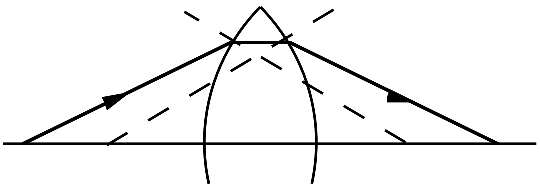
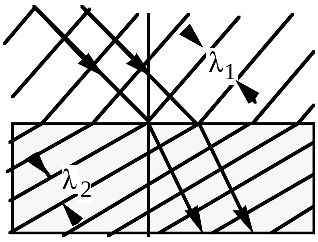
Physical optics (wave properties of light)

Refraction. Main application of diffraction is in lenses. Other examples are mirages, rainbows.

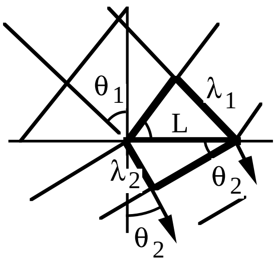


Refraction in the wave picture

Rays (→) are at right angles to wavefronts. Look at what happens when a ray bends at an interface



Let's look at this close-up:



Triangles: $\sin\theta_1 = \lambda_1/L$, $\sin\theta_2 = \lambda_2/L$

$$n_{21} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2}$$

But $\lambda = v/f$, so

$$n_{21} = \frac{v_1/f}{v_2/f} = \frac{v_1}{v_2}$$

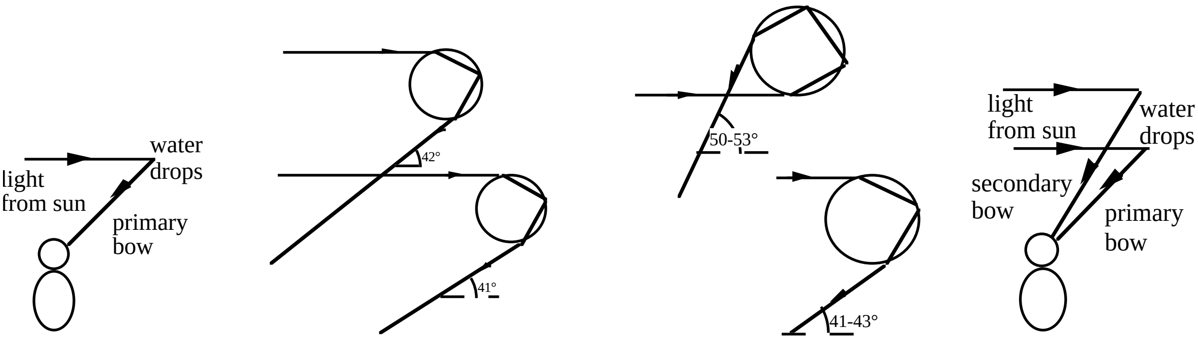
In particular, if (1) is vacuum, then

$$n_2 = \frac{c}{v_2}$$

For any material, $\frac{1}{n}$ is the factor of reduction in the speed in that medium.

$n = n(\lambda)$. n decreases with λ so blue refracts more

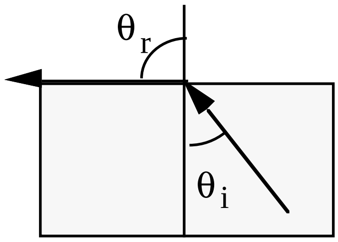
Example Rainbow



Total internal reflection

Going from high n to low n, $\theta_r > \theta_i$

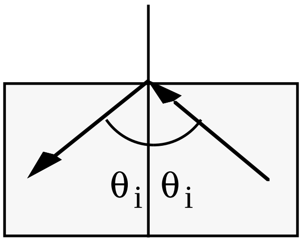
critical value of θ_i when $\theta_r = 90°$.



