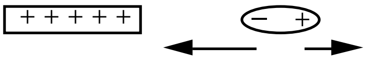
PHYS 1189-119 Notes by Joe Wolfe

Electricity (qualitative):

charge - causes electrical interactions
- can sometimes move →
- **conductors** & **insulators**

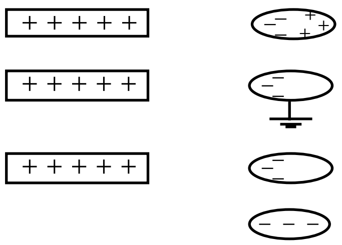
electric force - acts at a distance but decreases with distance
- comes in two sorts, called + and –
- like charges repel, unlike attract

Charged can attract neutral, because force on *dipole* ≠ 0



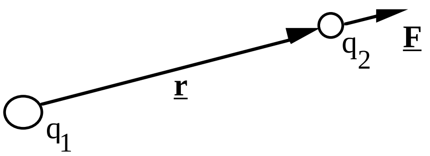
Charging by induction

i)
ii)
iii)
iv)



Coulomb's law

(Coulomb is SI unit of charge: symbol C)



Force on q_2 is

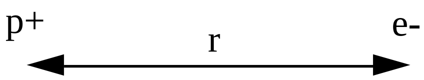
$$\underline{\mathbf{F}} = k\frac{q_1 q_2}{r^2}\hat{\underline{\mathbf{r}}} \qquad = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_1 q_2}{r^2}\hat{\underline{\mathbf{r}}}$$

$\frac{1}{4\pi\varepsilon_o}$ = k = Coulomb's constant = $9.0\ 10^{9}\ \mathrm{Nm^2C^{-2}}$

ε_o = permittivity of free space = $8.85\ 10^{-12}\ \mathrm{F.m^{-1}}$

Electric forces are (vectorially) **additive**.

Example What is ratio between electrical and gravitational attractions in H atom?



$$\frac{F_e}{F_g} = \frac{k\,\frac{q_e q_p}{r^2}}{-\,G\,\frac{m_e m_p}{r^2}}$$

$$= \frac{(9.0\ 10^{9}\ \mathrm{Nm^2C^{-2}})(-1.6\ 10^{-19}\ \mathrm{C})(1.6\ 10^{-19}\ \mathrm{C})}{-\,(6.67\ 10^{-11}\ \mathrm{Nm^2kg^{-2}})(9.1\ 10^{-31})(7.1\ 10^{-28})}$$

$= 5\ 10^{39}$

but gravity dominates because it is always attractive

Fields

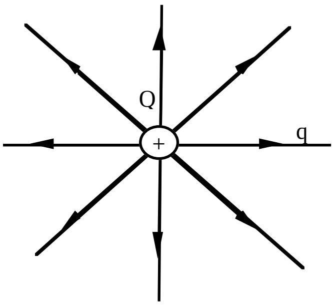
Define electric field:

$$\underline{\mathbf{E}} \equiv \frac{\underline{\mathbf{F}}_e}{q}$$

$\underline{\mathbf{F}}_e$ is total electric force on a small test charge q

then $\underline{\mathbf{F}}_e \equiv \underline{\mathbf{E}}q$

Lines of force: represent field by lines whose tangent is // $\underline{\mathbf{E}}$



$\underline{\mathbf{E}}$ due to point charge Q

$$\underline{\mathbf{E}} \equiv \frac{\underline{\mathbf{F}}}{q} = \frac{1}{q} k\frac{Qq}{r^2} \hat{\underline{\mathbf{r}}}$$

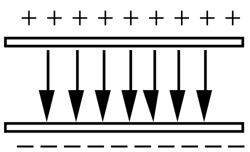
$$\underline{\mathbf{E}} = k\frac{Q}{r^2} \hat{\underline{\mathbf{r}}} \qquad = k\frac{Q}{r^2} \hat{\underline{\mathbf{r}}}$$

Note: $\dfrac{\text{no. of lines}}{\text{unit area}} = \dfrac{n}{4\pi r^2} \propto E$

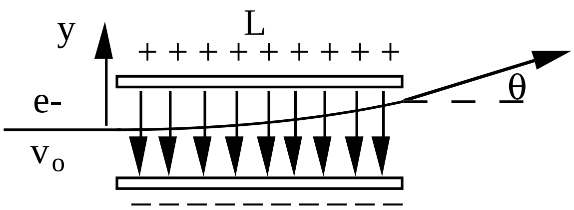
Fields are also additive

$$\underline{\mathbf{E}}_{\text{total}} = \Sigma \underline{E}_i$$

Field between two large plates, oppositely charged, far from edge:
approximately **uniform field**



Used in TV, monitors, injets etc.



