PHYS1221-1231 tutorials

Tutorial 10 Answers

Q1 $\Delta E.\Delta t = h/2\pi$. $E = -\left(\frac{me^4}{8\varepsilon_o^2 h^2}\right)\frac{1}{n^2}$

$\Delta E/E = \frac{h}{2\pi\Delta t.E}$ = 20 parts per billion. 8 significant figures only

Q2 $\Delta E.\Delta t = h/2\pi$. So $\Delta E/E = \frac{h}{2\pi\Delta t.E}$ = 2 parts in 10^{10}.

Q3 The photons must have $\lambda \sim 10$ pm, so they have $E = hc/\lambda = 120$ keV and $p = h/\lambda = 7\ 1 10^{-23}$ kgm.s^{-1}

The binding energy of the hydrogen atom is only 13 eV so interactions with these photons will ionise it.

The momentum of Sommerfeld electrons is $\frac{m}{2\varepsilon_o}\frac{e^2}{nh} = 2\ 10^{-25}$ kgm.s^{-1} << p_{photon} so the collisions would give the electrons large recoil velocities.

Q4 $\Delta f.\Delta t \geq 1/2\pi$. $\Delta f \geq 0.16$ Hz. For the second case, 1.6 Hz.

Answers for musicians: At this pitch, the uncertainty is half a semitone. (In practice, this is only 5 vibrations of the string, and it takes at least a few cycles for the bow to set up periodic Helmholtz motion, so it won't have a clear pitch at all. Musicians know this: in very fast runs in the bass you can get away with a lot: the ear will interpolate.

Serves the composer right for writing low demisemiquavers for the basses – but then Schubert never finished that symphony: Detective we're re-opening the case – I want you to interview all the bassists who were in Vienna on or around the night of 18 November 1828.)

Q5 For electrons, $Id\theta$ would be proportional to the number of electrons detected between the angles θ and $\theta+d\theta$. The integral of I from $\theta = -\pi/2$ to $\pi/2$ would be set equal to N, and this equation would define I_0, the maximum in the electron distribution function.

If you covered one slit, only half as many electrons would pass through. Further, there would be no interference, and only diffraction, so

$$I_{single}(\theta) = \frac{I_0}{2} \cos^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Q6 $n_e = \frac{2}{3}\frac{8\sqrt{2}\,\pi\, m_e^{3/2}}{h^3} E_F^{3/2} = 8.5\ 10^{28}$ electrons/m^3.

atom density = density/(atomic mass) = $\frac{(8920\ \text{kg.m}^{-3})}{(63.5\ \text{kg/kmol} / 6\ 10^{26}\ \text{atoms/kmol})} = 8.4\ 10^{28}$ atoms/m^3. So one conduction electron per atom.

$$v = \sqrt{\frac{2K}{m}} \cong \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 * 7.05\ \text{eV}}{9.11\ 10^{-31}\ \text{kg}}} = 1.6\ 10^3\ \text{km/s} = 0.5\%\ \text{of c}.$$

For a conductor, $i = nAvq$ so $v = i/nAq = 0.95$ mm/s.

If all moved at 0.5% of c, $i = nAvq \rightarrow 17$ GA

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E1 $P = \int \psi^2 \, dV$. In spherical symmetry, the volume element is $dV = 4\pi r^2 dr$.

$$P = \frac{4\pi}{\pi a_0^3} \int e^{-2r/a_0} \, r^2 dr = -\frac{2}{a_0^2} \left[e^{-2r/a_0}\left(r^2 + ra_0 + a_0^2/2\right) \right]_{r=0}^{r=R}$$

$$= 1 - \frac{2e^{-2r/a_0}}{a_0^2}\left(R^2 + Ra_0 + a_0^2/2\right)$$

Set $R = a_0$: $P = 1 - e^{-2}\frac{5}{2} = 0.66$

Set $R = 2a_0$: $P = 1 - e^{-4}\frac{13}{2} = 0.88$

Set $R = \infty$: $P = 1$

This last result is called normalisation: "everything has got to be somewhere", and in solving the Schrödinger equation, the factor $\frac{1}{\sqrt{\pi a_0^3}}$ comes from setting this integral = 1.

E2 $$E = \left(\frac{1}{2} mv^2\right) + \left(-\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r}\right)$$

but $$\frac{mv^2}{r} = F = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \qquad (1)$$

$\therefore$ (sustitute) $$E = -\frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r} \qquad (2)$$

Now $$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

$\therefore$ for $n = 1$, $$r = \frac{h}{2\pi mv} \qquad (3)$$

(1), (3) $\rightarrow$ $$v^2 = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{mr} = \frac{2\pi}{4\pi\varepsilon_o}\frac{ve^2}{h}$$

$$v = \frac{1}{2\varepsilon_o}\frac{e^2}{h}$$

$$\frac{c}{v} = \frac{2hc\varepsilon_0}{e^2} = 137$$ so $$\alpha = \frac{e^2}{2hc\varepsilon_0} = 0.0073$$

For a discussion, see http://www.phys.unsw.edu.au/einsteinlight/jw/module6_constant.htm