Question 1 (20 marks)

a) At $T = T_o$, $V_{tank} = V_{liquid} = V_o$,

At $T \neq T_o$, $V_{tank} = V_o(1 + \beta_m(T - T_o))$

$= V_o(1 + 3\alpha_m(T - T_o))$

$V_l = V_o(1 + \beta_l(T - T_o))$

Overflow $= V_l - V_{tank} = V_o(\beta_l - 3\alpha_m)(T - T_o)$

(If $\beta_l > \beta_m$, as is usual, overflow occurs for $T > T_o$. If $\beta_l < \beta_m$ overflow occurs on cooling, but the equation is valid in both cases.)

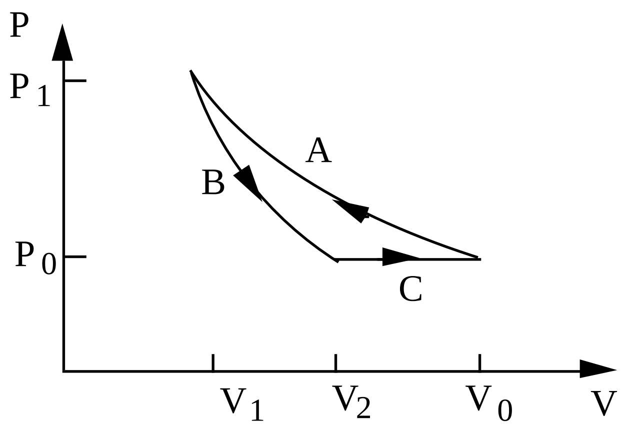
b) $PV = nRT$ $\quad \rho \equiv \dfrac{\text{mass}}{\text{volume}} = \dfrac{nN_A m}{V} = \dfrac{PN_A m}{RT}$

c) at constant pressure, $\beta = \dfrac{1}{V}\dfrac{dV}{dT} = \dfrac{d}{dT} \, ln \, V$

$PV = nRT \;\therefore\; ln\,P + ln\,V = ln\,n + ln\,R + ln\,T$

$\therefore\; d\,ln\,V = d\,ln\,T = \dfrac{dT}{T} \qquad \therefore\; \beta = \dfrac{1}{T}$

OR $\quad V = \dfrac{nRT}{P} \quad \therefore$ at constant pressure, $\beta = \dfrac{1}{V}\dfrac{dV}{dT} = \dfrac{1}{V} \cdot \dfrac{nR}{P} = \dfrac{1}{T}$



Step	Q	W	ΔU
A (isothermal compression)	-	-	0
B (adiabatic expansion)	0	+	-
C (isobaric)	+	+	+
Whole cycle (Σ)	-	-	0

Question 2 (21 marks)

i) $\bar{\varepsilon} = \frac{1}{2} m \overline{v^2} \equiv \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$

$\therefore \quad v_{r.m.s.} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{M/N_A}} = \sqrt{\frac{3 \times 1.38\ 10^{-23} \times 6\ 10^{23} \times 300}{0.032}}$

$= 480\ ms^{-1}$.

ii) $v_{r.m.s.} =$ ......

$= 1.9\ kms^{-1}$.

iii) Molecules have a wide range of energies, and some have considerably more than $v_{r.m.s.}$.

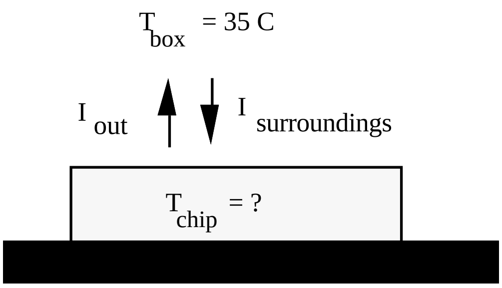
Over the age of the earth, enough hydrogen molecules would have had $\left(\frac{11}{1.9}\right)^2 = 33$ times the mean kinetic energy to escape from the atmosphere. Few oxygen molecules would have had $\left(\frac{11}{0.48}\right)^2 = 525$ times the mean kinetic energy, so we have retained O_2 and N_2, but no H_2.

(anything having most of these points scores 4)

(Not required in an answer: the proportion of molecules having more energy than $n\overline{K}$ is ~ exp(–n).

exp(–33) ~ 10^{-14}, while exp(–525) ~ 10^{-228}

a)



From the definition of k,

rate of heat loss $= kA\frac{\Delta T}{d} = 40$ mW

Radiation out – radiation in

$P = eA\sigma T^4 - eA\sigma T_s^4$

$= A\sigma(T^4 - T_s^4)$

$= 1 \times 10^{-4}\ m^2 \times 5.67\ 10^{-8}\ Wm^{-2}K^{-4}$

$\times ((323\ K)^4 - (303\ K)^4)$

= 62 mW – 48 mW = 14 mW

ii) Put a (thermally conducting but electrically insulated) heat sink on top of the chip. This conducts heat away and then loses it by radiation, or (if connected) by conduction to the existing heat sink.

