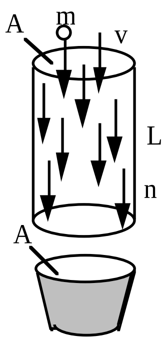


An analogy: How much rain falls in a bucket per second?

Bucket has area A. Raindrops of mass m, are falling at v. (*Why?*)

There are n drops per m^3.

Look at the cylinder. It contains:

n*volume = nAL raindrops,

so all this water leaves the cylinder in a time

$$t = L/v$$

(the time taken for the one just entering to get to the other end)

So rate of rain fall in bucket

$$I = \frac{\text{mass}}{\text{unit time}} = \frac{nALm}{L/v} = nAvm$$

$\propto$ number of mass carriers/cubic metre of air

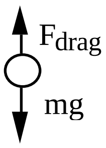
$\propto$ cross sectional area of 'conductor'

$\propto$ velocity of carriers

$\propto$ mass of carrier

'Drag' on drop depends on size air, etc, and on v

Put $F_{drag} = K_{drag}.v$



At terminal velocity,

$\underline{\mathbf{a}} = 0$, so $\Sigma\, \underline{\mathbf{F}} = 0$:

$F_{drag} = mg$

$$v = F_{drag}/K_{drag} = mg/K_{drag}$$

$$\therefore \quad I = \frac{\text{mass}}{\text{unit time}} = \frac{nAm^2g}{K_{drag}}$$

$\propto$ number of mass carriers/cubic metre

$\propto$ cross sectional area of 'conductor'

$\propto$ (mass of carrier)^2 squared because heavier fall faster

$\propto$ field that moves them

$\propto$ $1/K_{drag}$

$$V_{grav} \equiv \frac{U_{grav}}{m} = \frac{mgh}{m} = gh$$

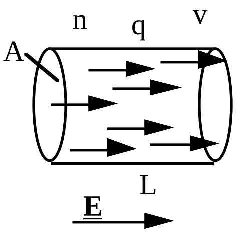
$$\frac{\Delta V_{grav}}{I} = gL\frac{K_{drag}}{nAm^2g} = \frac{\rho L}{A}$$

$$\text{where } \rho = \frac{K_{drag}}{nm^2}$$

$$\frac{\Delta V_{grav}}{I} = R \qquad \text{where } R = \frac{\rho L}{A}$$

Direct Currents (D.C.)

"drift speed" v in electric field E , $\frac{\text{n carriers}}{\text{unit volume}}$



$I \equiv$ \f(dq,dt) = \f(charge past point,per unit time)
n*vol carriers have charge
n.AL.q

Last charge gets to end after time $t = L/v$

$$\therefore \quad I = \frac{nALq}{L/v} = nAqv$$

$$v = \text{const.}*E = \text{const.}*\frac{V}{L}$$

$$I = nAqv = V.nq.\text{const.}\frac{A}{L}$$

$$\frac{V}{I} = \frac{1}{n.q.\text{const}}\frac{L}{A}$$

$$\frac{V}{I} = \rho\frac{L}{A} = R$$

R is the **resistance** of piece of contdutor,

ρ is the **resitivity** of the material

Alternatively

$$\frac{I}{V} = \sigma\frac{A}{L} = G$$

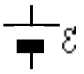
G is the **conductance** of this piece of conductor,

σ is the **conductivity** of the material

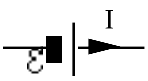
$$\sigma = n.q.\text{const} = \frac{1}{\rho}$$

Units

voltage →	$\frac{\text{joules}}{\text{coulomb}}$	≡ volt	V
current →	$\frac{\text{coulombs}}{\text{second}}$	≡ ampere	A
resistance →	$\frac{\text{volts}}{\text{ampere}}$	≡ ohm	Ω
conductance →	$\frac{\text{amperes}}{\text{volt}}$	= 'mho'	℧
resistivity			Ωm
conductivity			℧/m



An e.m.f. takes charge in at one terminal and puts it out at the other, with a different (usually higher) potential. It converts some energy into electrical potential energy.



