**PHYS 1189—1199**

# Particle dynamics

Aristotle: $\underline{\mathbf{v}} = 0$ is "natural" state

Galileo & Newton: $\underline{\mathbf{a}} = 0$ is "natural" state

## Newton's Laws

**First** "zero (total) force $\Rightarrow$ zero acceleration"

more formally:

If $\Sigma\, \underline{\mathbf{F}} = 0$, there exist reference frames in which $\underline{\mathbf{a}} = 0$.

called **Inertial frames**

observation: in these frames,
distant stars don't accelerate

## Mechanical equilibrium:

If $\Sigma\, \underline{\mathbf{F}} = 0$, body is in mechanical equilibrium,

$\therefore$ is not accelerating.

Either at rest, or at constant $\underline{\mathbf{v}}$

in intertial frames: **Newton's second law holds:**

$$\Sigma\, \underline{\mathbf{F}} = m\, \underline{\mathbf{a}}$$

*1st is special case of 2nd*

remember: 1 vector equation $\rightarrow$ n scalar equations

$(\Sigma\, F_x = ma_x \qquad \Sigma\, F_y = ma_y \qquad \Sigma\, F_z = ma_z)$

What does 2nd law mean?

$\Sigma\, \underline{\mathbf{F}} = m\, \underline{\mathbf{a}}$ and $\underline{\mathbf{W}} = m\, \underline{\mathbf{g}}$

is the m necessarily the same?

$\underline{\mathbf{F}} = m\, \underline{\mathbf{a}}$

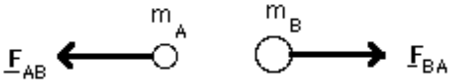
$\underline{\mathbf{a}}$ is already defined.

Does this law define m or $\underline{\mathbf{F}}$ ? or neither?

**Newton 3**: "To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

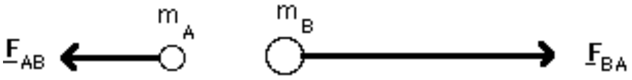
*Or*

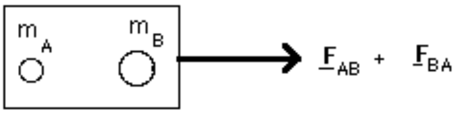
Forces always occur in pairs, $\underline{\mathbf{F}}$ and $-\underline{\mathbf{F}}$ , one acting on each of a pair of interacting bodies.




**Third** $\underline{\mathbf{F}}_{AB} = -\underline{\mathbf{F}}_{BA}$

Why so?

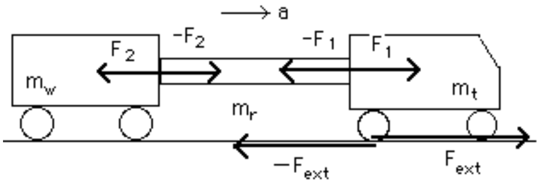


**"light" ropes etc.**

Truck ($m_t$) pulls wagon ($m_w$) with rope ($m_r$).

All have same **a**.




i) wagon: $-F_2 = m_w a$.

ii) rope: $F_1 - F_2 = m_r a$

iii) truck: $-F_1 + F_{ext} = m_t a$

(ii)/(i) $\rightarrow$ $\frac{F_1 - F_2}{-F_2} = \frac{m_r a}{m_w a}$

$\therefore$ if $m_r << m_w$, $F_1 = F_2$.

Forces at opposite ends of light ropes etc are equal and opposite.

**Mass and weight**

(inertial) mass m defined by F = ma

**observation:**

near earth's surface and without air,

all (?) bodies fall with same a ( = –g)

weight $W = -mg$

**Warning:** do not confuse mass and weight, or their units

kg → mass N → force ($kg.m.s^{-2}$)

kg wt = weight of 1 kg = mg = 9.8 N

If your mass is 70 kg, your weight is

$W = mg = 70kg*9.8ms^{-2} = 690$ N

**Mass and Density**

Density is mass per unit volume.

$\rho \equiv \frac{m}{V}$ units are $kg.m^{-3}$

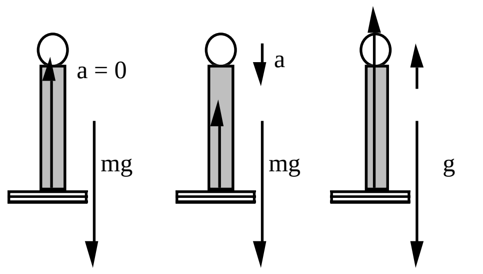
$\rho_{water} = 1000\ kg.m^{-3}$

$\rho_{seawater} = 1030\ kg.m^{-3}$

$\rho_{air} = 1.2\ kg.m^{-3}$

$\rho_{steel} = 7{,}800\ kg.m^{-3}$

**Example** In a lift accelerating downwards at 6 $ms^{-2}$, what is the force between your feet and the floor? What if it accelerates up at 6 $ms^{-2}$? Express your answer as a fraction of your weight.