$$n = \frac{\sin 90°}{\sin \theta_{crit}} \quad \therefore \quad \sin \theta_{crit} = \frac{1}{n}$$

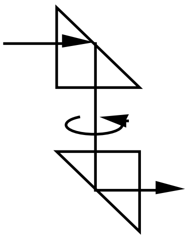
If $\theta_i > \theta_{crit}$ (ie if $\sin \theta_{crit} > \frac{1}{n}$)

Total internal reflection



Example

what is the minimum n for glass to be used in a periscope? (assume 45° prisms)



$\theta_i \sim 45°$

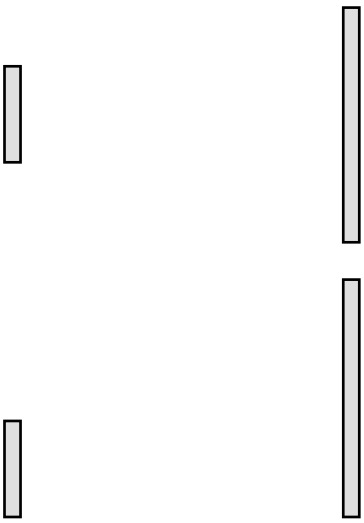
require $\sin \theta_i > \sin \theta_{crit} = \frac{1}{n}$

$$\therefore \quad n > \frac{1}{\sin 45°} = 1.4$$

Physical optics

Geometrical optics only works if size $>>\lambda$. Why?

Huygen's principle



Is light

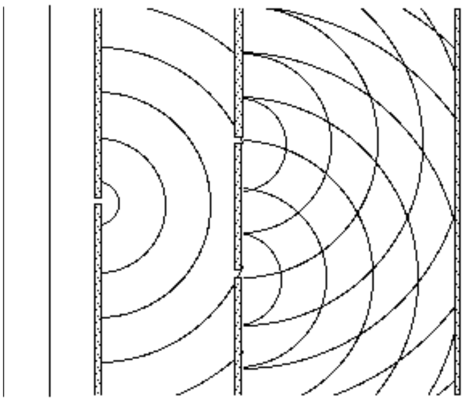
a) a particle (Newton) $\rightarrow$ geometric optics

b) a wave (Young) $\rightarrow$ physical optics

c) both

Young's experiment

Light through one slit (gives ***coherent source***) then **two slits** gives interference pattern on screen.



Electromagnetic radiation

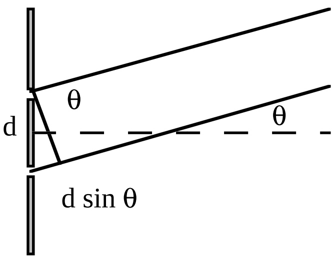
Speed $c = 3.0 \; 10^8 \; ms^{-1}$

(This is definition of metre)

λ: for visible light, $400 \text{ nm} < \lambda < 800 \text{ nm}$

Typical $f = \frac{c}{\lambda} \sim \frac{3 \; 10^8 \; ms^{-1}}{5 \; 10^{-7} \; m}$

$= 6 \; 10^{14}$ Hz $=$ 600 THz



Constructive interference if $d \sin\theta = m\lambda$

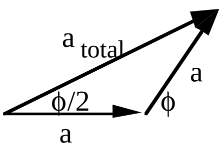
Destructive interference if $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

At an angle θ, the phase difference ϕ is

$$\frac{\phi}{2\pi} = \frac{\Delta \text{ pathlength}}{\lambda} = \frac{d \sin\theta}{\lambda}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda} d \sin\theta$$

how to add two sin waves? phasor diagrams



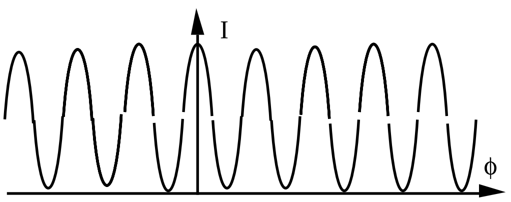
$a_{tot} = 2a \cos\beta$

$\beta = \phi/2 = \frac{\pi}{\lambda} d \sin\theta$

Intensity $\propto$ amplitude^2 $\therefore$ $I \propto 4a^2 \cos^2\beta$

I_{max} if $\beta = \phi/2 = 0$

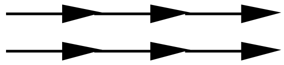
$$I = I_{max}\cos^2\beta \qquad \text{where } \beta = \frac{\pi}{\lambda} d \sin\theta$$



