

Waves and Sound for PHYS1169. Joe Wolfe, UNSW

Waves are moving pattern of displacements. May transmit energy and signals.

1169 Syllabus

Travelling waves, superposition and interference, velocity, reflection and transmission, harmonic waves, spherical and plane waves.

Sound.

Doppler effect, standing waves in strings and air columns, beats, decibel scale

Light.

Lab Ray approximation & geometric optics:

Lab Reflection and refraction, Huygen's pple, total internal reflection, mirrors, images, lenses, magnifier, compound microscope, telescope

Interference and Diffraction

Conditions for interference, Young's experiment, and interference pattern, phasor addition, reflection, thin films, diffraction

Mechanical waves

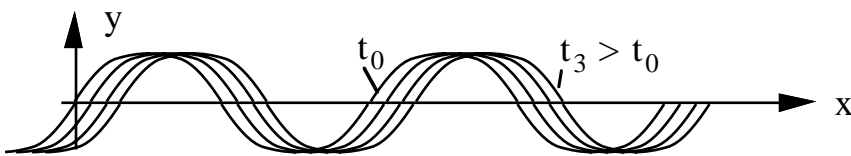
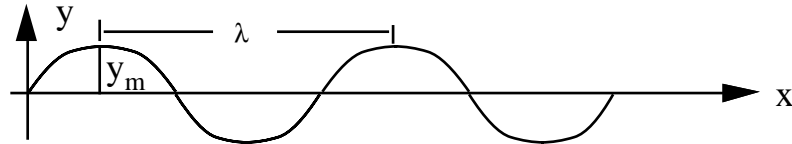
<i>example</i>	<i>type</i>	<i>restoring force</i>
Wave in string	transverse	tension in string
Water wave	transverse	gravity
Sound wave	longitudinal	air pressure

Only pattern travels, not medium.

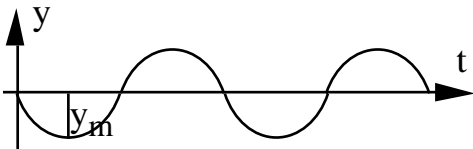
Travelling wave $f(x - vt)$ is a wave travelling at v in $+x$ dirⁿ:

An important example $y = y_m \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$

A sine wave travelling to the right



Each point: Simple Harmonic Motion with period T .



Harmonic waves

One cycle of SHM takes T , and wave travels λ , \therefore

$$v = \frac{\lambda}{T} = f\lambda$$

Write wave equation in various ways:

$$y = y_m \sin \left(\frac{2\pi}{\lambda} (x - vt) \right)$$

$$y = y_m \sin \left(2\pi \left(\frac{x}{\lambda} - ft \right) \right)$$

define angular frequency $\omega \equiv 2\pi f$ and

define wave number $k \equiv \frac{2\pi}{\lambda}$

$$y = y_m \sin (kx - \omega t)$$

You need practice with this: do some probs on tut 8!

Example A wave has $y = y_m \sin (kx - \omega t)$,

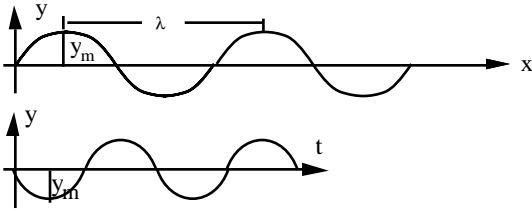
$$y_m = 10 \text{ nm}, k = 18.5 \text{ m}^{-1}, \omega = 6300 \text{ rad.s}^{-1}$$

- i) what is the speed of the wave?
 ii) What is (max) average speed of particles?

Only know k & ω . How to get v ? $v = \text{what?}$

$$\begin{aligned} v_{\text{wave}} &= f\lambda && \text{now use } k, \omega \\ &= \frac{\omega}{2\pi} \cdot \frac{2\pi}{k} = \frac{\omega}{k} = \dots = 340 \text{ ms}^{-1} \end{aligned}$$

what is v_{part} ?



$$\begin{aligned} v_{\text{part}} &= \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t} y_m \sin (kx - \omega t) \\ &= \omega y_m \cos (kx - \omega t) \end{aligned}$$

$$\begin{aligned} v_{\text{max}} &= \omega y_m \\ &= 6300 \text{ s}^{-1} \times 10 \text{ nm} = 63 \mu\text{m.s}^{-1} \end{aligned}$$

average speed, not individual speed, cf wind

Speed of wave

In a **string**, it depends on Tension (F) and the mass per unit length ($\mu = m/L$) of the string.