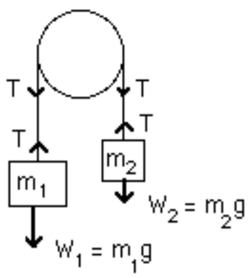

stationary    accelerating down    accelerating up

$$ma = \Sigma F = N - mg \qquad \text{(up as +ve)}$$

$$N = mg + ma$$

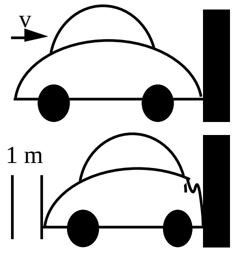
| | | |
|---|---|---|
| $a = 0$ | $a = -\ 6\ ms^{-2}$ | $a = +6\ ms^{-2}$ |
| $N = mg$ | $N = m(g - 6\ ms^{-2})$ | $N = m(g + 6\ ms^{-2})$ |
| $\frac{N}{mg} = 1$ | $\frac{N}{mg} = \frac{g - 6\ ms^{-2}}{g}$ | $\frac{N}{mg} = \frac{g + 6\ ms^{-2}}{g}$ |
| $\frac{N}{mg} = 1$ | $\frac{N}{mg} = 40\%$ | $\frac{N}{mg} = 160\%$ |

**Example** Grav. field on moon $g_m = 1.7\ ms^{-2}$. An astronaut weighs 800 N on Earth, and, while jumping, exerts 2kN while body moves 0.3 m. What is his/her weight on moon? How high does s/he jump on earth and on moon?




**Example**

Light pulley, light string. No friction on the axle. What is acceleration of the masses?




**Example.** A 800 kg car collides with 'immoveable' object and crumples a distance s = 60 cm. What is the average force on it? On a 70 kg person inside?

$v_o = 30\ km.hr^{-1}$.

**Contact forces**

**Normal force**: at right angles to surface

*(provided by deformation)*

Friction: complicated, but use this approximate law

If ∃ relative motion, **kinetic** friction $|F_f| = \mu_k N$

Direction opposes relative motion

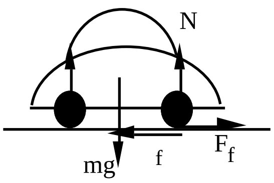
If ∃ no relative motion, **static** friction $|F_f| \leq \mu_s N$

Direction opposes applied force

i) Usually, $\mu_k < \mu_s$.

(It takes less force to keep sliding than to start sliding.)

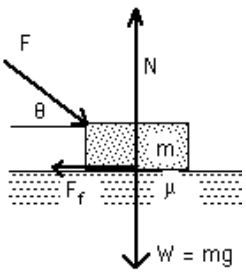
ii) $\mu_s$ and $\mu_k$ are approx. independent of N and of contact area.




**Example**. A car has equal weight on all wheels. $\mu_s = 0.9$

What is maximum acceleration?

maximum decelleration?



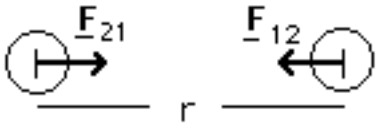

**Example.**

Apply force F at θ to horizontal. Mass m on floor, coefficients $\mu_s$ and $\mu_k$.

For any given θ, what F is required to make the mass move?

*Eliminate 2 unknowns N and* $F_f \rightarrow F(\theta, \mu_s, m, g)$

**Newton's law of gravity**:




$$F = -G\frac{m_1 m_2}{r^2}$$

Negative sign: $\underline{\mathbf{F}} \,//\, -\underline{\mathbf{r}}$

$$\underline{\mathbf{F}}_{12} = -\underline{\mathbf{F}}_{21}$$ *Newton 3*

**Gravity near Earth's surface**

$$|F_g| = G\frac{M_e m}{r_e^2}$$ proportional to m

$$|F_g| = m\left(\frac{GM_e}{r_e^2}\right) = mg_o$$

*$g_o$ is accel$^n$ in an inertial*
*(non-rotating) frame: $g_o \cong g$*
*also Earth not uniform*
*Earth not spherical*

$$g_o = \frac{GM_e}{r_e^2}$$

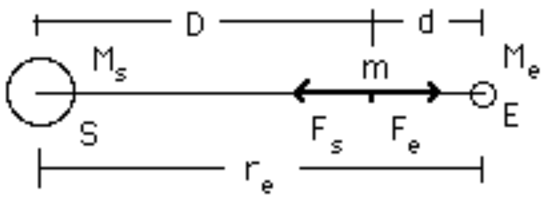
We can measure g, we can measure $r_e$, but how to find G or $M_e$?

**Gravitational field.** Ratio of force on a thing to some property of the thing. For gravity, **mass** is the property:

$$\frac{\underline{\mathbf{F}}_{grav}}{m} = \underline{\mathbf{g}} = \underline{\mathbf{g}}(\underline{\mathbf{r}})$$ is a vector field

*cf* electric field $\frac{\underline{\mathbf{F}}_{elec}}{q} = \underline{\mathbf{E}}(\underline{\mathbf{r}})$

**Puzzle:** How far from the earth is the point at which the gravitational attractions towards the earth and that towards the sun are equal and opposite? Compare with distance earth-moon (380,000 km)




$$F_e = F_s$$