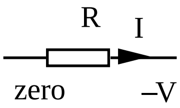
Power in an emf:

current I flows through an emf for time t and delivers charge q

$$\text{work done} = \Delta U \equiv q\Delta V = q\mathcal{E}$$

$$\text{Power} \equiv \frac{\text{work}}{\text{time}} = \frac{q\mathcal{E}}{t} \equiv I\mathcal{E}$$

$$\text{Power produced} = \mathcal{E}I$$ *if I is -ve, P is -ve*



Power in a resistor:

current I flows through a resistor for time t and delivers charge q over a voltage *drop* V

$$\text{energy lost} = |\Delta U| = |qV| = q|V|$$

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{q|V|}{t} \equiv I|V|$$

where V is the voltage *drop*, in the direction of the current. Now I = V/R or V = IR, so

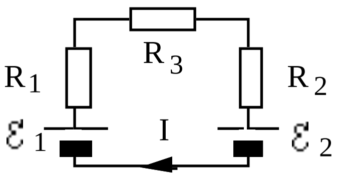
Power lost in resistor $\quad P_r = IV = I^2R = \dfrac{V^2}{R}$

Conservation of energy: An electron is unchanged in one voyage around the circuit, so its potential energy is unchanged

Σ rises in U = Σ falls in U

Σ rises in V = Σ falls in V

$\Sigma\, \mathcal{E} = \Sigma\, IR$ **Kirchoff's Loop Rule**



In this circuit consider current flowing clockwise:

$\Sigma\, \mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2$

-ve because in clockwise sense it is **fall** in V

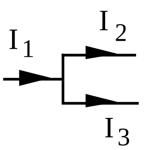
$\Sigma\, IR = IR_1 + IR_2 + IR_3$

$\mathcal{E}_1 - \mathcal{E}_2 = IR_1 + IR_2 + IR_3$

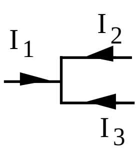
$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + R_3}$$

Conservation of charge:

At a circuit junction, charge doesn't disappear or appear, e.g.



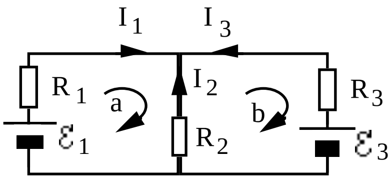
or



$I_1 = I_2 + I_3$ or $I_1 + I_2 + I_3 = 0$

$\Sigma\, i_{in} = \Sigma\, i_{out}$ Kirchchoff's Junction Rule

Example: Circuits with more than one loop



Loop a: $\Sigma\,\mathcal{E} = \Sigma\, IR$

clockwise $\mathcal{E}_1 = +I_1R_1 - I_2R_2$ (1)

+ve because rise in V in chosen dirn.	+ve because is V drop in chosen dirn.	–ve because is V *rise* in chosen dirn.

Loop b: $\Sigma\,\mathcal{E} = \Sigma\, IR$

clockwise $-\mathcal{E}_3 = +I_2R_2 - I_3R_3$ (2)

–ve because drop in V in chosen dirn.	+ve because is V drop in chosen dirn.	–ve because is V *rise* in chosen dirn.

2 equations in 2 unknowns. (3rd loop just gives the sum of these two.) Use the junction rule:

$I_1 + I_2 + I_3 = 0$ (3)

(2) & (3) eliminate I_3: $-I_3 = I_1 + I_2$

$\therefore\quad -\mathcal{E}_3 = I_2R_2 + I_1R_3 + I_2R_3$ (2')

$-R_3(1) + R_1(2')$ to eliminate I_1

$$-R_3\mathcal{E}_1 - R_1\mathcal{E}_3 = +I_2R_2R_3 + I_2R_1R_2 + I_2R_1R_3$$

$$I_2 = -\frac{R_3\mathcal{E}_1 + R_1\mathcal{E}_3}{R_1R_2 + R_2R_3 + R_1R_3}$$

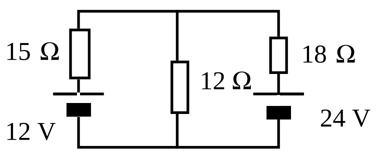
substitute in (1) →

$$I_1 = \frac{R_2\mathcal{E}_1 + R_3\mathcal{E}_1 - R_2\mathcal{E}_3}{R_1R_2 + R_2R_3 + R_1R_3}$$ *units okay*

substitute in (2) or (3) →

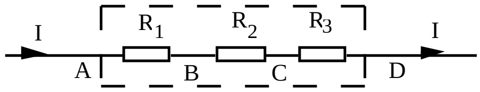
$$I_3 = \frac{R_2\mathcal{E}_3 + R_1\mathcal{E}_3 - R_2\mathcal{E}_1}{R_1R_2 + R_2R_3 + R_1R_3}$$ *units okay symmetric with I_1*

But note: this is a case where the problem is easier if one uses values right from the beginning.