Reduce the emissivity of the surroundings by coating them with a finish that has low emissivity e.

(any reasonably suggestion gets 3)

Question 3 (13 marks)

a) i) $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = 1.33$

The length of Bruce's ship, measured in Annes frame, is $L_o/\gamma = 5.6$ m.

ii) Either 5 or 6.

iii) "You didn't fire them simultaneously: you fired the one from the stern first, and it went past my bow, then you fired the next at bit later and so on, until you fired the last one after I was past you. That's why the first and the last ones missed."

iv) Simultaneity is relative . (Observers travelling at sufficiently high relative velocity may disagree with the time order of events which occur outside each other's light cone.)

v) "You didn't fire them simultaneously: you fired the from the stern first, and it went past my bow, then you fired the last one after I was past you. That's why one when ahead and one went behind." (Simultaneity is relative.)

Question 4 (10 marks)

i) $U_i + K_i = U_f + K_f$ (of course no marks to anyone who puts "KE = PE")

$K_f = U_i - U_f = e\Delta V = 20$ MeV

ii) $E^2 = p^2c^2 + m^2c^4$

$(mc^2 + K)^2 = p^2c^2 + m^2c^4$

$p = \sqrt{\dfrac{(mc^2 + K)^2}{c^2} - m^2c^2} =$ $\quad$ $kgms^{-1}$

iii) The e could lose < all its K in the collision.

Energy of photon = hf < K, f < K/h

$\lambda = \dfrac{c}{f} > \dfrac{ch}{K} =$ $\quad$ nm

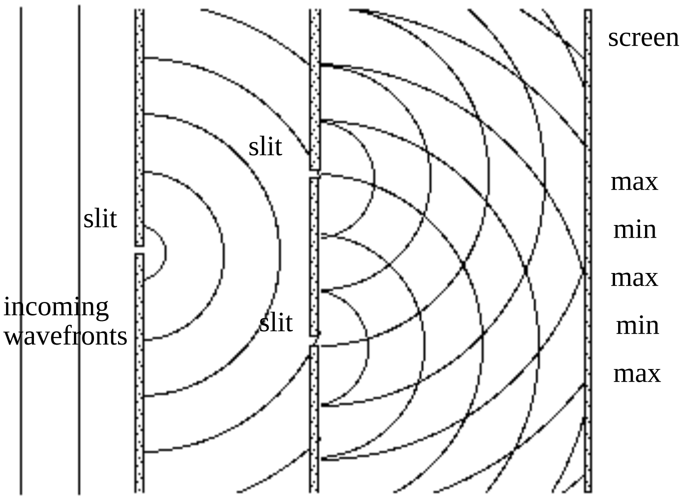
iv) If electron and positron are both annihilated:

$E = 2(K + mc^2)$. To conserve momentum, (at least) two photons must be created, so

$\lambda = \dfrac{c}{f} > \dfrac{ch}{(K + mc^2)} =$ $\quad$ pm

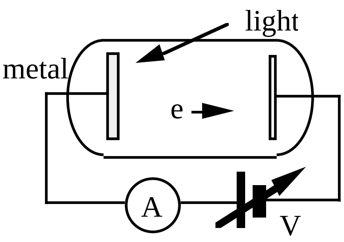
Question 5 (13 marks)

i)



e.g. Young's experiment. Light passes through one slit (gives *coherent source*) then two slits. This gives an interference pattern on a screen. Waves arriving in phase give interference maxima, those arriving out of phase produce destructive interference. Interference is a phenomenon characteristic of waves.

ii)



e.g. The photoelectric effect. The apparatus uses a variable potential difference to turn back ejected electrons, and thus to measure their energy. Electrons are ejected by photons as soon as the metal electrode is exposed, whatever the intensity. The energy of an ejected electron depends on the wavelength, but not on the intensity. This suggests interaction with a single electron and therefore localisation. Localisation of energy in a collision is characteristic of particles.

iii) de Broglie introduced the idea to explain the quantisation of the energy of electron orbitals. In the Bohr model of the atom (small nucleus with classical circular electron orbits), the observed discrete emission spectrum can be explained if only certain orbits are possible. If the electrons have wavelengths, then they are stable only if the electron in an orbit interferes constructively with itself. (This condition, the Maxwell momentum expression, Newtonian mechanics and electrostatics, gives a model in good agreement with much of the observed spectrum of hydrogen.)