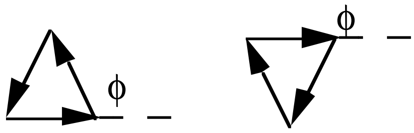
$\phi = \frac{2\pi}{\lambda} d \sin\theta$

Example 3 narrow slits radiate uniformly and in phase. Interference pattern on screen. (a) Show phase diagrams for (i) central maximum (ii) 1st order maximum (iii) minima between i and ii.
(b) Sketch $I(\phi)$

i) $\phi = 0$, central max
ii) $\phi = 2\pi$, 1st order max

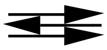


iii)

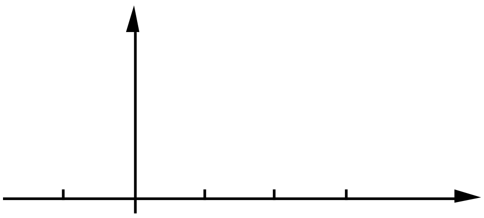


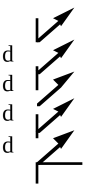
$\phi = 2\pi/3$ $\phi = 4\pi/3$ both $\rightarrow$ 0 resultant

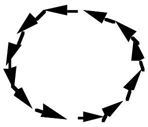
b) if $\phi = \pi$,



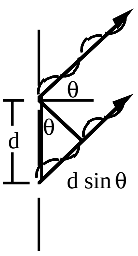
amplitude = 1/3 central max
$\therefore$ intensity = $I_{max}/9$







Diffraction grating
has very many slits. Used to measure λ very accurately.
If there are N slits per unit lenth, d = 1/N.
The first minimum is *very* close (small ϕ to close polygon), ie very narrow maxima



For constructive interference

$$d \sin\theta = m\lambda$$

$m = 1 \rightarrow$ 1st order spectrum
$m = 2 \rightarrow$ 2nd order spectrum

$$\theta_{red} = \sin^{-1}\frac{m\lambda_{red}}{d}$$

$$\theta_{blue} = \sin^{-1}\frac{m\lambda_{blue}}{d} \quad \text{etc}$$

Coherence length
Wave trains have finite length - coherence length

Only interfere if Δ pathlength < l
Examples
Radio transmitter $E = E_m \sin(kx - \omega t)$

E_m, ω, k vary slowly

Hot body (e.g. lamp) l ~ 1 m, but different regions have different, random phase. $\therefore$, to get interference, use a pin hole and keep Δ path << 1 m
LASER (**L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) l >> km

Interference in thin films

Light in a medium travels at c/n.

$$\lambda_{medium} = \lambda/n$$

$$\Delta\phi = 2\pi \frac{\Delta \text{ path}}{\lambda_{med}} = 2\pi \frac{n \, \Delta \text{ path}}{\lambda}$$

Define **Optical path length** $\equiv$ n . pathlength

$$\Delta\phi = 2\pi \frac{\Delta \text{ optical pathlength}}{\lambda}$$

Reflections *remember reflections in strings*

From less dense to more dense

$\Delta\phi = \pi$

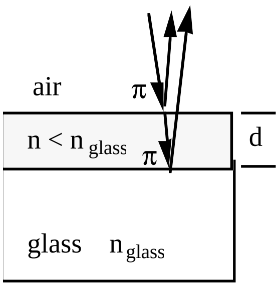
From more dense to less dense

$\Delta\phi = 0$

transmitted wave
has no phase change

Example: Non reflective coating

useful in camera lens etc



Coating has $1 < n < n_{glass}$

How thick should it be to give minimum reflection?

