1231 end of year test 2002

The following equations may be used with proof.

$PV = NkT = nRT$ $\quad P = \frac{1}{3}\rho\overline{v^2}$ $\quad I = e\sigma T^4$ $\quad \bar{\varepsilon} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{PV}{N} = \frac{3}{2}$ kT

$x' = \gamma(x - vt)$ $\quad t' = \gamma(t - vx/c^2)$ $\quad u'_x = \frac{u_x - v}{1 - u_x v/c^2}$ $\quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\quad E^2 = p^2c^2 + m^2c^4$

$\lambda_{max}T = 2898\ \mu m.K$ $\quad \lambda - \lambda' = \frac{h}{m_e c}(1 - \cos\theta)$ $\quad E_n = -\frac{13.6\ eV}{n^2}$ $\quad p = h/\lambda$

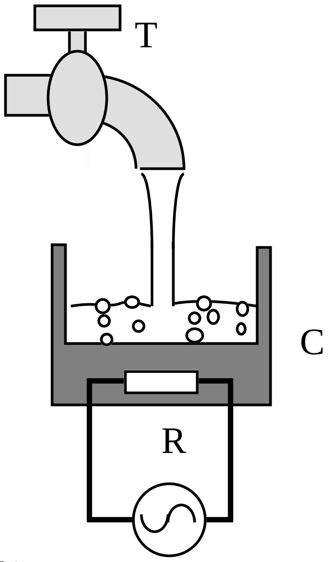
$m_e = 9.1\ 10^{-31}$ kg $\quad e = 1.6\ 10^{-19}$ C $\quad m_n = 1.68\ 10^{-27}$ kg

$h = 6.63\ 10^{-34}$ Js $\quad k = 1.38\ 10^{-23}\ JK^{-1}$ $\quad \sigma = 5.67\ 10^{-8}\ Wm^{-2}K^{-4}$ $\quad G = 6.67\ 10^{-11}\ m^3s^{-2}kg^{-1}$

$\mathbf{F}_e = -\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}$ $\quad \mathbf{F}_G = -G\frac{m_1m_2}{r^2}\hat{\mathbf{r}}$

Question 1 (18 marks)

a)



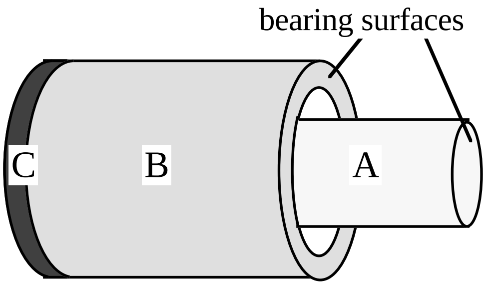
Water at temperature 20 °C flows from a tap T into a heated container C. The container has a heating element (a resistor R) which is supplied with electrical power P, that may be varied.

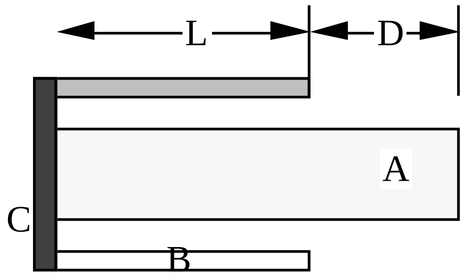
The rate of water flow is F = 0.020 litres per minute. The electrical power is sufficient that the water in the container is boiling. What is the minimum power P that must be supplied in steady state so that the amount of liquid water in the container neither increases nor decreases with time? (Neglect other losses of heat, such as conduction from the container to the air.)

For water, $c = 4.2\ \mathrm{kJ.kg^{-1}K^{-1}}$, $L_{vap} = 2.3\ \mathrm{MJ.kg^{-1}}$,

$\rho = 1000\ \mathrm{kg.m^{-3}}$

b)





The diagrams show a sketch (top) and cross section of a low expansion mounting. It is designed so that the two bearing surfaces remain separated by a constant distance D, independent of temperature. Part A is a rod, which has length L_o+D_o at reference temperature T_o and is made of material with linear coefficient of thermal expansion α_A. Part B is a hollow cylinder which has length L_o at T_o and is made of material with linear coefficient of thermal expansion α_B. Both are mounted on a rigid plate C.

i) Showing all working, derive an expression for the length D as a function of temperature, in terms of the parameters given above.

ii) Give an expression for the value of ratio D_o/L_o which produces the result that D is independent of temperature.