Dimensional analysis:

$$v \propto T^a (\mu)^b$$

$$\text{dimensions: } lt^{-1} = (mt^{-2})^a (mt^{-1})^b$$

equate the powers:

$$t: \quad -1 = -2a \quad \therefore a = \frac{1}{2}$$

$$m: \quad 0 = a + b \quad \therefore b = -a = -\frac{1}{2}$$

$$l: \quad (\text{check}) \quad 1 = a - b = \frac{1}{2} + \frac{1}{2} \rightarrow \text{consistent}$$

$$v_{\text{string}} = \sqrt{\frac{T}{\mu}}$$

In general, $v_{\text{wave}} = \sqrt{\frac{\text{springy thing}}{\text{inertial thing}}}$

$$v_{\text{sound}} = \sqrt{\frac{\text{elastic const}}{\text{density}}} = \sqrt{\frac{\gamma P}{\rho}}$$

$$c = \sqrt{\frac{k_{\text{elec}}}{k_{\text{mag}}}} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

$$\gamma = \frac{c_P}{c_V} \quad \text{is ratio of specific heats } \frac{c_P}{c_V}$$

Example The bulk modulus B of elasticity of water is 2.0 GPa. How fast is sound in water?

$$v = \sqrt{\frac{\text{springy thing}}{\text{inertial thing}}}$$

$$\text{for string it was } v = \sqrt{\frac{\text{Tension}}{l D \text{ density}}}$$

try $v = \sqrt{\frac{B}{\rho}}$

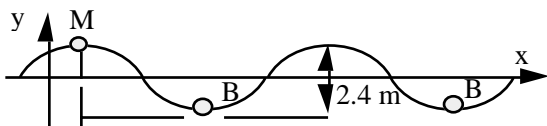
$$\text{units: } = \sqrt{\frac{\text{Pa}}{\text{kg} \cdot \text{m}^{-3}}} = \sqrt{\frac{\text{F/area}}{\text{kg} \cdot \text{m}^{-3}}} = \sqrt{\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \frac{1}{\text{m}^2} \frac{\text{m}^3}{\text{kg}}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

what is ρ ?

$$v = \sqrt{\frac{2 \cdot 10^9 \text{ Pa}}{10^3 \text{ kgm}^{-3}}} = 1.4 \text{ km} \cdot \text{s}^{-1}$$

Example Bob and Mary float in water deep enough for waves to be sinusoidal. The wave speed is 10 ms^{-1} . Their vertical position varies 2.4 m, Mary gets to the top just as Bob gets to the bottom. They experience maximum accelerations of 0.015 g . How far apart are they?



we need λ . let's write wave equation for a start

$$y = y_m \sin(kx - \omega t) \quad \text{but we know } a_m$$

$$a = \frac{\partial^2 y}{\partial x^2} = -y_m \omega^2 \sin(kx - \omega t)$$

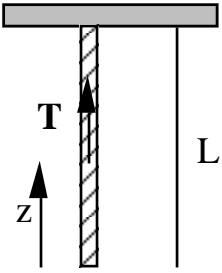
$$a_{\text{max}} = y_m \omega^2$$

$$.015 \text{ g} = 0.15 \text{ ms}^{-2} = \frac{2.4 \text{ m}}{2} \omega^2$$

$$\omega = 0.35 \text{ s}^{-1} \rightarrow f = 0.056 \text{ Hz} \quad \rightarrow \lambda, T$$

$$\lambda = \frac{v}{f} = 180 \text{ m.}$$

Example. A rope of length L hangs vertically. How long does it take a wave to travel from one end to the other?



well we'll need the wave speed...

$$v = \sqrt{\frac{T}{\mu}} \quad \text{where } \mu = \frac{m}{L}$$

but T varies

T = weight below that point

$$= (\mu z)g$$

$$v = \frac{dz}{dt} = \sqrt{\frac{\mu z g}{\mu}}$$

$$dt = \frac{1}{\sqrt{g}} \cdot z^{-1/2} dz$$

$$dt = \int dt = \int_{z=0}^L \frac{z^{-1/2} dz}{\sqrt{g}}$$

$$= \left[\frac{2}{\sqrt{g}} \cdot z^{1/2} \right]_0^L$$

$$= 2\sqrt{\frac{L}{g}} \quad \text{check units}$$

Reflection:

Going from less dense to more dense, waves are reflected with a phase change of π .

e.g. reflection at a 'fixed' end
thin string to thick string,
air to water

From more dense to less dense, no phase change

e.g. reflection at 'free' end, etc

Superposition

In a linear medium, waves superpose linearly, *i.e.* their displacements simply add.

Most media linear for *small* amplitude waves.