Just like projectiles: $a = \dfrac{F}{m} = \dfrac{Eq}{m}$

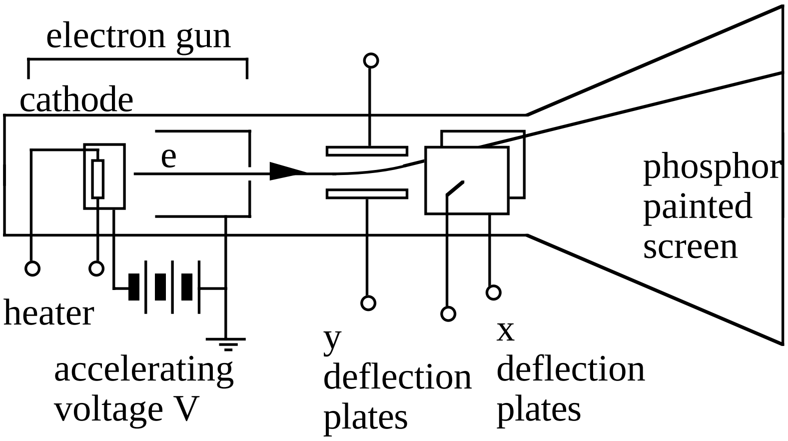
$$v_y = v_{yo} + a_y t = 0 + \frac{Eq}{m} t$$

$$v_x = \text{constant} = v_o \quad \rightarrow \quad t = \frac{L}{v_o}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{Eq}{m}\frac{L}{v_o}\frac{1}{v_o} = \frac{EqL}{m v_o^2}$$

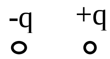
Cathode Ray Tube (CRT)

used in oscilloscopes, TVs etc



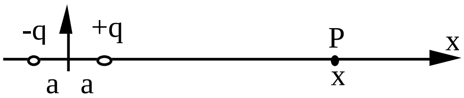
- cathode is hot (heating coil) and several kV negative
- electrons are colimated (& focussed) to form a beam
- beam is deflected horizontally by the field between the x plates and vertically by that between the y plates
- phosphor on screen glows when the beam strikes

Dipole: equal and opposite charges.



Example: radio antenna, molecule in a field... lots!

Far field due to a dipole (on axis) at point P, x>>a



E due to + $\quad E_+ = k\dfrac{q}{(x-a)^2}$ (i.e. to right)

E due to – $\quad E_- = k\dfrac{-q}{(x+a)^2}$ (i.e. to left)

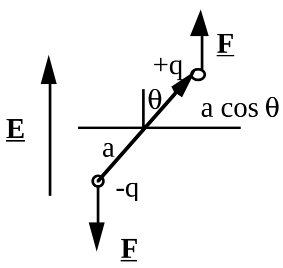
$$E_{total} = kq\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right)$$

$$= kq\frac{x^2 + 2ax + a^2 - x^2 + 2ax - a^2}{(x+a)^2(x-a)^2}$$

$$(x>>a) \quad \rightarrow \quad \cong kq\frac{4ax}{x^4} = k\frac{4aq}{x^3}$$

We usually define the dipole $p \equiv 2aq \rightarrow k\dfrac{2p}{x^3}$

Example: energy of dipole in a field



Take the energy to be zero when dipole and field are perpendicular. (Choice of zero is arbitrary)

$dU = -F{*}ds_E$ component of ds // **E**

$U_{+q} = -Eq{*}a\cos\theta \qquad U_{-q} = -Eq{*}a\cos\theta$

$U_{total} = -2Eqa\cos\theta = -Ep\cos\theta$

Example. Molecules have thermal energies of ~ k_BT where k_B is Boltzmann's constant. How big must E be if $U_{elec} \cong k_BT$?