HOMEWORK
Find the current in the 12 Ω resistor:

Resistors in series



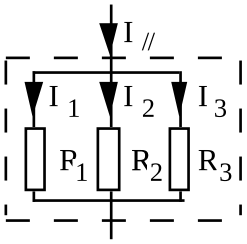
$$R_{series} \equiv \frac{V_{AD}}{I}$$

$$= \frac{V_{AB} + V_{BC} + V_{CD}}{I}$$

$$= \frac{IR_1 + IR_2 + IR_3}{I}$$

$$R_{series} = R_1 + R_2 + R_3$$

Resistors in parallel



Voltage is same for all

$$I_{//} = I_1 + I_2 + I_3$$

Recall $G \equiv I/V$

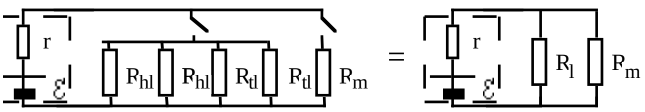
$$G_{//} = \frac{I_{//}}{V}$$

$$= \frac{I_1 + I_2 + I_3}{V}$$

$$= G_1 + G_2 + G_3$$

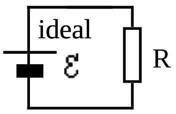
$$G_{//} = G_1 + G_2 + G_3 \qquad \textit{or} \qquad \frac{1}{R_{//}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Example The starter motor in a car is rated at 1.0 kW at 12 V. The headlights and tail lights together are together rated at 200 W at 12 V. The internal resistance of the 12 V car battery is 0.10 Ω.



What power do the lights consume running only on the battery? What power do they consume if left on when the engine is being started?

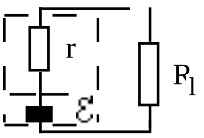
what are the resistances of lights and motor? Use rated power.



Circuit used for rating.

$P = V^2/R, \ \therefore \ R = V^2/P$

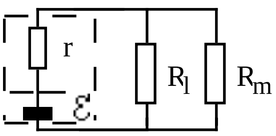
$R_l = 0.72\ \Omega, \quad R_m = 0.144\ \Omega$



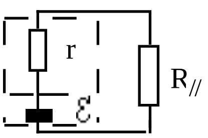
Lights only. $I = \mathcal{E}/(R_l + r)$

$P_l = I^2R_l$

$= 154$ W *(< 200 W)*



Motor and lights. R_l & R_m are in //.



$$R_{//} = \frac{1}{1/R_m + 1/r}$$

$$= 0.088\ \Omega$$

$I = \mathcal{E}/(R_{//} + r) = 64$ A $\qquad V_l = \mathcal{E} - Ir = 5.6$ V

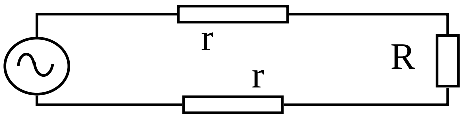
$$P_l = \frac{V_l^2}{R_l} = 44 \text{ W}$$

t

Example What is R of Cu cable 2 cm diameter and 10 km long? $\rho_{Cu} = 17\ 10^{-8}\ \Omega m$

$$P \equiv \frac{\rho L}{A} = \frac{1.7\ 10^{-8}\ 10^{4}}{\pi(10^{-2})^{2}}\ \Omega = 0.54\ \Omega$$

Use 2 such cables to deliver 1 MW at (a) 240 V and (b) 100 kV. What are the efficiencies?



240 V or 1 kV
P = 1 MW

a) $i = P/V = 1\ \text{MW}/240\ \text{V} = 4\ \text{kA}$ *large magnetic field*

P lost in r and r: $P = 2i^{2}r = \ldots = 17\ \text{MW}$

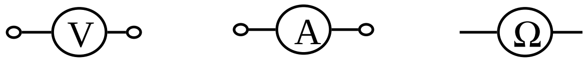
$$\text{efficiency} = \frac{1\ \text{MW}}{1\ \text{MW} + 17\ \text{MW}} = 6\%$$

b) $i = P/V = 1\ \text{MW}/100\ \text{kV} = 10\text{A}$ *but large electric field*