But beware water waves when $y_m \rightarrow \text{depth}$

sound waves when $p_m \rightarrow P_{\text{atmos}}$

\rightarrow non linear

Superpose incident & reflected waves \rightarrow standing waves

Standing waves

$$y_1 = y_m \sin(kx - \omega t)$$

$$y_2 = y_m \sin(kx + \omega t)$$

$$y_T = y_m \left(\begin{array}{l} \sin kx \cos \omega t - \cos kx \sin \omega t + \\ \sin kx \cos \omega t + \cos kx \sin \omega t \end{array} \right)$$

$$= 2y_m \sin kx \cos \omega t$$

stationary wave simple harmonic motion

1. Boundary fixed (no displacement)

phase change of 180°

e.g. reflection at a hard surface

node at boundary

2. Boundary free (any displacement)

phase change of zero

e.g. reflection at bell of trumpet


anti-node at boundary

Standing waves




$$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

(fundamental)




$$f_2 = \frac{v}{\lambda} = \frac{2v}{2L} = 2f_1$$

(2nd harmonic)



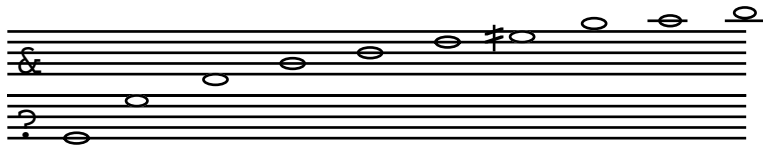
$$f_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_1$$

(3rd harmonic)



$$f_4 = \frac{v}{\lambda} = \frac{4v}{2L} = 4f_1$$

(4th harmonic)



300

400

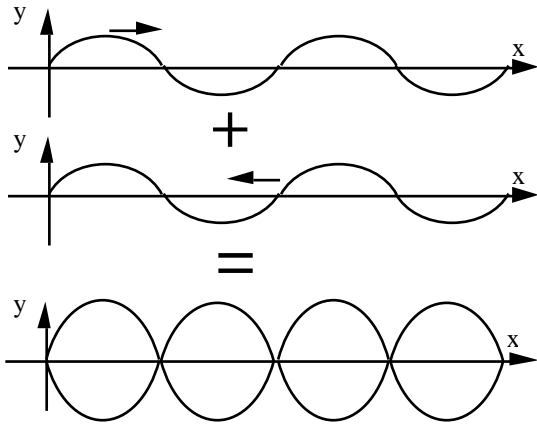
500

Example

Write the equations for two travelling waves which together in superposition could produce a standing wave.

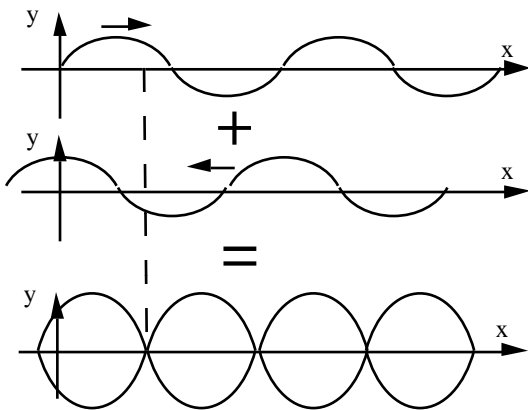
$$y_1 = A \sin(kx - \omega t) \text{ and}$$

$$y_2 = A \sin(kx + \omega t + \phi) \quad (\phi \text{ may have any value including zero})$$



$\phi = 0$, but also:

ϕ of second wave = 90°



Example A cable is 8 m long, 8 mm in diameter, and subject to a tension of 7.0 kN under some conditions. The cable has $\rho = 5,600 \text{ kg}\cdot\text{m}^{-3}$.

- i) Estimate the first 5 resonant frequencies of the cable.
- ii) So what?
- i) The possible standing wave resonances have

$$\lambda = 2L, L, 2L/3, \dots \lambda_n = 2L/n$$

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

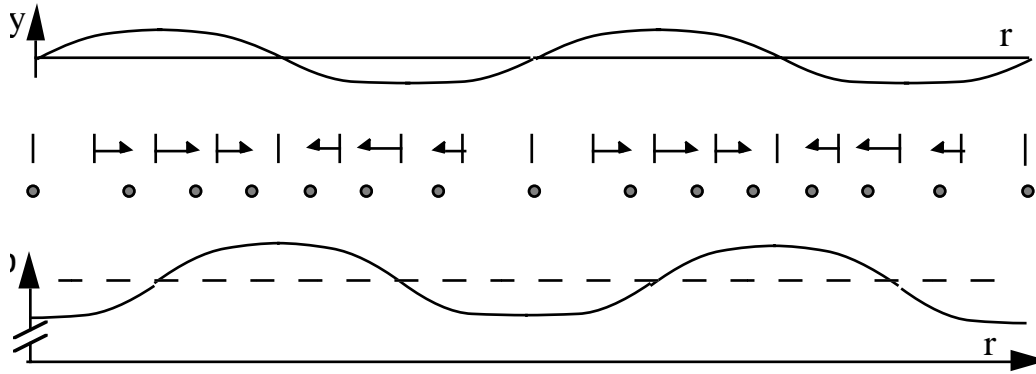
$$\mu = \frac{m}{L} = \frac{\pi r^2 L \rho}{L} = \rho \pi r^2.$$

