

PHYS1221-1231 tutorials

PHYS1231 Tutorial 9 answers

$$Q1 \quad \lambda = h/p = h/mv = \frac{h}{v\rho \cdot \frac{4}{3}\pi r^3} = 2 \cdot 10^{-30} \text{ m}$$

A slit with width comparable to $2 \cdot 10^{-30}$ m would not let molecules through, let alone raindrops.

$$Q2 \quad \text{Energy conservation: } \Delta E + \Delta K = 0, \text{ so } eV = \frac{1}{2}mv^2 \text{ so } v = \sqrt{2eV/m} = 60,000 \text{ km/s.}$$

$$\lambda = h/p = 12 \text{ pm.}$$

The relativistic factor $\gamma = 1.02$. So, for this problem with only one or two significant figures, it's not important. However, for the purposes of getting the focussing right, yes, it must be considered.

$$Q3 \quad \text{Photons: } E = hc/\lambda = 400 \text{ eV (soft X-rays).}$$

Electrons: $E = p^2/2m = h^2/2\lambda^2m = .3 \text{ eV}$. Accelerating voltage 0.3 V. (In practice EM uses typically many kV, but note that λ goes only as \sqrt{E} and that focussing technologies are far from ideal.

Sound: $f = v/\lambda = 100 \text{ GHz}$. (We don't know how to make solid objects vibrate that quickly. Remember the limited resolution of ultrasound pictures: it's not easy to tell the sex of a foetus.)

$$Q4 \quad \text{Energy conservation: } \Delta E + \Delta K = 0, \text{ so } eV = \frac{1}{2}mv^2 \text{ so } v = \sqrt{2eV/m} = 60,000 \text{ km/s.}$$

$$\lambda = h/p = h/\sqrt{2m \cdot e \cdot V} = 9 \text{ pm.}$$

$E_{\text{photon}} = hc/\lambda = 140 \text{ keV}$ (hard Xrays). Compare with 20 keV for the electron microscope. Xrays are difficult to focus (what would you use for a lens or a mirror). However, they are widely used for diffraction studies, provided that the object is crystalline.

$$Q5 \quad mv = p = h/\lambda \quad v = h/m\lambda = 1.3 \text{ km/s}$$

$$\frac{1}{2}mv^2 = K_e = qV \quad \text{so } V = 5 \mu\text{V}$$

$$Q6 \quad E = \frac{p^2}{2m} = \frac{p_x^2 + p_y^2 + p_z^2}{2m} = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2) = n^2 \frac{h^2}{8mL^2}$$

$$\text{Here: } n_x^2 + n_y^2 + n_z^2 \leq \frac{8mL^2E}{h^2} = 12.8. \text{ The } n_i \text{ are integers, so } n_x^2 + n_y^2 + n_z^2 \leq 12.8.$$

The allowable levels are (1,1,1), (1,1,2), (1,2,1), (2,1,1), (1,2,2), (2,1,2), (2,2,1), (2,2,2), (1,1,3), (1,1,3), (3,1,1), So there are 11 levels (and thus 22 states, because the electrons may have two values of spin).

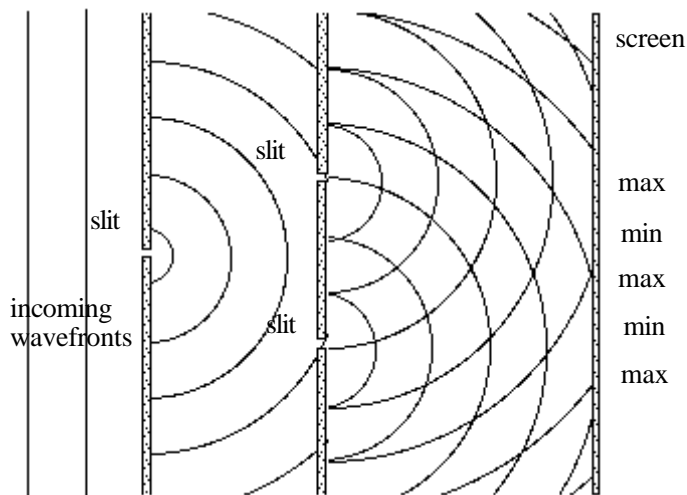
Q7 Compare: A particle detector will detect many particles per unit time where $\psi\psi^*$ is large, an antenna or microphone will detect a large intensity where E^2 or p^2 is large. At very low particle densities and at very low intensities, the first sentence must be interpreted statistically: ie the detectors will, on average, make larger signals where $\psi\psi^*$, E^2 or p^2 is large.

PHYS1221-1231 tutorials

Contrast: we usually use the QM picture when dealing with small numbers of particles (usually fermions such as electrons, that cannot share quantum numbers). With EM and acoustic waves, we usually have very large numbers of photons and phonons, often in phase, so we don't notice the statistical aspect. At extremely low intensities, individual photon and phonon effects are seen. (eg a human photoreceptor needs about 7 photons to respond, under optimal conditions). (Bose-Einstein condensates are to some extent analogous to multi-photon and multi-phonon standing waves.) The parenthetical sentences are beyond the syllabus.

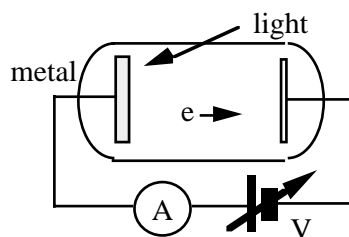
Answer to past test question

i)



e.g. Young's experiment. Light passes through one slit (gives *coherent source*) then two slits. This gives an interference pattern on a screen. Waves arriving in phase give interference maxima, those arriving out of phase produce destructive interference. Interference is a phenomenon characteristic of waves.

ii)



e.g. The photoelectric effect. The apparatus uses a variable potential difference to turn back ejected electrons, and thus to measure their energy. Electrons are ejected by photons as soon as the metal electrode is exposed, whatever the intensity. The energy of an ejected electron depends on the wavelength, but not on the intensity. This suggests interaction with a single electron and therefore localisation. Localisation of energy in a collision is characteristic of particles.

E1 $\Delta S = S_2 - S_1 = k \ln \frac{W_2}{W_1} = k \ln \frac{W_1^2}{W_1} = k \ln W_1$. The energy of dissociation is finite, consequently the free energy of dissociation is *extremely* negative – in an infinite universe it would be infinitely negative. Or, to put it informally: in the 'entropy-energy balance' that characterises chemical equilibrium, the entropy wins. According to this argument, all molecules are unstable. Breathe deeply.

However, the rate of dissociation includes a Boltzmann term $\exp(-E_{\text{dissoc}}/kT)$, so the rate is negligible, unless UV photons are available. In interstellar space, would you expect to find more oxygen (or hydrogen) in molecular or atomic form?

E2 If the photons can undergo interference, their coherence length must be long (much longer than the thickness of the film) or, if you prefer, their wavelength must be well defined. HUP says that if the wavelength is well defined, the photon is not confined in space,. Either way, the image of a photon 'arriving' at the interface is misleading. Even a single photon is a wave. And it is either reflected or not, according to the boundary conditions that it samples in all of the space over which it is distributed.