c) A spacecraft in deep space is to be made of metal, coated with an insulating layer. For the comfort of the inhabitants, the spacecraft will be heated by generating heat so that the total rate of heat production in the spacecraft is P. An engineer is asked to determine the required value of P. The metal walls have an area $A = 40\ \mathrm{m^2}$ and, in steady state, they are to be maintained at temperature $T_1 = 20$ °C. The insulating layer has an external temperature T_2, an emissivity of 1.0, a thickness t = 2 cm, and a thermal conductivity k. The outside of the spacecraft radiates heat into space where the radiation temperature will be, on average, $T_b = 4$ K, and approximates that of a black body.

i) Derive or state an equation relating P, T_1 and T_2.

ii) Derive or state an equation relating P, T_2 and T_b.

iii) Explain in one or two sentences how you could use your answers to (i) and (ii) to determine the required heating rate P. ***Note:*** you are not required to solve for P.

a) In time t, the mass of water added is $m = F\rho t$

In steady state, energy added = heat to raise T + heat to biol water

$$Pt = mc\Delta T + mL = F\rho t(c\Delta T + L)$$

$$P = F\rho(c\Delta T + L)$$

$$= \frac{.020}{60} \text{ l/s} * 1 \text{ kg/l} * (80 \text{ K}*4.2 \text{ kJ.kg}^{-1}\text{K}^{-1} + 2.3 \text{ MJ.kg}^{-1}) = 880 \text{ W} \qquad (5)$$

b)

i) $D = L_A - L_B = (L_o+D_o)(1 + \alpha_A(T - T_o)) - D_o(1 + \alpha_B(T - T_o))$

$=$ (various simplifications possible, but not asked for) (3)

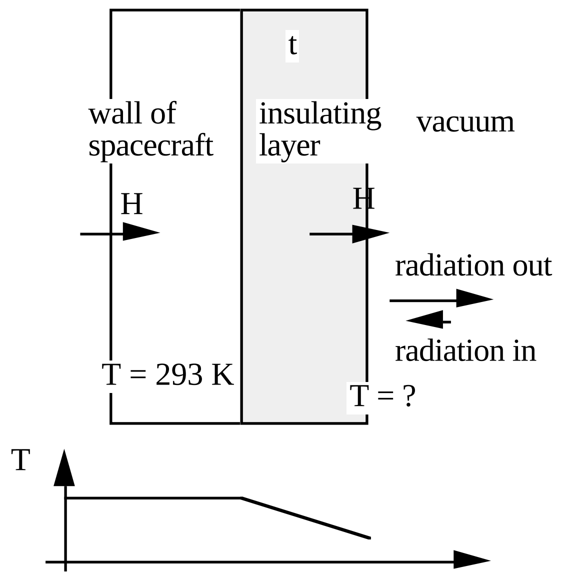
ii) $D = (L_o + D_o)\alpha_A T - D_o \alpha_B T$ + constant terms

if independent of T, $(L_o + D_o)\alpha_A = D_o \alpha_B$

$L_o\alpha_A = D_o (\alpha_B - \alpha_A)$

$D_o/L_o = \alpha_A/(\alpha_B - \alpha_A)$. (4)

c)



i) In steady state,

$$P = H_{conduction} = kA\frac{T_1 - T_2}{t} \qquad (2)$$

ii) In steady state,

$$P = H_{radiation} = e\sigma A(T_2^4 - T_b^4) \qquad (2)$$

iii) Solve (i) and (ii) for T_2, substition in either then gives P. (2)

Question 2 (16 marks)

a) An ideal gas, initially with pressure P_o, volume V_o and temperature T_o expands adiabatically to P_1, V_1 and T_1 (step A). The gas then expands isobarically to P_1, V_2 and T_2 (step B). It then returns isothermally to its orginal state P_o, V_o and T_o (step C).

i) Sketch a P,V diagram for this process. On the axes, indicate P_o, P_1, V_o, V_1 and V_2. Also label the steps A, B and C and indicate their direction with arrows.