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\rho \pi r^2}} = \dots = n * 9.9 \text{ Hz}$$

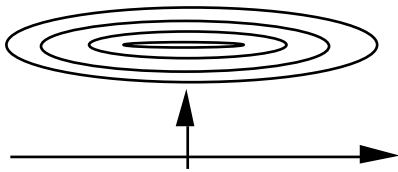
$$f_n \text{s} \cong 10, 20, 30, 40, 50 \text{ Hz}$$

- ii) Lots of energy stored at resonance! vibrations could be structurally serious.

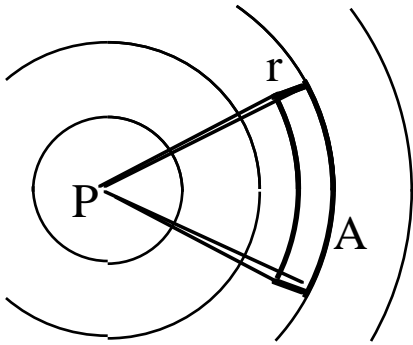
Sound is a compression wave - longitudinal
Write displacement $y(r,t)$ but y is in r direction



Radiation in 2 dimensions



Radiation in 3 dimensions



Intensity $I \equiv \frac{\text{power}}{\text{area}}$

Source, power P , radiates isotropically in 3D

$$I = \frac{P}{4\pi r^2}$$

Power in a wave: energy in spring $U_s = \frac{1}{2} k_s y^2$

Energy $\propto y^2 \therefore$ Intensity $\propto (\text{displacement})^2$

Example

An air duct is closed at one end but open at the other. It is 3.4 m long. What are its resonant frequencies? So what?

It can have a displacement node at closed end, antinode at the open end

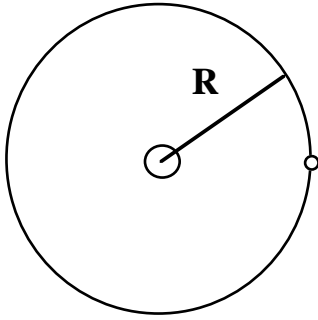
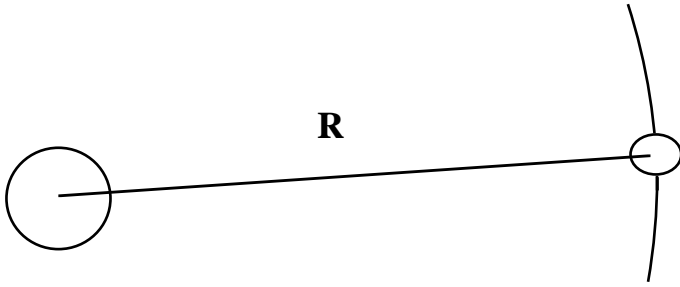
$$\lambda = 4L, \frac{4L}{3}, \frac{4L}{5}, \frac{4L}{7} \text{ etc}$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}, \frac{3v}{4L}, \frac{5v}{4L}, \dots$$

$$= 25 \text{ Hz}, 75 \text{ Hz}, 125 \text{ Hz}$$

Can store large amounts of energy at these frequencies: noise & structural vibration.

Example: What is the intensity of solar radiation? $P_{\text{sun}} = 3.9 \cdot 10^{26}$ W. Earth is 150 million km from sun.



$$I = \frac{P}{4\pi r^2} = \dots = 1.38 \text{ kWm}^{-2}$$

above atmosphere, \perp radiation

Intensity and sound level

Sense of loudness is also \sim logarithmic, \therefore define

Sound intensity level:

$$L_I \equiv 10 \log_{10} \frac{I}{I_0} \quad \text{where } I_0 = 10^{-12} \text{ W.m}^{-2} \quad (L_I \text{ in decibels})$$

Note: Power \propto displacement² \propto pressure²

$$L_2 - L_1 = 10 \left(\log_{10} \frac{I_2}{I_0} - \log_{10} \frac{I_1}{I_0} \right) = 10 \log_{10} \frac{I_2}{I_1}$$

Examples of p , I , L_p , L_I

p_2/p_1	ΔL_p	I_2/I_1	ΔL_I
$\sqrt{2}$	3 dB	2	3 dB
2	6 dB	4	6 dB
$\sqrt{10}$	10 dB	10	10 dB
10	20 dB	100	20 dB

Sound radiation:

If sound radiates uniformly in three dimensions:

$$\text{Intensity} \equiv \frac{\text{power}}{\text{unit area}}$$

$$\text{But note: } I \propto p^2, \therefore p \propto \frac{1}{r}$$

Uniform spherical radiation:

r_2/r_1	I_2/I_1	p_2/p_1	$L_2 - L_1$
2	1/4	1/2	6 dB
10	1/100	1/10	20 dB

Example. If sound level $L_I = 3$ dB at 10 cm from a source radiating uniformly, what is the acoustic power of the source? $L \rightarrow I, I \& r \rightarrow P$

$$3 \text{ dB} = L_I \equiv 10 \log \frac{I}{I_0}$$

$$0.3 = \log \frac{I}{I_0}$$

$$I/I_0 = \text{antilog } 0.3 = 10^{0.3} = 2$$

$$I = 2 I_0 = 2 \cdot 10^{-12} \text{ Wm}^{-2}$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

$$\begin{aligned} P &= 4\pi r^2 I \\ &= 4\pi (0.10\text{m})^2 (2 \cdot 10^{-12} \text{ Wm}^{-2}) \\ &= 0.25 \text{ pW} \quad (2.5 \cdot 10^{-13} \text{ W}) \end{aligned}$$

Effect of boundaries:

A completely reflecting wall absorbs no energy:

$$I = \frac{P}{A} = \frac{P}{2\pi r^2} = 2 \times \begin{array}{l} \text{intensity from} \\ \text{free radiation} \end{array}$$

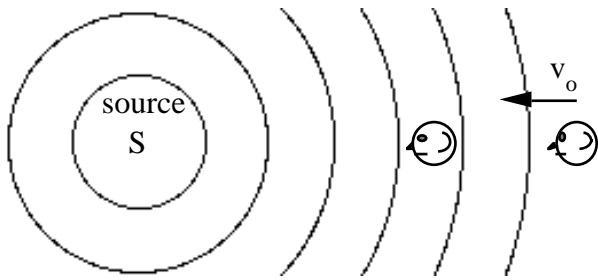
\therefore busker standing next to a wall gains 3 dB (but so does some of the background noise).

What if you stand in a (reflecting) corner?

Example. A loudspeaker at floor level produces 1 W of acoustic power. What sound level does it produce at a distance of 3 m?

$$\begin{aligned} L_I &\equiv 10 \log \frac{I}{I_0} \\ &= 10 \log \frac{P/2\pi r^2}{I_0} = \dots \\ &= 103 \text{ dB} \end{aligned}$$

Doppler effect. 1. Stationary source



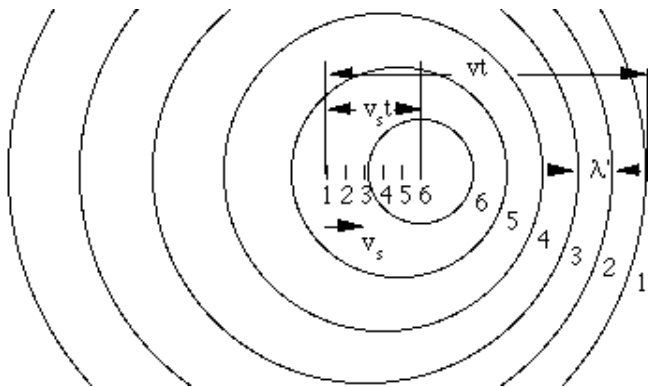
Observer at rest receives v/λ crests/unit time. Moving observer crosses v_o/λ extra crests/unit time. \therefore Observer hears

$$f' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v + v_o}{v/f}$$

$$f' = f \left(1 + \frac{v_o}{v} \right)$$

Note convention: v_o is positive if approaching.

Doppler effect. 2. Moving source



Wavefront 1 emitted from position 1, etc
(ft) crests are spread over $(vt - v_s t)$

For observer, $\lambda' = \frac{vt - v_s t}{ft}$

$$f' = \frac{v}{\lambda'} = f \left(\frac{v}{v - v_s} \right)$$

For both moving: $f' = f \left(\frac{v + v_o}{v - v_s} \right)$

v_o and v_s are positive for approaching
measure all velocities with respect to medium

Example. How fast must you run (bicycle?) to reduce the pitch by one semitone (6%)?

$$0.94 f = f' = f \left(\frac{v + v_o}{v - 0} \right)$$

$$0.94 = 1 + \frac{v_o}{v}$$

$v = 340 \text{ ms}^{-1}$ $v_o = -20 \text{ ms}^{-1}$

Example. i) Car approaches stationary observer at 150 kph. What doppler shift? **ii)** What if observer approaches stationary source at 150 kph?

