks)

ii) From the equation of state, derive an expression for $\rho(T)$, the density of air in the tank as a function of temperature. Express your answer as a fractional increase in volume, $\Delta\rho/\rho_o$, where ρ_o is the volume at T_o. Simplify the expression. (5 marks)

iii) Take $\alpha = 2.0\ 10^{-5}\ K^{-1}$, $T_o = 293$ K and $\Delta T = 20$ K. Calculate the percentage change in the mass of gas in the tank due to (a) the change in volume of the tank alone (ie neglect density change) and (b) the change in density of the gas alone (ie neglect volume change). State the sign of the change in each case. (4 marks)

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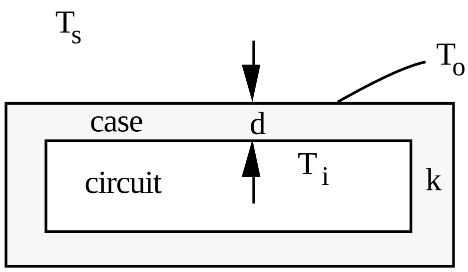
Take the average temperature of the Earth's atmosphere to be 290 K.

iv) Calculate the root mean square velocity v_{rms} for Nitrogen (molar mass 28 kg/kmol) and Hydrogen (2 kg/kmol) in the Earth's atmosphere. (5 marks)

v) The escape velocity for the Earth is 11 $km.s^{-1}$. Comment briefly on the significance of your results for part (iv) for the composition of the atmospheres of the Earth, the Earth's moon, and Jupiter. (Three four clear sentences should suffice.) (3 marks)

Question 2 (16 marks)

a)



A circuit is operating in a vacuum, and you are worried about overheating, so you have installed an infrared camera to observe it.

You constructed the circuit inside a sealed, black steel case, with thickness d = 1.2 mm, area A = 0.24 m^2 and thermal conductivity k = 14 $W.K^{-1}.m^{-1}$. The thickness d is much smaller than $\sqrt{A}$, so you may assume that the inside and outside areas of the case are the same. A fan inside the case cirulates air and keeps the internal temperature uniform.

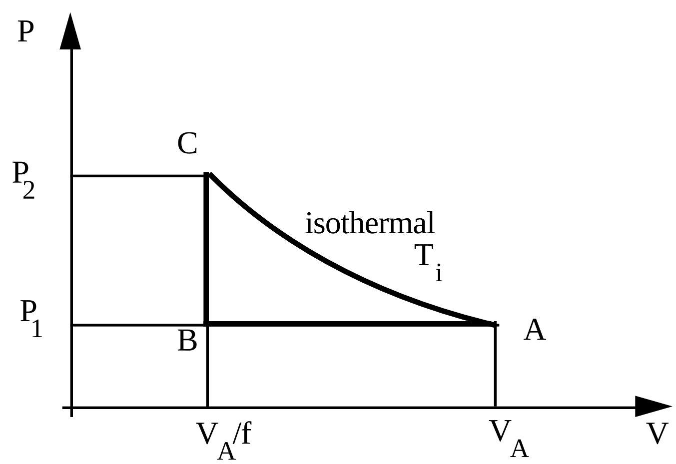
From the spectra of the infrared camera, you can tell that

- the outer suface of the case is at temperature T_o = 45 °C
- that the surroundings are at temperature T_s = 28 °C and
- that these are not changing with time.

Both the surroundings and the case have emissivity of 1.0. Showing your working, determine the internal temperature T_i. (4 marks)

b)

A rather impractical closed cycle heat engine has been constructed solely for the purpose of an examination question. n moles of an ideal gas form the working substance that undergoes the cycle $A \rightarrow B \rightarrow C \rightarrow A$ as shown in the pressure-volume PV diagram at right, where the step CA is isothermal at temperature T_i. The volume of the isochoric step BC is a factor f smaller than the volume at A.

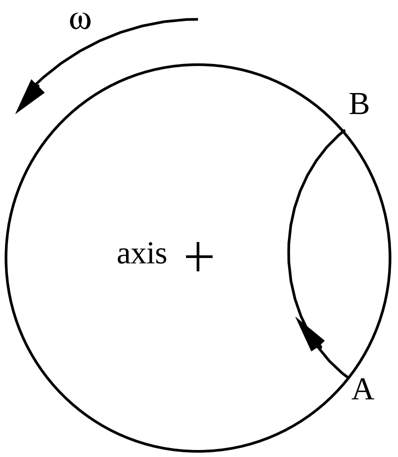


i) Derive expressions for the work done W in each step of the cycle (W_{AB}, W_{BC}, W_{CA}). **Important:** express your answers in terms of n, R, T_i and f only. (8 marks)

ii) For each of the steps, state whether heat is added to the gas or heat is lost from the gas. (3 marks)

Question 3 (16 marks)

a)



