## Question 1:

a) $\mathrm{PV}=\mathrm{nRT}$

$$
\rho \equiv \frac{\text { mass }}{\text { volume }}=\frac{\mathrm{nN}_{\mathrm{A}} \mathrm{~m}}{\mathrm{~V}}=\frac{\mathrm{PN}_{\mathrm{A}} \mathrm{~m}}{\mathrm{RT}}
$$

b) at constant pressure, $\quad \beta=\frac{1}{\mathrm{~V}} \frac{\mathrm{dV}}{\mathrm{dT}}=\frac{\mathrm{d}}{\mathrm{dT}} \ln \mathrm{V}$

$$
\begin{aligned}
\mathrm{PV}=\mathrm{nRT} & \therefore \ln \mathrm{P}+\ln \mathrm{V}=\ln \mathrm{n}+\ln \mathrm{R}+\ln \mathrm{T} \\
& \therefore \mathrm{~d} \ln \mathrm{~V}=\mathrm{d} \ln \mathrm{~T}=\frac{\mathrm{dT}}{\mathrm{~T}} \quad \therefore \beta=\frac{1}{\mathrm{~T}}
\end{aligned}
$$

OR $\quad \mathrm{V}=\frac{\mathrm{nRT}}{\mathrm{P}} \quad \therefore$ at constant pressure, $\quad \beta=\frac{1}{\mathrm{~V}} \frac{\mathrm{dV}}{\mathrm{dT}}=\frac{1}{\mathrm{~V}} \cdot \frac{\mathrm{nR}}{\mathrm{P}}=\frac{1}{\mathrm{~T}}$
c) $\frac{\Delta \mathrm{V}}{\mathrm{V}} \equiv \beta \Delta \mathrm{T}$
$\Delta \delta \mathrm{x} \approx \Delta \mathrm{V}_{\mathrm{fl}}-\Delta \mathrm{V}_{\text {sphere }}=\beta_{\mathrm{f} 1} \mathrm{~V} \Delta \mathrm{~T}-\beta_{\mathrm{g} 1} \mathrm{~V} \Delta \mathrm{~T}$
$\frac{\Delta x}{\Delta T}=\frac{V}{a}\left(\beta_{f 1}-\beta_{\mathrm{gl}}\right)$
d)

i) As shown at left.
ii) $\quad(\mathrm{C}-\mathrm{A})$ is an expansion. Adiabats $\left(\mathrm{PV}^{\gamma}=\right.$ constant) are steeper than isotherms ( $\mathrm{PV}^{1}$ $=$ constant) because $\gamma>1$. Therefore the volume $V_{1}>V_{3}$, as shown.
iii) A-B. When a gas is compressed isothermally ( $\mathrm{U}=$ constant), work is done on the gas so, from the first law, it must lose heat.
iv) Work ( $(\mathrm{PdV})$ is done when the volume increases with $\mathrm{P}>0$. So work is done by the gas in $\mathrm{B}-\mathrm{C}$ and $\mathrm{C}-\mathrm{A}$.

## Question 2

a)


In steady state, the temperature in the coatings is not changing. It follows that the heat flow H entering a given area of one side of either coating equals the heat flow H leaving the other side. Setting the heat flows through a given area $A$ of the coatings to be equal and using the definition of the thermal conductivity k gives:

$$
\begin{aligned}
& \mathrm{H} \equiv \mathrm{k}_{1} \mathrm{~A} \frac{\mathrm{~T}_{\mathrm{a}}-\mathrm{T}}{\mathrm{~d}_{1}}=\mathrm{H} \equiv \mathrm{k}_{2} \mathrm{~A} \frac{\mathrm{~T}-\mathrm{T}_{\mathrm{b}}}{\mathrm{~d}_{2}} \\
& \mathrm{k}_{1} \frac{\mathrm{~T}_{\mathrm{a}}-\mathrm{T}}{\mathrm{~d}_{1}}=\mathrm{k}_{2} \frac{\mathrm{~T}-\mathrm{T}_{\mathrm{b}}}{\mathrm{~d}_{2}} \\
& \mathrm{k}_{1} \mathrm{~d}_{2}\left(\mathrm{~T}_{\mathrm{a}}-\mathrm{T}\right)=\mathrm{k}_{2} \mathrm{~d}_{1}\left(\mathrm{~T}-\mathrm{T}_{\mathrm{b}}\right) \\
& \mathrm{k}_{1} \mathrm{~d}_{2} \mathrm{~T}_{\mathrm{a}}+\mathrm{k}_{2} \mathrm{~d}_{1} \mathrm{~T}_{\mathrm{b}}=\left(\mathrm{k}_{2} \mathrm{~d}_{1}+\mathrm{k}_{1} \mathrm{~d}_{2}\right) \mathrm{T} \\
& \mathrm{~T}=\frac{\mathrm{k}_{1} \mathrm{~d}_{2} \mathrm{~T}_{\mathrm{a}}+\mathrm{k}_{2} \mathrm{~d}_{1} \mathrm{~T}_{\mathrm{b}}}{\mathrm{k}_{2} \mathrm{~d}_{1}+\mathrm{k}_{1} \mathrm{~d}_{2}}
\end{aligned}
$$

b)