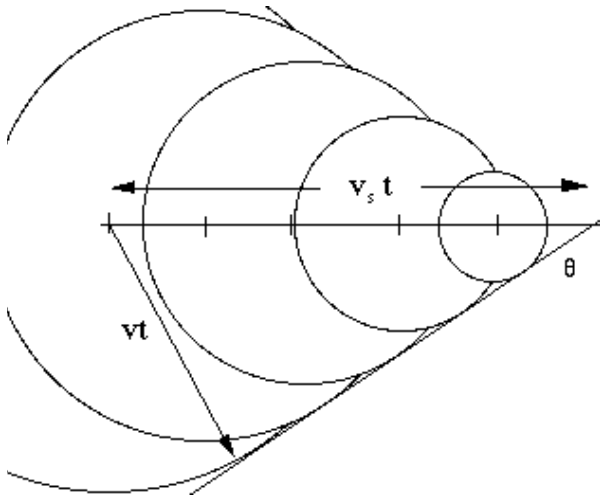
Moving source Moving observer

$$f' = f \left(\frac{v + 0}{v - v_s} \right) \qquad f' = f \left(\frac{v + v_o}{v - 0} \right)$$

1.14 f 1.10 f

What if $v_s > v$?

Shock wave

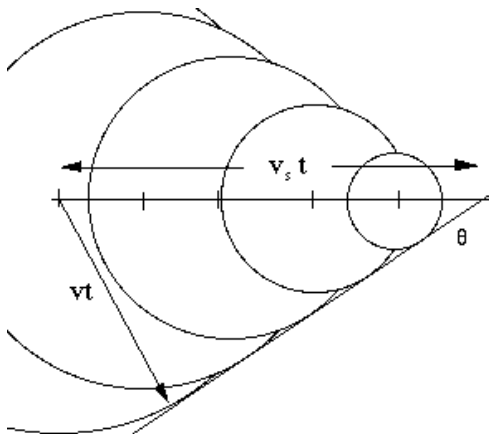


Crests combine to form a shock wave. Cone has half-angle θ where

$$\sin \theta = \frac{v}{v_s}$$

$\frac{v_s}{v}$ is called the Mach number

Example Plane travelling at 2000 km.hr and height 5 km. Where is the plane when you first hear it?



$$\sin \theta = \frac{v_{\text{sound}}}{v_{\text{plane}}} \quad \theta = 38^\circ$$

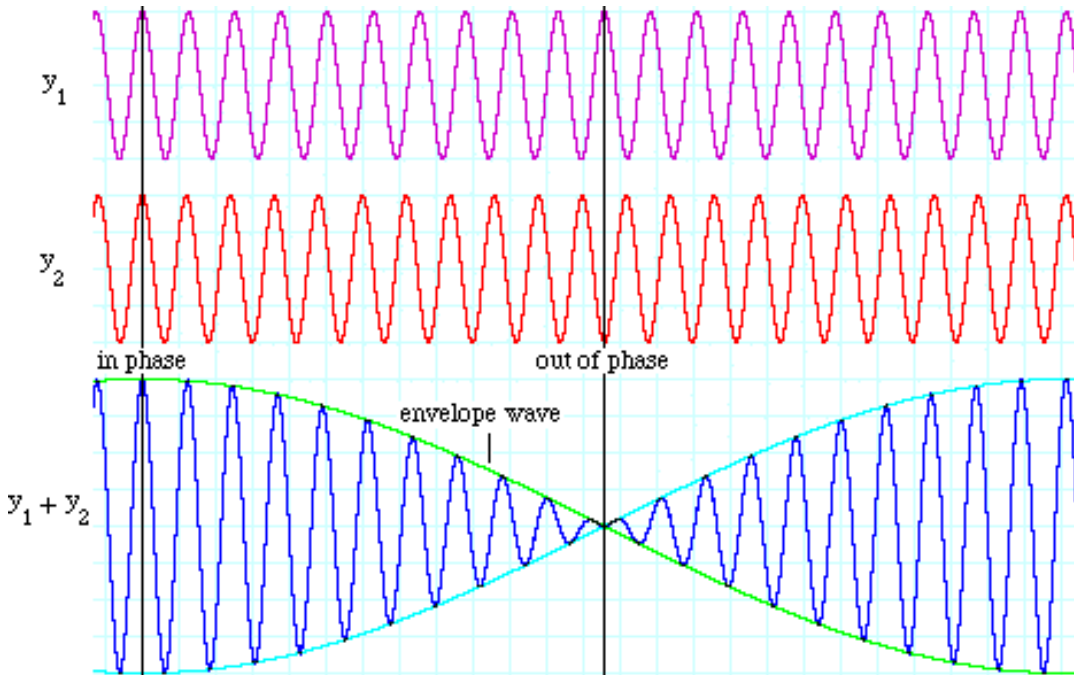
$$\frac{h}{D} = \tan \theta, \quad \dots \quad D = 6.5 \text{ km}$$

Beats <http://www.phys.unsw.edu.au/~jw/beats.html>

Add two sine waves of similar frequencies

$$y_1 = A \cos 2\pi f_1 t$$

$$y_2 = A \cos 2\pi f_2 t$$



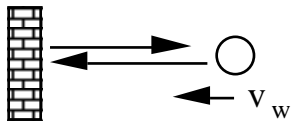
$$\cos A + \cos B = 2 \cos \frac{A-B}{2} \cos \frac{A+B}{2}$$

$$y_1 + y_2 = 2A \cos 2\pi \frac{f_1+f_2}{2} t \cos 2\pi \frac{f_2-f_1}{2} t$$

average frequency

Beat frequency is $f_2 - f_1$.