$$\Delta U_{dipole} \sim pE$$

p = length * charge ~ 10^{-10} m * 1.6 10^{-19} C

$$E \sim \frac{\Delta U}{p} = \frac{1.38\ 10^{-23}\ JK^{-1}\ 300\ K}{10^{-10}\ m * 1.6\ 10^{-19}\ C}$$

$$= 3\ 10^{8}\ NC^{-1}$$

($> E_{lightning}$)

Electrical Potential V

$$V \equiv \frac{\text{Electrical Potential Energy}}{\text{Unit Charge}}$$

W_{AB}: work done *against* electrical forces going from A to B

$$V_B - V_A \equiv \frac{W_{AB}}{q}$$

S.I. Unit: 1 Volt ≡ 1 $J.C^{-1}$

Electric potential and Field

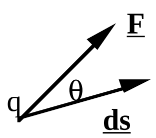
$$dW_{against\ E} = -\,dW_E$$

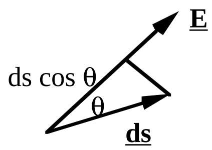
$$\equiv -\,F_E\,ds\cos\theta$$

$$\therefore \quad dV = \frac{dW_{against\ E}}{q} = -\frac{F_E\,ds\cos\theta}{q}$$

$$= -\frac{Eq\,ds\cos\theta}{q}$$

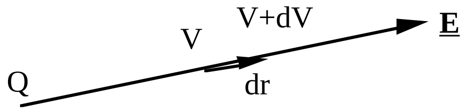
$$dV = -\,E\,ds\cos\theta \qquad \textit{E * compt of ds // E}$$





Conversely: $E = -\dfrac{dV}{dr}$ where dr is //E

Potential due to a point charge:



$$dV = -\,E\,ds\cos\theta = -\,E\,dr$$

$$dV = -\,kQ\,.\frac{dr}{r^2}$$

$$V = -\,kQ \int_{reference}^{r} \frac{dr}{r^2}$$

choose $r_{ref} = \infty$

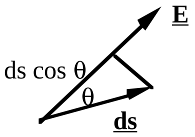
$$= kQ\left[\frac{1}{r}\right]_{r=\infty}^{r}$$

$$V(r) = kQ\frac{1}{r}$$

for many charges $V = \dfrac{1}{4\pi\varepsilon_o}\Sigma\dfrac{q_i}{r_i}$

check: $E = -\dfrac{dV}{dr}$ where dr is //E → .....

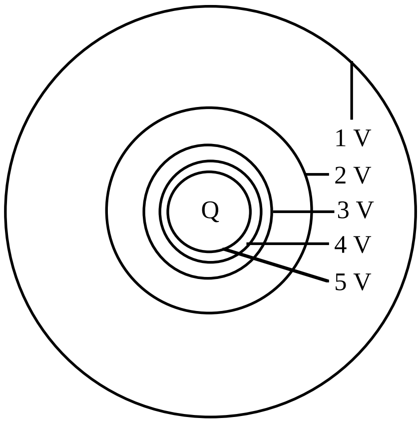
Equipotential Surfaces have constant value of V

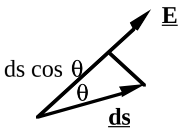


$$dV = -E\, ds \cos\theta$$

If dV = 0, then either E = 0 or $\theta = 90°$

∴ **E** perpendicular to equipotentials.
e.g. V(r) for point charge





$$dV = -E\, ds \cos\theta$$

Corollary 1: In equipotential surface, dV = 0.

∴ $\underset{\sim}{E}$ perpendicular to surface.

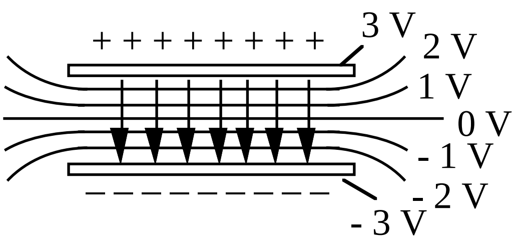
Corollary 2: Ideal conductor is equipotential.

Example: uniform field in x direction

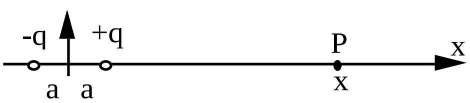
$dV = -E\, ds \cos\theta$ *E * compt of ds // E*

$dV = -E\, dx$

Field is approximately uniform between // plates



Example: V due to dipole



$$V(r,\theta) = V_+ + V_-$$

$$= \left(\frac{kq}{x_1} - \frac{k\,q}{x_2}\right)$$

$$= kq\,\frac{x_2 - x_1}{x_1 x_2}$$

if x>> a $\quad\cong kq\,\dfrac{2a}{x^2}$

$$\therefore \quad V = k\,\frac{p}{x^2}$$

Capacitance

remember $V = \sum V_i = k \sum_i \frac{q_i}{r_i}$

$\therefore$, for given geometry: $V \propto Q$.