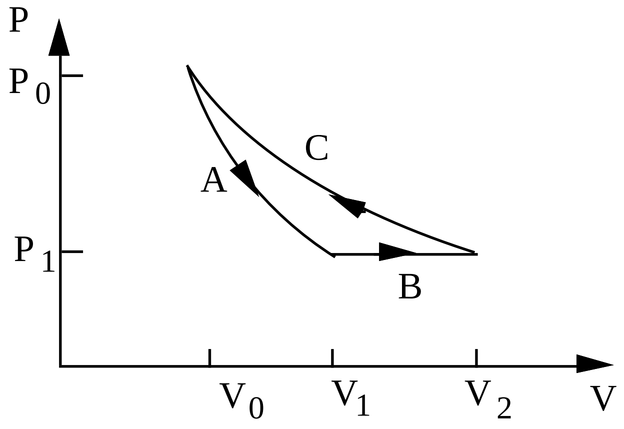
ii) Q is the heat added to the gas, W is the work done by the gas, and ΔU is the change in its internal energy. In the table provided, indicate with the symbols +, – and 0 whether the terms are positive, negative or zero for each step, and also for the whole cycle ($Q\Sigma$).

Step	Q	W	ΔU
A (adiabatic expansion)			
B (isobaric expansion)			
C (isothermal)			
Cycle (Σ)			

b) For a gas, which is larger? c_V (the specific heat at constant volume) or c_P (the specific heat at constant pressure)? **Explain** your answer in a few clear sentences,. A couple of equations may serve to clarify your answer.

Question 2

a)



(6 for diagram)

Step	Q	W	ΔU
A (adiabatic expansion)	0 (definition)	+	-
B (isobaric expansion)	+	+	+
C (isothermal)	-	-	0
Cycle (Σ)	-	-	0

(6 for the table)

b) For a gas, the internal energy is a function of T alone and gases expand with increasing T. To raise T in a gas at constant pressure, work $\int PdV > 0$ must be done.

from the definition and from the first law: $c \equiv \frac{\delta Q}{\delta T} = \frac{\delta U + \delta W}{\delta T}$

δW is > 0 for c_P, and = 0 for c_V, so $c_P > c_V$.

(4 marks.*This is a rather complete answer. Much briefer but clear answers could score full marks.*)

Question 3 (19 marks)

a) State the principle of Galilean or Newtonian relativity

b) State Einstein's principle of Special Relativity.

c) Two spaceships are travelling in opposite directions towards each other. According to an observer on Earth, spaceship A is moving at a speed of 0.80c in the +x direction and spaceship B is moving at 0.60c in the –x direction. The observer also measures the separation of the spaceships to be 4.0×10^{12} m at t = 0. Both spaceships are 100m in length, according to the Earth observer.

   i) calculate the respective proper lengths of the spaceships.

   ii) calculate the relative speed of the spaceships, as observed from one of the spaceships.

   iii) calculate the length of *each* spaceship as measured by the *other* spaceship.

   iv) calculate the time until collision according to the Earth observer.

   v) calculate the time until collision according to spaceship A.

Question 3 (19 marks)

a) Various statements are possible, such as:

If Newton's Laws of mechanics hold in one frame of reference, then in another frame of reference moving with constant, uniform relative velocity with respect to the first the laws of mechanics also hold. (Such frames are called inertial frames.) (3)

a) Various statements are possible, such as:

If Newton's Laws of mechanics and Maxwell's laws of electromagnetism hold in one frame of reference, then in another frame of reference moving with constant, uniform relative velocity with respect to the first the laws of mechanics *and the laws of electromagnetism* also hold. (3)

c) i) $L' = \dfrac{L_o}{\gamma}$ $\quad L_o = \gamma L'$ For v = 0.80c, $\gamma = \dfrac{1}{0.60}$ $\quad$ For v = 0.60c, $\gamma = \dfrac{1}{0.80}$ (3)

L_A = 170 m, L_B = 130 m.

ii) Let A travel at v = 0.80c, B travels with $u_x = -0.60c$, both according to Earth

For A, B travels with $u_x' = \dfrac{u_x - v}{1 - \dfrac{vu_x}{c^2}} = \dfrac{u_x - v}{1 - \dfrac{vu_x}{c^2}} = -0.95$ c. (3)

(and for A, B travels with speed 0.95 c also.)

iii) For each of them, the γ to be applied to the other is $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{u'^2_x}{c^2}}} = 3.1$

A sees B as having length L_B/γ = 41 m

B sees A as having length L_A/γ = 54 m (3)

iv) According Earth, their relative speed is 1.40 c, so the collision is due in 9,500 s. (1)

v) Suppose that they crash at the origin in the Earth frame, and let's take t = 0 at collision. Time is now −9,500 s, as measured by Earth. So A is now (Earth's 'now') at $x = -(0.8/1.4)(4.0\ 10^{12}\ \text{m})$ and B is at $x = +(0.6/1.4)(4.0\ 10^{12}\ \text{m})$. So the Earth observer calculates that A's clock now (Earth's 'now') reads

$$t' = \gamma_A\left(t - \frac{v\,x}{c^2}\right) = \gamma\left(-9500\ \text{s} - 0.8\ \frac{(-0.8/1.4)(4.0\ 10^{12}\ \text{m})}{c} \supseteq\right) = -2000\ \text{s}. \quad (3)$$