Example. You walk towards a wall, blowing a whistle at $f = 500$ Hz. You hear beats at 5 Hz between your whistle and the reflected sound. How fast are you walking?



You hear your own whistle at frequency f .

The wall receives

$$f' = f \frac{v + v_o}{v - v_s} = f \frac{v + 0}{v - v_w}$$

This is the source of the reflection. You hear

$$f'' = f' \frac{v + v_o}{v - v_s} = f' \frac{v + v_w}{v - 0}$$

$$f'' = f \frac{v + v_w}{v - v_w}$$

$$f''(v - v_w) = f(v + v_w)$$

$$(f'' - f)v = (f'' + f)v_w$$

$$v_w = v \frac{f'' - f}{f'' + f} = v \frac{5}{1005} = 1.7 \text{ ms}^{-1}$$

Example

An air duct is closed at one end but open at the other. It is 3.4 m long. What are its resonant frequencies?

It can have a displacement node at closed end, antinode at the open end

$$\lambda = 4L, \frac{4L}{3}, \frac{4L}{5}, \frac{4L}{7}$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}, \frac{3v}{4L}, \frac{5v}{4L},$$

250 Hz, 750 Hz, 1250 Hz etc

Example

An organ pipe is closed at one end. It is tuned to 440 Hz (*called A4*). Another is tuned so that it makes beats with the first at 2 Hz. What is the difference in length? (You may neglect end effects)

$$L \cong \frac{\lambda}{4} = \frac{v}{4f}$$

$$L_2 - L_1 = \frac{v}{4} \left(\frac{1}{f_2} - \frac{1}{f_1} \right)$$

$$f_1 = 440 \text{ Hz. } f_2 = 438 \text{ Hz or } 442 \text{ Hz}$$

$$\Delta L = 0.9 \text{ mm}$$

Example

How much must I increase the voltage applied to a loudspeaker in order to get an increase in sound level of 80 dB?

$$V \rightarrow P \rightarrow I \rightarrow L$$

$$L \equiv 10 \log_{10} \frac{I_1}{I_0} \rightarrow$$

$$L_2 - L_1 = 10 \left(\log_{10} \frac{I_2}{I_0} - \log_{10} \frac{I_1}{I_0} \right) = 10 \log_{10} \frac{I_2}{I_1}$$

Speakers ~ resistors:

$$I \equiv \frac{P}{A} = \frac{V^2/R}{A} \quad R \text{ \& A constant}$$

$$L_2 - L_1 = 10 \log_{10} \frac{V_2^2}{V_1^2}$$

$$\log_{10} \frac{V_2^2}{V_1^2} = \frac{L_2 - L_1}{10} = 8$$

$$\frac{V_2^2}{V_1^2} = 10^8$$

Must increase voltage by factor of 10^4

Example. A guitarist tunes the A string of her guitar to 110 Hz. She then wants to tune the E string to $\frac{3}{4} 110 = 82.5$ Hz. How to do this

A string:



$$f_{3A} = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_A$$

(3rd harmonic on A string)

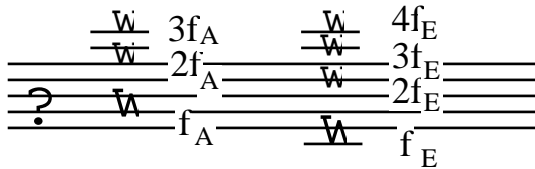


$$f_{4E} = \frac{v}{\lambda} = \frac{4v}{2L} = 4f_E$$

(4th harmonic on E string)

When $3f_A = 4f_E$, $\frac{f_E}{f_A} = \frac{3}{4}$

So get the strings approx in tune and then **remove the beats**



Example Which is faster, sound in air, or sound in Aluminium?

$$v = \sqrt{\frac{\text{elastic constant}}{\text{inertial constant}}}$$

Elastic constant for pressure deformation:

$$\frac{\Delta V}{V} \equiv -\frac{p}{\kappa}$$

where κ is the bulk modulus of elasticity.