$\therefore$ define $C \equiv \frac{Q}{V}$

S.I. Unit: Farad $\equiv$ Coulomb/Volt

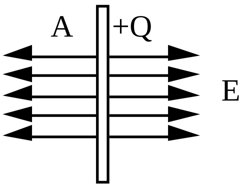
how much charge something stores at a given potential

Example: C of isolated sphere.

$V = \frac{kQ}{r} \quad \therefore \quad C \equiv \frac{Q}{V} = \frac{r}{k}$

Field due to large plate, far from edge

Charge Q, area A $\rightarrow$ Uniform charge density Q/A

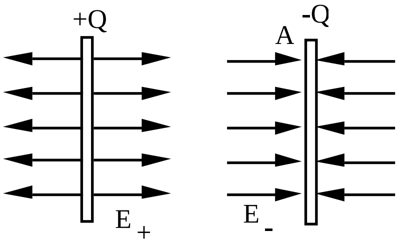


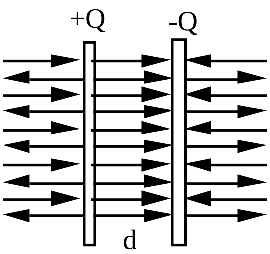
For infinite plate, from symmetry, the field is normal to the plane, and equal on both sides

Coulomb's law and integration gives

$$E = 2\pi k \frac{Q}{A} \qquad \text{on both sides}$$

Field between two (large) **parallel plates**





Fields cancel outside, reinforce between them

$$E = 4\pi k \frac{Q}{A} = \frac{Q}{\varepsilon_o A}$$

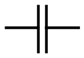
Need A >> d^2 to neglect edge effects

$V = -\int E.ds \qquad$ E is uniform,

$\therefore \qquad V = Ed = \frac{Q}{\varepsilon_o A} d$

Capacitors used for storing energy (particularly for brief applications, AC$\rightarrow$DC), for filtering and for timing applications.

Parallel place capacitors

Circuit symbol: 

$$C \equiv \frac{Q}{V} = Q.\frac{\varepsilon_o A}{Qd}$$

$$C = \frac{\varepsilon_o A}{d}$$

ε_o is 8.85 10^{-12} Fm^{-1} permittivity of space

ε_o = 8.85 pFm^{-1}: to get large C, d often of molecular size

e.g. Two plates, 1 m^2, separated by 1 mm.

$$C = \frac{8.85\ pFm^{-1}}{1\ 10^{-3}\ mm}$$

$$= 9\ 10^{-9}\ F = 9\ nF$$

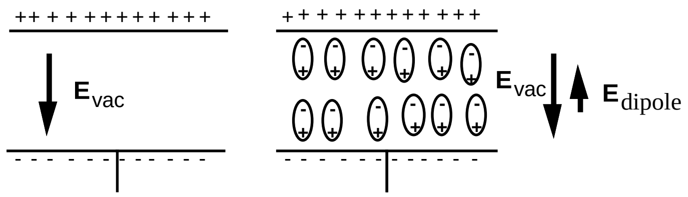
Put ± 1 microcoulomb on the plates:

$$V \equiv \frac{Q}{C} = \frac{1\ 10^{-6}}{9\ 10^{-9}} = 110\ V$$

$$E = V/d = 110\ kV.m^{-1}$$

Dielectrics

put insulator in elec field:



molecular dipoles innate or induced

E_{dipole} oppose E_{vac} $\therefore$ $E_{resultant} < E_{vac}$

Dielectric constant $K \equiv \frac{E_{vac}}{E_{resultant}} > 1$

Intrinsic dipoles, K large e.g. $K_{water} = 78$

Nonpolar media K smaller, but > 1 $K_{plastic} \sim 3$

Parallel plate capacitor with dielectric:

$$C = \frac{Q}{V}$$

$$= \frac{Q}{Ed}$$

$$= \frac{KQ}{E_{vac}d} = \frac{A\varepsilon_o KQ}{Qd}$$

$$\therefore \quad C = \frac{K\varepsilon_o A}{d} = \frac{\varepsilon A}{d}$$

where $\varepsilon \equiv K\varepsilon_o$ is permittivity of medium.