Question 4 (16 marks)

a) In free space, a mass m is in a circular gravitational orbit about a mass M (M>>m).

i) By considering the centripital acceleration (or otherwise), derive a relation between the radius R of the orbit and the speed v of the mass m. (Neglect relativistic effects: v << c)

ii) Using de Broglie's wave hypothesis for the mass m, determine the allowed values of R for which the matter wave of mass m gives constructive interference around the circular orbit.

iii) Using the results above, calculate the minimum radius of the orbit of a ***neutron*** around a small black hole of mass of 100 kg (again, you may neglect relativistic effects).

iv) What is the speed of the neutron in the case (iii)?

v) What is the rms speed of neutrons in thermal equilibrium with the background radiation of the universe, which has a radiation temperature of 3 K?

vi) Compare the answers for (iv) and (v) and comment.

[$m_n = 1.68\ 10^{-27}$ kg $h = 6.63\ 10^{-34}$ Js $k = 1.38\ 10^{-23}$ JK^{-1} $G = 6.67\ 10^{-11}$ m^{3}s^{-2}kg^{-1}]

Question 4

i) $$\frac{mv^2}{R} = F_{centrip} = G\frac{Mm}{R^2}$$

$$v = \sqrt{\frac{GM}{R}}$$ (3)

ii) de Broglie: $\rightarrow$ $p = h/\lambda$ constructive interference $\rightarrow$ $2\pi R = n\lambda$

$2\pi R = n\dfrac{h}{mv}$ but substitution from (i) gives

$$2\pi R = n\frac{h}{m}\sqrt{\frac{R}{GM}}$$

$$\sqrt{R} = n\frac{h}{2\pi m\sqrt{GM}}$$

$$R = n^2\frac{h^2}{4\pi^2 G m^2 M}$$ (5)

iii) $$R = 1^2\frac{(6.63\ 10^{-34}\ \text{Js})^2}{4\pi^2(6.67\ 10^{-11}\ \text{m}^3\text{s}^{-2}\text{kg}^{-1})(1.68\ 10^{-27}\ \text{kg})^2(100\ \text{kg})}$$

$= 590$ nm (2)

iv) (i) $\rightarrow v = \sqrt{\dfrac{GM}{R}} = 0.11$ m.s^{-1}

(1)

v) $\dfrac{1}{2}m\overline{v^2} = \dfrac{3}{2}\dfrac{PV}{N} = \dfrac{3}{2}kT$ $\rightarrow$ $v_{rms}^2 = \dfrac{3kT}{m}$ $\rightarrow$ $v_{rms} = 300$ ms^{-1}

(3)

vi) The thermal velocities are much greater than the velocities at which quantum effects occur. Consequently, the latter could not be observed. (2)

Question 5 (11 marks)

a) Choose a physical phenomenon which may be explained by the Heisenberg Uncertainty Principle, and then briefly (a few sentences, perhaps including an equation) explain it.

b) de Broglie proposed that electrons could have a wavelength. Explain briefly the phenomenon that he explained using this wavelength. (Your explanation could be several sentences, ***or*** it could be in point form. In either case, a diagram may be useful.)

Question 5 (11 marks)

a) Any suitable examples, including:

Virtual particles: a virtual particle-antiparticle pair, each of mass m, may appear and interact with other particles for a time t provided that $2mc^2t \lesssim h$.

Diffraction: photons passing through a slit of width Δx have an uncertainty in the x component of momentum $\Delta p_x \gtrsim h/\Delta x$, hence a diffraction angle $\sin\theta = \frac{\delta p_x}{p} \sim \frac{\lambda}{\delta x}$. (5)

b) de Broglie introduced the idea to explain the quantisation of the energy of electron orbitals. In the Bohr model of the atom (small nucleus with classical circular electron orbits), the observed discrete emission spectrum can be explained if only certain orbits are possible. If the electrons have wavelengths, then they are stable only if the electron in an orbit interferes constructively with itself. (This condition, the Maxwell momentum expression, Newtonian mechanics and electrostatics, gives a model in good agreement with much of the observed spectrum of hydrogen.) (6)