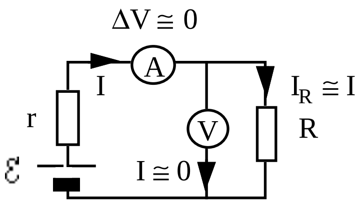
P lost: $P = 2i^{2}r = \ldots = 108\ \text{W}$

efficiency = ... = 99.99%

Voltmeters, ammeters and ohmmeters


measure potential difference, current & resistance respectively

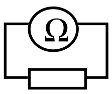
Use of meters



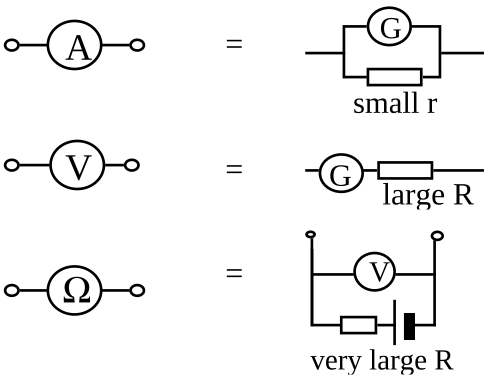
An **ammeter** measures that current flowing through the meter, ∴ must be connected in ***series***. It shouldn't perturb circuit, so its internal resistance should be ***low***. (Ideal case $r_A = 0$).

An **voltmeter** measures that potential difference between the leads of the meter, ∴ must be connected in ***parallel*** It shouldn't perturb circuit, so internal resistance should be ***high***. ('Ideal' case $r_V = \infty$).

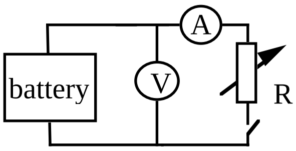
To measure resistance, disconnect the resistor from the circuit first.



In the old days meters were made using galvanometers (very senstive meters)



Internal resistance in emfs.



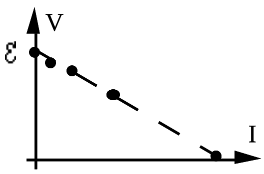
Experiment: battery and variable resistor. Measure terminal voltage V and output current.

Assume (V) is ideal

If R = ∞, (switch open), no current flows. The voltage measured here is called the emf of the battery. ie,

emf = terminal voltage when I = 0.

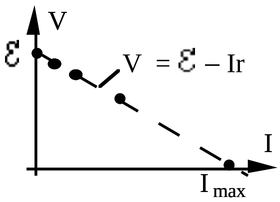
As R gets smaller, I gets bigger and V gets smaller



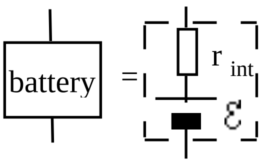
Often the resulting graph is a straight line. The intercept is by definition $\mathcal{E}$.

Its slope is negative and has units V/A = Ω. This slope defines the internal resistance, i.e.

$$r_{int} \equiv -\frac{dV}{dI} \quad \text{in this experiment}$$

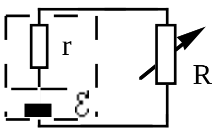


This gives us an equivalent circuit for the battery



A real emf can be represented as an ideal emf in series with an internal resistance

Example. What is the value of R for which the power dissipated in R is a maximum?



Loop rule gives

$$\mathcal{E} = Ir + IR$$

$$I = \frac{\mathcal{E}}{R + r}$$

$$P_R = I^2R \qquad I = \frac{\mathcal{E}}{R + r}$$

$$\therefore \quad P_R = \frac{\mathcal{E}^2}{(R + r)^2} R$$

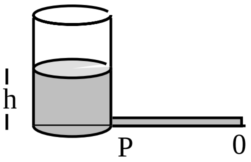
max when $\frac{dP}{dR} = 0$

$$\frac{dP}{dR} = 0 = -\frac{2\mathcal{E}^2}{(R + r)^3} R + \frac{\mathcal{E}^2}{(R + r)^2}$$

$\times\ (R+r)^2/\mathcal{E}^2 \ \rightarrow \quad 0 = -2R + (R + r)$

$$R = r$$

called the maximum power transfer theorem



Hydraulic analogy—not in syllabus.

