PHYS1221-1231 tutorials

PHYS1231 Tutorial 8 Answers

Q1 $E_{photon} = hf = hc/\lambda = \dfrac{(6.63\ 10^{-34}\ \text{Js})(3.00\ 10^{8}\ \text{m.s}^{-1})}{350\ \text{nm}} = 5.7\ 10^{-19}\ \text{J} = 3.55\ \text{eV}$

$E_{max} = E_{photon} - \phi = 1.25$ eV for Li.

Set $E_{max} = \frac{1}{2} mv^2 \rightarrow v = 660{,}000\ \text{m.s}^{-1}$ (<< c so classical mechanics is okay).

The other metals do not exhibit the photoelectric effect for this wavelength.

Q2 $\dfrac{1}{\lambda} = -R.\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$ here take $n_1 = \infty$ for complete ionisation and $n_2 = 2$ for first excited state.

$\dfrac{1}{\lambda} > \dfrac{R}{4} \quad \therefore \quad \lambda < \dfrac{4}{R} = 360$ nm. $\left(\text{or else derive from } E = -\left(\dfrac{me^4}{8\varepsilon_o^2 h^2}\right)\dfrac{1}{n^2}\right)$

Q3 centripital force $\dfrac{mv^2}{r} = F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{e^2}{r^2}$ (1)

$rv^2 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{e^2}{m}$ (2)

de Broglie $2\pi r = n\lambda = \dfrac{nh}{p} = \dfrac{nh}{mv}$

$\therefore \quad rv = \dfrac{nh}{2\pi m}$ (3)

Solve (2) & (3) $\quad r = \dfrac{n^2h^2}{\pi m}\dfrac{\varepsilon_0}{e^2} \quad \rightarrow \quad n = 4$

Q4 $U = -\dfrac{e^2}{4\pi\varepsilon_0 r} \qquad F_{centrip} = \dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r} = \dfrac{2K}{r}$

Multiply RH equation by $r \rightarrow U = -2K$

So $E = U + K = -K$, so $U/E = -2$ and $K/E = -1$

Q5 The sails are shiny, so they reflect the light, so the momentum transferred to the sail is twice the incident.

Radiation force $= \dfrac{2\Delta p}{t} = \dfrac{2E}{ct} = \dfrac{2P}{c} = \dfrac{2IA}{c}$ where I is the intensity and A the sail area.

Gravitational force $= \dfrac{Gm_{sun}m_{sail}}{r^2} = \dfrac{Gm_{sun}\rho Ad}{r^2}$ where d is the thickness.

Setting these equal, $d = \dfrac{2Ir^2}{cGm_{sun}\rho} = 1.6\ \mu\text{m}$.

It doesn't vary with distance: both weight and radiation go as $1/r^2$. If you know of keel for the solar sailor, don’t tell us – tell NASA!

Q6 Nucleus has charge 2e, single electron still has -e

As before, mechanical energy = kinetic + potential

$E = \left(\frac{1}{2}mv^2\right) + \left(-\dfrac{1}{4\pi\varepsilon_o}\dfrac{\boldsymbol{2}e^2}{r}\right)$ bold italic shows effect of extra proton hereafter.

centripital force $\dfrac{mv^2}{r} = F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{\boldsymbol{2}e^2}{r^2}$ (1)

$\therefore$ (sustitute) $E = -\frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{\boldsymbol{2}e^2}{r}$ (2)

Now $2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$

$\therefore$ $v = \frac{nh}{2\pi rm}$ (3)

and (1) $\Rightarrow$ $\frac{1}{2}mv^2 = \frac{\boldsymbol{2}ke^2}{2r}$

solve (1) & (3) for r, substitute in (2)

$$\therefore \quad E = -\left(\frac{\boldsymbol{4}me^4}{8\varepsilon_o^2h^2}\right)\frac{1}{n^2}$$

$$\therefore \quad \frac{1}{\lambda} = \frac{\Delta E}{hc} = -\boldsymbol{4}R.\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)$$

where R = Rydberg's constant = $1.10\ 10^7\ m^{-1}$

So, looking at the terms in bold italic, the energies are 4 times and the wavelengths 1/4 those for hydrogen, respectively.

Q7 An accelerated charge radiates EM waves. This is how a transmitting antenna works: electrons are accelerated backwards and forwards along it. This energy would be taken from the mechanical energy of the electron's orbit (see Q4)

According to one popular theory of gravitation (Einstein's General Theory of Relativity), the accelerating Earth does indeed radiate gravitational energy. However, the radition emitted by orbiting planets and stars is predicted to be so feeble that it is no surprise that the equisitely sensitive gravity radiation detectors (including one at UWA) have thus far not been able to detect it. So don't worry, the sun will engulf the earth (when the former becomes temporarily a red giant) long before the earth falls into the sun.

Answer to past test question

i) $\frac{mv^2}{R} = F_{centrip} = G\frac{Mm}{R^2}$

$$v = \sqrt{\frac{GM}{R}} \qquad (3)$$

ii) de Broglie: $\rightarrow$ $p = h/\lambda$ constructive interference $\rightarrow$ $2\pi R = n\lambda$

$2\pi r = n\frac{h}{m_pv}$ but substitution from (i) gives

$$2\pi r = n\frac{h}{m_p}\sqrt{\frac{r}{GM}}$$

$$\sqrt{r} = n\frac{h}{2\pi m_p\sqrt{GM}}$$

$$r = n^2\frac{h^2}{4\pi^2Gm_p{}^2M}$$

iii) $E_1 = U_g/2 = -\frac{GMm_p}{2r_\supseteq} = -\frac{2\pi^2G^2M^2m_p{}^3}{h^2\supseteq} = E_H = -13.6\text{ eV} = -13.6 \times 1.6\ 10^{-19}$ J.

$$M = \sqrt{\frac{U_Hh^2}{2\pi^2G^2m_p{}^3}} = \sqrt{\frac{13.6 \times 1.6\ 10^{-19}\ J \times (6.63\ 10^{-34}\ Js)^2}{2\pi^2(6.67\ 10^{-11}\ m^3s^{-2}kg^{-1})^2(1.67\ 10^{-27})^3}} = 48{,}000 \text{ tonnes}$$

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E1 Force $= \frac{\Delta p}{t} = \frac{E}{ct} = \frac{P}{c} = \frac{120\ \text{W}}{3\ 10^8\ \text{ms}^{-1}} = 400$ nN

Power = Fv = ... 12 μW.

a) The generator requires more than 120 W to power these lamps, which (via a belt to the crankshaft) takes a power of more than 120 W from the engine.

b) The air resistance is greater. From the ideal gas equation, a 20 K fall in temperature increases the density of air by 20K/300K = 7%. The air resistance ($C_d\rho Av^2/2$) is typically 100 N at 100 kph (it increases if you add bull bars, spoilers etc), so the increased air resistance requires a few hundred extra watts.

c) For the same reason, the air pressure in the tires falls (slightly) at night, and this increases the rolling resistance.

d) On the other hand, the air conditioner is less likely to be on at night and the widows are more likely to be closed. AC and open windows both require extra power.

E2 To be permanently emitted from the sphere, the electron must have a total energy of 0 after emission. Its potential energy at emission is –Ve, where V is the potential of the sphere. So the energy of the photon is

$E = hc/\lambda = \phi + Ve$

For a spherical capacitor, $C = 4\pi\varepsilon_0 R$, so

$Q = CV = 4\pi\varepsilon_0 R(hc/\lambda - \phi)/e = 47$ pC