Dielectric Breakdown

For any insulator ∃ a value of E

where molecules → ions , ∴ conducts.

This value called **dielectric strength** (E_{max})

air: ~3 MVm^{-1}

mica: 160 MVm^{-1}

Energy Stored in C:

Charge up from q=0 to q=Q

v=0 to v=V

$$dW = v\,dq = \frac{q}{C}\,dq$$

$$W = \int dW = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_o^Q$$

$$\therefore \quad W = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

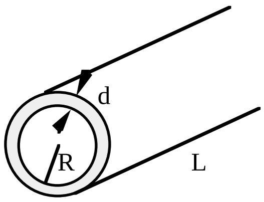
Energy density

$$\frac{W}{\text{volume}} = \frac{1}{2}CV^2 \cdot \frac{1}{Ad} = \frac{\varepsilon A}{2d} \cdot V^2 \cdot \frac{1}{Ad}$$

but $\frac{V^2}{d^2} = E^2$

so $\frac{W}{\text{vol}} = \frac{1}{2}\varepsilon E^2$

Example:



A nerve cell is ~ cylindrical, L = 20 mm long and R = 5 μm diameter. It has a fatty membrane d = 4 nm thick, separating salt solutions (conductors) inside and out. What is its capacitance? What is its capacitance per unit area? Usually, the inside is at – 90 mV with respect to the outside. What charge does it store? How much energy? (Take $K_{fat} = 3$)

Example:

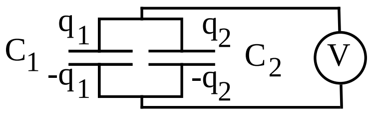
Capacitor, A = 10m^2, d = 10nm, K = 3.0, diel. strength = 10 MV m^{-1}. What is max W stored?

Example: capacitor with K = 3, diel. strength = 10^8 Vm^{-1}. How big must it be to store 1 kW.hr ?
(1 kW.hour = 1 kW. 3600 s = 3.6 MJ)

But energy can be stored in & obtained from capacitors very quickly, easily and efficiently.

e.g. Flash in cameras, power regulation, timing

Capacitors in parallel



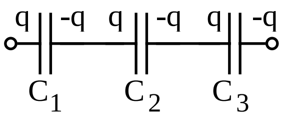
$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$C_{//} = \frac{q_1 + q_2}{V} = \frac{q_1}{V} + \frac{q_2}{V}$$

$$\therefore \quad C_{//} = C_1 + C_2$$

Don't compare with resistors,
Compare with conductors $G \equiv 1/R$

Capacitors in series

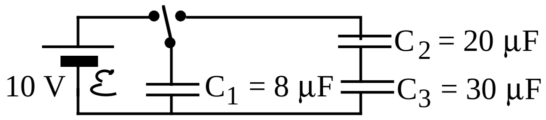


$V = V_1 + V_2 + V_3$

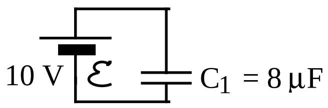
$$\therefore \quad \frac{q}{C_{ser}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Example



Switch moved from left to right. What are the final values of V_1 V_2 V_3 ?

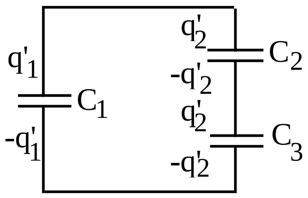


Switch to left: capacitor charges until $V_1 = \varepsilon$

$V_1 = 10$ V

$q_1 = C_1 V_1$

$= C_1\varepsilon = \; = 80\ \mu$C



Switch to right:

charges on C_2 & C_3 are equal.

$V'_1 = V'_2 + V'_3$

$\therefore \quad q_1 = q_1' + q_2'$ charge conservation

From the definition of C, $V = q/C$, $\therefore$

$$\rightarrow \quad \frac{q_1'}{C_1} = \frac{q_2'}{C_2} + \frac{q_2'}{C_3}$$

$$\frac{q_1 - q_2'}{C_1} = \frac{q_2'}{C_2} + \frac{q_2'}{C_3}$$