κ is **large** for solids!

$$\kappa_{Al} = 70 \text{ GPa}$$

$$\rho_{Al} = 2698 \text{ kg.m}^{-3}$$

$$v_{Al} = \sqrt{\frac{\kappa_{Al}}{\rho_{Al}}} = 5 \text{ km.s}^{-1}$$

Example A wave travels in a stretched string. Derive an expression for the ratio of the speed of the *string* to the slope of the string at any point.

$$y = A \sin (kx - \omega t)$$

$$\text{where } k = \text{wave number} \equiv \frac{2\pi}{\lambda} \quad \omega = 2\pi f$$

$$\text{Slope of string} = \frac{\partial y}{\partial x} \quad (\text{t const})$$

$$= Ak \cos (kx - \omega t)$$

$$\text{Speed of a particle in the string} = \frac{\partial y}{\partial t} \quad (\text{x const})$$

$$= -A\omega \cos (kx - \omega t)$$

$$\frac{\text{speed}}{\text{slope}} = \frac{-A\omega \cos (kx - \omega t)}{Ak \cos (kx - \omega t)}$$

$$= 2\pi f \cdot \frac{\lambda}{2\pi}$$

$$= f\lambda$$

$$= v$$

Example

$$y_1 = (.003) \sin (10 x - 20 t) \quad (\text{SI units})$$

$$y_2 = (.003) \sin (15 x - 30 t)$$

What is the phase difference at $(x,t) = (0.10, 2.0)$?

Where does $y_1 + y_2 = 0$ at $t = 2.00$ s?

Note: they have different k and ω , so different λ and f . But

$$\omega/k = 2\pi f \cdot \frac{\lambda}{2\pi} = f\lambda = v \text{ is the same.}$$

$$\begin{aligned} \phi_1 &= (10 x - 20 t) = (10*0.10 - 20*2.0) \\ &= -39.0 \text{ rad} \end{aligned}$$

$$\phi_2 = \dots = -48.5 \text{ rad}$$

$$\Delta\phi = 9.5 \text{ radians} = 1 \text{ cycle} + 3.2 \text{ rad}$$

$$y_1 + y_2 = 0$$

$$(.003) \sin (10 x - 20 t) = - (.003) \sin (15 x - 30 t)$$

$$\sin \alpha = - \sin \beta \text{ when}$$

$$\alpha = \beta \pm n\pi \text{ where } n \text{ is odd integer}$$

$$\begin{aligned} \therefore \quad 10 x - 20 t &= 15 x - 30 t \pm n\pi \\ t &= 2.00 \text{ s} \end{aligned}$$

$$\therefore \quad x = (4.0 \mp n\pi) \text{ m} \quad n \text{ odd}$$

Example How much would the pitch of my voice rise (all else equal) if I filled my vocal tract with helium?

$$v_{\text{sound in gas}} = \sqrt{\frac{K_{\text{adiabatic}}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

where $\gamma = \frac{C_p}{C_v}$ see later

Now P will be the same in the two cases, and γ is not very different.

Example

Cheap ear-plugs reduce the sound level at the ear by 26 dB. How much do they reduce the sound power and the sound pressure transmitted to the ear?

$$L_2 - L_1 \equiv 10 \log_{10} \frac{I_2}{I_1} \equiv 20 \log_{10} \frac{p_2}{p_1}$$

$$\text{i) } 26 \text{ dB} \equiv 10 \log_{10} \frac{I_2}{I_1}$$

$$\therefore \frac{I_2}{I_1} = 10^{26/10} = 400$$

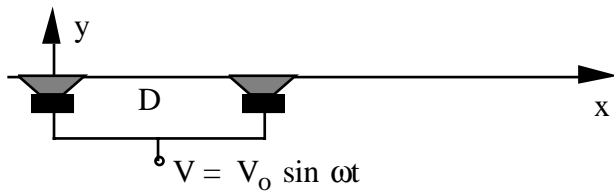
$$3 \text{ dB} = *2$$

$$10 \text{ dB} = *10$$

$$\text{so } 26 \text{ dB} = 2*2*10*10 = 400$$

$$\text{ii) } \frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = 20$$

Example. Sinusoidal signal drives two identical loudspeakers a distance D apart. How does the sound level vary along the line between them?



Between speakers ($0 < x < D$)

from left speaker (L) $p_L = A_L \sin(kx - \omega t)$

from right $p_R = A_R \sin(k(x-D) + \omega t)$

but $A = A(x)$. How does amplitude vary with x ?

$$I \propto p^2 \text{ and } I \propto \frac{1}{r^2}$$

$$\text{so } p \propto \frac{1}{r}$$

$$p_L = \frac{P_m}{x} \sin(kx - \omega t)$$

from right $p_R = \frac{P_m}{x - D} \sin(k(x-D) + \omega t)$

Near middle, $A_L(x) \cong A_R(x) \rightarrow$ standing wave.

Very near left speaker, $p_{\text{total}} \cong p_L$