Both reflections have π phase change. As $d \to 0$, $\to$ constructive interference.

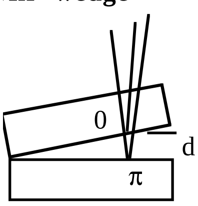
For destructive, we want

Δ optical pathlength $= \lambda/2$

$2nd = \lambda/2$

$$d \sim \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4 \quad 1.2} \sim 100 \text{ nm}$$

Air wedge



Denstructive interference if

$2d = m\lambda$

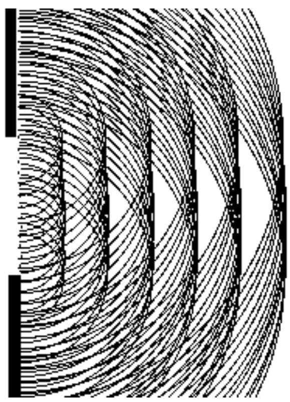
Constructive interference if

$$2d = \left(m + \frac{1}{2}\right)\lambda$$

At L.H. end, destructive (dark), then count fringes to get thickness.

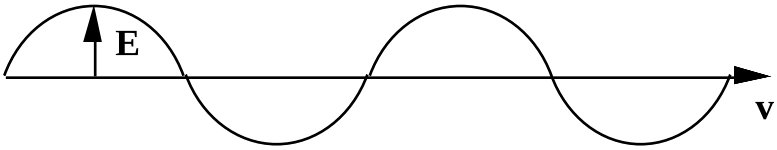
Also: Newton's rings, oil slicks

Diffraction from a slit (Later from a circle)



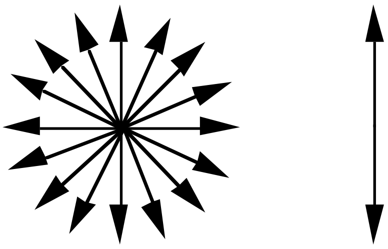
Use Huygen's construction:
beam of finite width,
interference if $d \sim \lambda$

Polarisation.

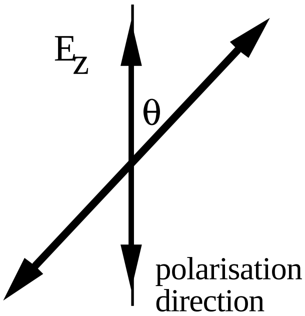


EM waves are **transverse** waves, ∴ can be polarised. Usually light has waves with **E** in all directions