How quickly does water flow out of this tank?

volume $Q = Ah$

flow $F = -\dfrac{dQ}{dt}$

"Resistance" of pipe $R \equiv \dfrac{\Delta P}{F}$

$$\frac{dQ}{dt} = -F = -\frac{P}{R} = -\frac{\rho g h}{R} = -\frac{\rho g Q}{RA}$$

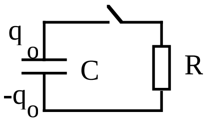
"Capacitance" of tank $C \equiv \dfrac{Q}{P} = \dfrac{A}{\rho g} = \text{const.}$

$$\frac{dQ}{dt} = -\frac{Q}{RC} \qquad \therefore \ \ln Q = -\frac{t}{RC} + \text{const.}$$

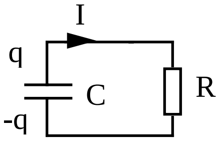
$t = 0$, $Q = Q_o$ therefore const. $= \ln Q_o$

$$Q = Q_o e^{-t/RC}$$

RC circuits



What happens when you close the switch? (close switch at t = 0)



Current flows *off* capacitor, so

$$I = -\frac{dq}{dt}$$

Across capacitor: $V_C = \dfrac{q}{C}$ $\qquad C \equiv \dfrac{q}{V}$

Across resistor: $V_R = IR = V_c$

$$\therefore \quad \frac{dq}{dt} = -I$$

$$= -\frac{V}{R}$$

$$= -\frac{q}{RC}$$

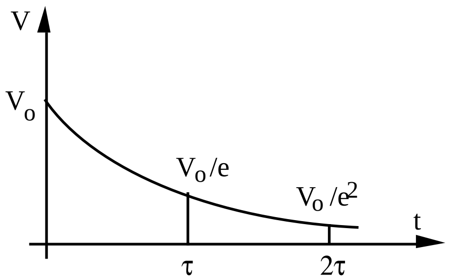
so $$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

$$\textit{ln}\, q = -\frac{t}{RC} + \text{const.}$$

$t = 0,\ q = q_o \qquad \therefore\ \textit{ln}\, q_o = \text{const.}$

$$\textit{ln}\, q - \textit{ln}\, q_o = -\frac{t}{RC}$$

$$\therefore \quad q = q_o\, e^{-t/RC} \qquad V = q/c \ \therefore$$

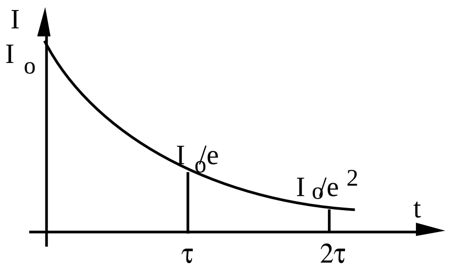


$$V = V_o e^{-t/RC}$$

$\tau = RC =$ time constant

Units: $\Omega F = \dfrac{V}{A} \cdot \dfrac{C}{V} = \dfrac{s}{C} \cdot C$

$= s$ (time)



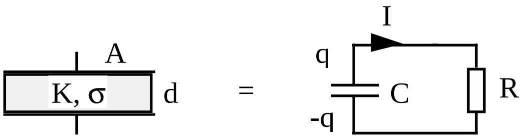
$$I = -\frac{dq}{dt}$$

$$= \frac{q_o}{RC}\, e^{-t/RC} = \frac{V_o}{R}\, e^{-t/RC}$$

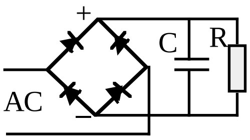
$$I = I_o\, e^{-t/\tau}$$

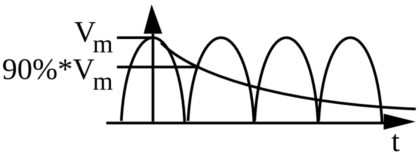
Example

A real C uses dielectric with dielectric const. 4.0 and conductivity $\sigma = 2\ 10^{-15}\ \Omega^{-1}m^{-1}$. It is charged then left disconnected for 1 hour. What fraction of charge remains?



Example Electronic appliance requires 20 W at 40 V DC, with max variation of 10%. What capacitance needed to store energy between cycles? (*circuit not in syllabus!*)





AC has 50 cycles per second, $\therefore$ turns off 100 times/second, ie every 10 ms.

$$\frac{V}{V_m} = e^{-t/RC} \cong 0.90$$

R is the effective resistance of the circuit.

We know V and P, so use $P = \dfrac{V^2}{R}$

Example the membrane of a "pacemaker" neurone of the heart has a capacitance of 10 pF and a conductance G = 3.0 pΩ^{-1}. Initially the potential difference across this membrane is –80 mV. The neurone begins a pulse when the potential difference reaches –60 mV. How long before it begins a pulse? Neglecting the duration of the electrical impulse itself, what is the pulse rate?

The "leakiness" of the membrane is now suddenly increased by a factor of 2. How long between electrical impulses? and what is the approximate pulse rate, again neglecting the duration of the inidividual impulses.