$\Delta$ momentum $=2 \mathrm{mv}_{\mathrm{X}}$
collide every $2 \mathrm{~L} / \mathrm{v}_{\mathrm{X}}$
$|\overline{\mathrm{F}}|=\left|\frac{\Delta \mathrm{p}}{\Delta \mathrm{t}}\right|=\frac{2 \mathrm{mv} v_{\mathrm{X}}}{2 \mathrm{~L} / \mathrm{v}_{\mathrm{X}}}$
$=\frac{\mathrm{mv}_{\mathrm{X}}{ }^{2}}{\mathrm{~L}}$
$\mathrm{F}_{\mathrm{all}}=\mathrm{PA}=\frac{\mathrm{Nm} \overline{\mathrm{v}_{\mathrm{x}}{ }^{2}}}{\mathrm{~L}}$
$P=\frac{F_{\text {all }}}{A}=\frac{N m \overline{V_{x}{ }^{2}}}{A L}=\frac{N m \overline{v_{x}{ }^{2}}}{\mathrm{~V}}$

Question 3 (marks)
a) Various statements are possible, such as:

If Newton's Laws of mechanics hold in one frame of reference, then in another frame of reference moving with constant, uniform relative velocity with respect to the first the laws of mechanics also hold. (Such frames are called inertial frames.)
a) Various statements are possible, such as:

If Newton's Laws of mechanics and Maxwell's laws of electromagnetism hold in one frame of reference, then in another frame of reference moving with constant, uniform relative velocity with respect to the first the laws of mechanics and the laws of electromagnetism also hold.
c).

(4 for the diagram)
i) $t_{a}=2 \mathrm{w} / \mathrm{c}$, where c is the speed of light.
ii) (From Einstein's principle of Special Relativity, both observers agree that the speed of light is c in all inertial frames.)

$$
\text { Pythagoras: } \quad w^{2}+\left(\mathrm{vt}_{\mathrm{b}} / 2\right)^{2}=\left(\mathrm{ct}_{\mathrm{b}} / 2\right)^{2}
$$

$$
\mathrm{w}^{2}=\mathrm{t}_{\mathrm{b}}{ }^{2}\left((\mathrm{c} / 2)^{2}-\left(\mathrm{vt}_{\mathrm{b}} / 2\right)^{2}\right)
$$

$$
\mathrm{t}_{\mathrm{b}}=\frac{2 \mathrm{w}}{\sqrt{\mathrm{c}^{2}-\mathrm{v}^{2}}}
$$

iii) $\quad \frac{\mathrm{t}_{\mathrm{b}}}{\mathrm{t}_{\mathrm{a}}}=\frac{1}{\sqrt{1-\mathrm{v}^{2} / \mathrm{c}_{2}}}$
iv) b observes that processes (as exemplified by the round trip of light) occur more slowly than in b's own frame

$$
\left(\text { by a factor } \frac{1}{\sqrt{1-v^{2} / c_{2}}}\right)
$$

v) From symmetry, a must also observe that processes occur more slowly in b's frame than in a's own frame

$$
\left(\text { by a factor } \frac{1}{\sqrt{1-v^{2} / c_{2}}}\right)
$$

## Question 4

a) The electron has a wavelength $\lambda=\mathrm{h} / \mathrm{p}$, where h is Planck's constant and p is the electron momentum. For constructive interference, an integral number of wavelengths must equal the circumference of the 'electron orbit'.
b) The condition described in (a) is only possible for certain discrete values of momentum and radius of orbit, and therefore the electron-proton system can have only certain discrete total energies. When light interacts with the H atom, only photons having these discrete energies may be absorbed.
c) Any suitable examples, including:

Virtual particles: a virtual particle-antiparticle pair, each of mass m, may appear and interact with other particles for a time $t$ provided that $2 \mathrm{mc}^{2} \mathrm{t} \lesssim \mathrm{h}$.
Diffraction: photons passing through a slit of width $\Delta x$ have an uncertainty in the $x$ component of momentum $\Delta \mathrm{p}_{\mathrm{x}} \gtrsim \mathrm{h} / \Delta \mathrm{x}$, hence a diffraction angle $\sin \theta=\frac{\delta \mathrm{p}_{\mathrm{x}}}{\mathrm{p}} \sim \frac{\lambda}{\delta \mathrm{x}}$.

Question 5 (13 marks)
i)

e.g. Young's experiment. Light passes through one slit (gives coherent source) then two slits. This gives an interference pattern on a screen. Waves arriving in phase give interference maxima, those arriving out of phase produce destructive interference. Interference is a phenomenon characteristic of waves.
ii)

e.g. The photoelectric effect. The apparatus uses a variable potential difference to turn back ejected electrons, and thus to measure their energy. Electrons are ejected by photons as soon as the metal electrode is exposed, whatever the intensity. The energy of an ejected electron depends on the wavelength, but not on the intensity. This suggests interaction with a single electron and therefore localisation. Localisation of energy in a collision is characteristic of particles.
iii) de Broglie introduced the idea to explain the quantisation of the energy of electron orbitals. In the Bohr model of the atom (small nucleus with classical circular electron orbits), the observed discrete emission spectrum can be explained if only certain orbits are possible. If the electrons have wavelengths, then they are stable only if the electron in an orbit interferes constructively with itself. (This condition, the Maxwell momentum expression, Newtonian mechanics and electrostatics, gives a model in good agreement with much of the observed spectrum of hydrogen.)