Unpolarised Plane polarised



Polaroid materials allow **E** in only one dirn



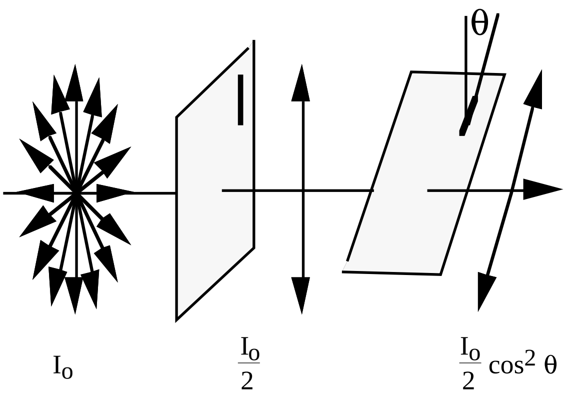
$E_{transmitted} = E \cos\theta$

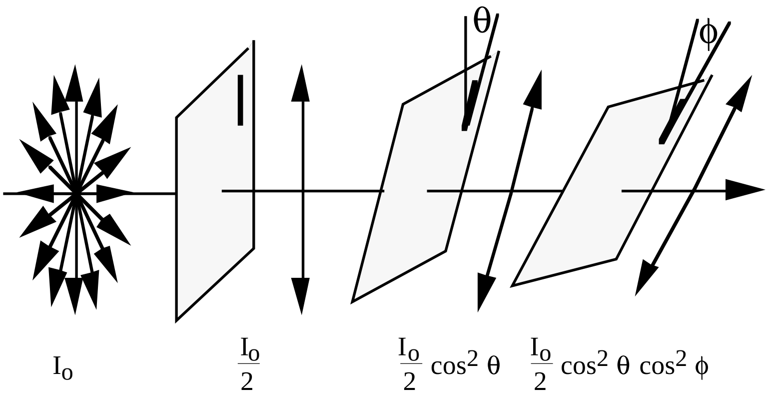
Malus' Law:

$I_{trans} = I_{in} \cos^2\theta$

Average of $\cos^2\theta$ over all angles is 1/2 ∴

Unpolarised plane polarised plane polarised

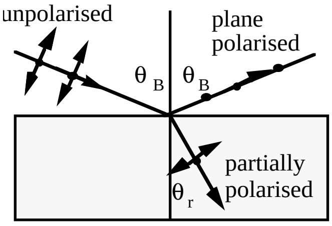




Note: even if $\theta + \phi = 90°$, $I \neq 0$

Polarisation by reflection

For wave in medium, **E** of light causes oscillation // **E**. This oscillation can produce (only) transverse waves, hence polarisation of reflected wave.



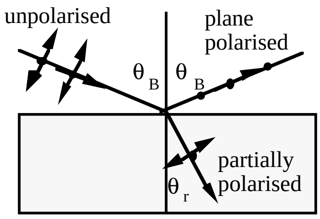
When refracted ray ⊥ reflected → plane polarised reflected wave (Brewster's angle θ_B).

→ polaroid sunglasses (*see your optics kit*)

Scattering of light also polarises

More effective for short λ, → blue sky

Example. What is Brewster's angle for a medium with n = 1.40?



Refraction: $n = \dfrac{\sin\theta_B}{\sin\theta_r}$

If refracted and reflected are at 90°,

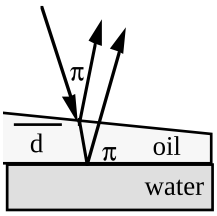
$$\theta_B + \theta_r = 90°$$

so $$\sin\theta_r = \cos\theta_B$$

$$n = \frac{\sin\theta_B}{\cos\theta_B} = \tan\theta_B$$

$$\theta_B = \tan^{-1} 1.4 = 54°$$

Example. An oil slick (n = 1.20) floats on water. What are the thicknesses for which red light ($\lambda \cong 700$ nm) is reflected weakly? What does the slick look like at its thinnest point?



$n_{water} > n_{oil}$

Constructive interference if

$\Delta\ OPL = m\lambda$

Destructive interference if

$\Delta\ OPL = \left(m + \frac{1}{2}\right)\lambda$

i) If red has destructive interference,

$$\Delta\ OPL = 2nd = \left(m + \frac{1}{2}\right)\lambda_{red}$$

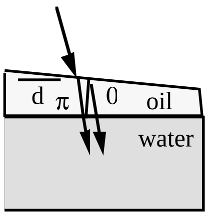
$$d = \frac{\lambda_{red}}{2n}\left(m + \frac{1}{2}\right)$$

m = 0, m = 1, m = 2 .....

= 150 nm, 440 nm, 730 nm etc

ii) If $d \ll \lambda$, π phase difference on both paths so constructive interference for all λ, so it looks bright and 'white'.

Example. Same problem, but for scuba diver!



Destructive interference if

$\Delta\ OPL = m\lambda$

Contructive interference if

$\Delta\ OPL = \left(m + \frac{1}{2}\right)\lambda$