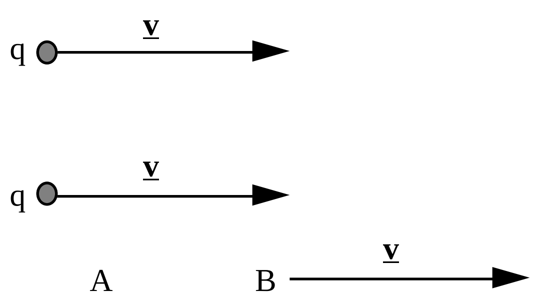
A carnival ride comprises a closed cylinder that rotates about its axis, which is vertical. (The diagram is a view from above.) Riders stand inside the cylinder, against the vertical wall. One of these riders throws a ball from point A towards the axis. The thrower of the ball sees it follow the trajectory AB whose projection on the horizontal plane is shown in the diagram.

i) Explain how the ball thrower's observations seem to at variance with Newton's laws, as applied in this frame of reference. (One or two clear sentences should suffice.) (2 marks)

ii) Explain how the observed motion can be accounted for using Newton's laws. (A diagram and a couple of clear sentences should suffice.) (2 marks)

iii) What can the thrower of the ball deduce about her frame of reference from the observed trajectory? (One or two clear sentences should suffice.) (1 mark)

b)



Two equal positive charges q travel to the right at speed v with respect to observer A, as shown. A calculates the force between the charges.

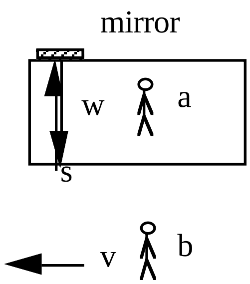
Observer B moves to the right with speed v with respect to observer A. B calculates the force between the charges.

i) Do their calculations for the force give the same value? If not, whose value is greater? (1 mark)

ii) Explain your answer to part (i). (One clear sentence should suffice.) (2 marks)

iii) How is the theory of special relativity involved? (One or two clear sentences should suffice.) (1 mark)

c) (7 marks)



An observer a sees a beam of light travel from source s to a mirror and return to the source, as shown. The mirror and the source are at rest with respect to observer a, and are separated by a distance w.

i) State the time t_a light takes to travel from source to mirror and back again, as measured by observer a.

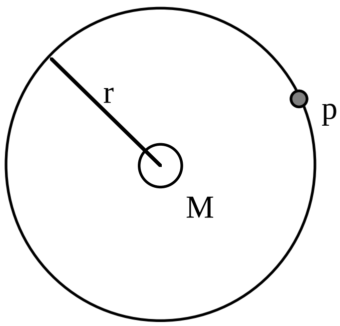
Observer a observes that observer b is travelling at speed v to the left, and at right angles to the path of the light, as shown. Both observers agree that the distance between the source and the mirror, in the frame of a, is w, and that the magnitude of their relative speed is v.

ii) Draw a sketch of the path of the light ray, as observed by b. Using this sketch, determine the time t_b that light takes to travel from source to mirror and back again, as measured by observer b.

iii) Write an equation for t_a/t_b, as determined by observer b.

iv) What observation can b make about the rate of physical processes in a's frame of reference?

Question 4 (14 marks)



UNSW astronomers have discovered a cloud of interstellar gas. The gas has the exactly the same absorption spectrum as normal atomic hydrogen. However, from the motion of the gas in known magnetic fields, the astronomers determine that the atoms are positively charged. An early theory, which proposes that the gas is made of He^+ ions, is discarded. First, the spectrum has the wrong wavelengths. Second, the charge/mass ratio of the atoms is much, *much* too small for He^+.

Eccentric scientist Josef Lupus proposes that each atom is in fact a small, uncharged, black hole of mass M, about which a normal proton (proton mass m_p << M) follows a circular orbit. Lupus' model resembles the Bohr-Sommerfeld model for hydrogen, but, because the 'nucleus' is uncharged, the attraction in this new 'atom' is purely gravitational, instead of electrostatic.,

Your job is to calculate the mass M of the black hole and the radius r of the orbit necessary for this 'atom' to have the same absorption spectrum as normal atomic hydrogen.

i) Derive a relation between the radius r of the orbit and the speed v of the proton (mass m_p) in gravitational orbit around the mass M. (You may assume that both special and general relativistic effects are negligible.) (3 marks)

ii) Use the de Broglie wavelenth to derive an expression for the values of the radius of the orbit at which the de Broglie's waves for the proton give constructive interference. (5 marks)

iii) Using the above results, or otherwise, determine the value required for M such that the spectrum of this 'atom' has the same wavelength lines as normal atomic hydrogen. The ground state of normal hydrogen is –13.6 eV. You may use without proof the expression for the gravitational potential energy of two masses M and m at separation r: $U_g = -GMm/r$. (6 marks)

[m_p = 1.67 10^{-27} kg   h = 6.63 10^{-34} Js   e = 1.6 10^{-19} C   G = 6.67 10^{-11} $m^3s^{-2}kg^{-1}$]

Question 5 (14 marks)

a) i) State the Heisenburg Uncertainty Principle as it applies to energy. If your statement is an equation, then define all terms in the equation.

ii) What are virtual particles? (Your explanation should refer to an expression from the Special Theory of Relativity and to Heisenberg's Uncertainty Principle. Four or five clear sentences.)

iii) Why is the range of the strong nuclear force finite? (Hint: you may refer to your answer to part ii)

b) de Broglie proposed that electrons could have a wavelength. Explain briefly the phenomenon that he explained using this wavelength. (Your explanation could be several sentences, ***or*** it could be in point form. In either case, a diagram may be useful.)