PHYS1221 end of year test 2004 answers

Question 1 (14 marks)

i) Temperature is that quantity that is equal in two bodies in thermal equilibrium. (1)

ii) The heat capacity C of a body is the heat required to raise its temperature by unit temperature, or

Heat capacity $C = \frac{Q}{\Delta T}$ where Q is the heat added and ΔT is the temperature change resulting, or equivalent. (1)

iii) The specific heat c of a substance is the heat required to raise the temperature of unit mass (*or one mole*) of the substance by unit temperature, or

Specific heat $c = \frac{Q}{m\Delta T}$ where Q is the heat added, m is the mass (*or , for the molar heat capacity, the number of moles. Either to get a mark*) and ΔT is the temperature change resulting, or equivalent. (1)

iv) $\Delta V = \beta_w V\Delta T$ so $A\Delta h = \beta_w A h\Delta T$ so $\Delta h = \beta_w h\Delta T = 2.1\ 10^{-4}\ °C^{-1} * 4000\text{m} * 2°C = 1.7$ m.

(5)

v) Let the initial volume of gas contain n moles. $P_A V_0 = nRT_0$, where R is the gas constant.

At the new thermal equilibrium, the pressure P satisfies $PV = PAh_0 = nRT_0$.

Mechanical equilibrium requires $mg = (P - P_A)A$ so $P = P_A + mg/A$

Combining these equations and rearranging: $h_0 = \frac{nRT_0}{PA} = \frac{P_A V_0}{PA} = \frac{P_A V_0}{(P_A + mg/A)A}$

$h_0 = \frac{P_A V_0}{P_A A + mg} = \frac{V_0/A}{1 + mg/P_A A}$ or an equivalent expression. (6)

Question 2 (29 marks)

i) a) 20% efficient, so rate of heat production = 4 times rate of mechanical power.

$H = 4\frac{d}{dt}mgh = 4mg\frac{dh}{dt} = 4*(80\text{ kg})*(9.8\text{ms}^{-2})(0.55\text{ m.s}^{-1}) = 1.7\text{ kW}$ (4)

b) Rate of heat lost from skin by radiation – rate of heat absorbed by skin from surroundings

$H_{rad} = A\sigma\left(e_o T_o^4 - e_s T_s^4\right)$

$= (1.8\text{ m}^2)(5.67\ 10^{-8}\text{ Wm}^{-2}\text{K}^{-4})*0.8*\left((307\text{ K})^4 - (303\text{ K})^4\right)$

$= 37$ W (3)

c) Energy to evaporate water $= L.m_{sweat}$ where m_{sweat} is the mass of sweat evaporated

Rate of heat lost by evaporation $H = L\frac{dm_{sweat}}{dt}$

Rate of sweating = H/L = $(1.7\text{ kW})/(2.5\text{ MJ.kg}^{-1}) = 7.8\ 10^{-4}\text{ kg.s}^{-1}$

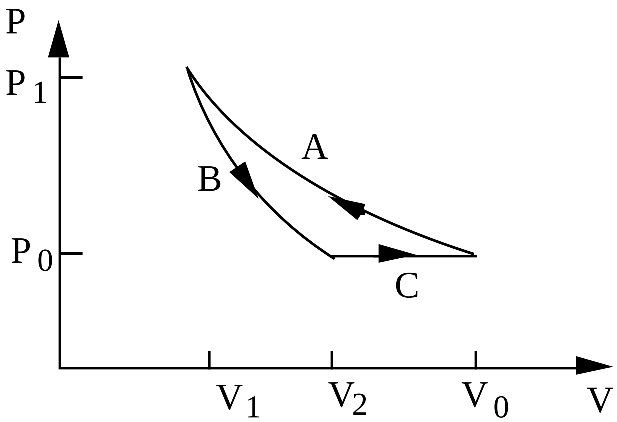
= 2.8 kg/hr = 2.8 litres/hour. (4)

d) Rate of heat produced – rate of heat lost $= H_{new} = H - 500\text{ W} = 1.2$ kW. (2)

Definition of heat capacity: $Q \equiv mc\Delta T \qquad Q = H_{new}t$ (2)

$$t = \frac{Q}{H_{new}} = \frac{mc\Delta T}{H_{new}} = \frac{(80\ \text{kg})(4\ \text{kJ.kg}^{-1}.°\text{C}^{-1})(2°\text{C})}{1.2\ \text{kW}} = 500\ \text{s} = 9\ \text{minutes} \qquad (3)$$

ii)

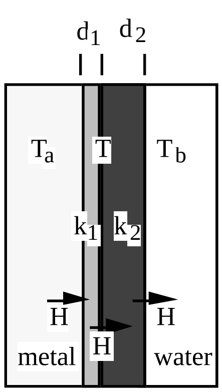


(4)

Step	Q	W	ΔU
A (isothermal compression)	-	-	0
B (adiabatic expansion)	0	+	-
C (isobaric)	+	+	+
Whole cycle (Σ)	-	-	0

(6)

iii)



In steady state, the temperature in the coatings is not changing. It follows that the heat flow H entering a given area of one side of either coating equals the heat flow H leaving the other side. Setting the heat flows through a given area A of the coatings to be equal and using the definition of the thermal conductivity k gives:

$$H \equiv k_1 A\frac{T_a - T}{d_1} = H \equiv k_2 A\frac{T - T_b}{d_2}$$

$$k_1\frac{T_a - T}{d_1} = k_2\frac{T - T_b}{d_2}$$

$$k_1 d_2 (T_a - T) = k_2 d_1 (T - T_b)$$

$$k_1 d_2 T_a + k_2 d_1 T_b = (k_2 d_1 + k_1 d_2)T$$

$$T = \frac{k_1 d_2 T_a + k_2 d_1 T_b}{k_2 d_1 + k_1 d_2} \qquad (5)$$

Question 3 (14 marks)

i) For the judges, using Pythagoras' theorem, the light beam travels $2\sqrt{(L/2)^2 + w_n^2}$. It travels at c, so the time taken by the car

$$t_{judge} = \frac{2\sqrt{(L/2)^2 + w_n^2}}{c} = \frac{\sqrt{L^2 + 4w_n^2}}{c}.$$ (simplification not required for marks) (3)

ii) For the judges, $v = L/t_{judge}$, so

$$v = \frac{cL}{\sqrt{L^2 + 4w_n^2}} = \frac{c}{\sqrt{1 + 4(w_n/L)^2}}$$ (ditto) (2)

iii) $1 + 4(w_n/L)^2 = (c/v)^2$ so $L/w_n = \dfrac{2}{\sqrt{(c/v)^2 - 1}}$ (3)

iv) From Jane's point of view, the light beam travels at right angles to the car: it goes out to the right, strikes the reflector when the car is at the halfway point, and arrives at the detector at the finish. She is an inertial observer, so, according to the priniciple of special relativity, light travels at c with respect to her. Hence the time taken is

$t_{Jane} = 2w_n/c$ (2)

iv) Jane would conclude that, from her frame of reference, the course is actually shorter than L: it has suffered a relativistic length contraction due to its speed v, relative to her. (2)

v) The judges would conclude that Jane's clock is running slow. It has suffered a relativistic time factor due to its speed v, relative to them. (Note that the judges can measure the proper length L.) (2)

Question 4 (8 marks)

i) (Iron is a very stable nucleus.) Because iron is stable, it takes energy to "pull its nucleus apart" into its components. Adding this energy (the binding energy E) to the components increases their mass. So the component nucleons are more massive by an amount $\delta m = E/c^2$. (3)

ii) The proper time lifetime of the muons equals that measured at negligible speed, ie

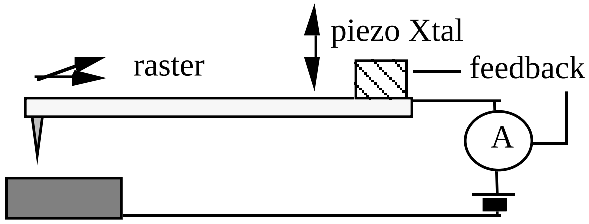
$t_p = 2.2\ \mu s$. For the high speed muons, $t = 16\ \mu s$.

$$\frac{t'}{t} = \sqrt{1 - \frac{v^2}{c^2}} \qquad \left(\frac{t'}{t}\right)^2 = 1 - \frac{v^2}{c^2} \qquad 1 - \left(\frac{t'}{t}\right)^2 = \frac{v^2}{c^2}$$

$$c\sqrt{1 - \left(\frac{t'}{t}\right)^2} = v = 0.99\ c$$ (5)

Question 5 (13 marks)

i) In the Scanning Tunnelling Electron Microscope (STEM), a very sharp point is held very close to the sample. The point scans over the sample (in a raster of x&y). An applied voltage allows electrons to tunnel across the gap (usually a vacuum). The electric current is measured, which gives the rate of tunnelling. This is a strong function of how close the tip is to the sample. (In the usual mode of operation, a feedback loop is applied in the z direction to keep the tunnelling current constant.) (*The level of detail expected is less than this: any comments about measuring the tunnelling current across the small gap to get the distance should score well. There may also be descriptions of a tunnel diode. Again, a reasonable explanation should do well.*)



(4)

ii) a) A particle-antiparticle pair have opposite charge and spin, and the same mass, m. To create them from nothing requires an energy $E > 2mc^2$. Creating them for an indefinite time without this energy is impossible. However, for a time $t < \hbar / E$, conservation of energy is not violated because of Heisenberg's Uncertainty Principle. So a pair of virtual particle and antiparticle can spontaneously exist for such a time. (5 marks)

b) The virtual particles that mediate the strong force have finite mass and therefore limited lifetimes. They cannot travel further than c times their lifetime, so the range is finite. Virtual photons, which are massless, mediate the electrical interaction, so the range is potentially infinite. (4 marks)