Mechanics and forces

Aristotle: $\mathbf{v} = 0$ is "natural" state

Galileo & Newton: $\mathbf{a} = 0$ is "natural" state

Newton's Laws

First "zero (total) force $\Rightarrow$ zero acceleration"

more formally:

If $\Sigma\,\mathbf{F} = 0$, $\exists$ reference frames in which $\mathbf{a} = 0$

called **Inertial frames**

In such frames:

Second $\Sigma\,\mathbf{F} = \frac{d}{dt}\mathbf{p}$

$$\frac{d}{dt}\mathbf{p} \equiv \frac{d}{dt}m\mathbf{v} \equiv \frac{dm}{dt}\mathbf{v} + m\frac{d\mathbf{v}}{dt}$$

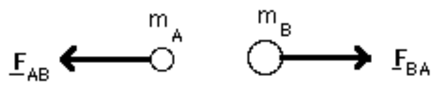
if m is constant $\Sigma\,\mathbf{F} = m\mathbf{a}$

$(\Sigma\,F_x = ma_x \qquad \Sigma\,F_y = ma_y \qquad \Sigma\,F_z = ma_z)$

Third: "To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

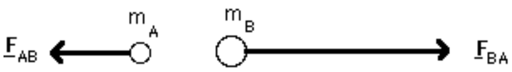
Or

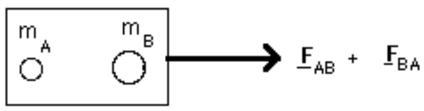
Forces always occur in pairs, $\underline{\mathbf{F}}$ and $-\underline{\mathbf{F}}$, one acting on each of a pair of interacting bodies.



Third $\underline{\mathbf{F}}_{AB} = -\underline{\mathbf{F}}_{BA}$

Why so?





Work and energy

Work defined as $dW \equiv \mathbf{F}.d\mathbf{s}$

Work energy theorem follows from (is another way of stating) the 2nd law:

If m = constant

$$\mathbf{F} = \frac{d}{dt}\mathbf{p} \equiv \frac{d}{dt}m\mathbf{v} \quad m\frac{d}{dt}\mathbf{v}$$

$$\therefore \quad \text{work done} = \int_i^f \mathbf{F}.d\mathbf{s} = \int_i^f m\frac{d\mathbf{v}}{dt}.d\mathbf{s}$$

$$= m\int_i^f \frac{d\mathbf{s}}{dt}.d\mathbf{v} = m\int_i^f \mathbf{v}.d\mathbf{v}$$

Total work done $= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

If work is done by a conservative force, define the potential energy U

Work done against conservative force $= \Delta\,U$

$\therefore$ If only conservative forces act $U_i + \frac{1}{2}mv_i^2 = U_f + \frac{1}{2}mv_f^2$

Important forces:

$$\mathbf{F}_{grav} = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

On surface of the earth: $\mathbf{F}_{grav} = -G\frac{M_{earth}m}{r_{earth}^2}\,\hat{\mathbf{up}}$

$$= m\left(\frac{GM_{earth}}{r_{earth}^2}\right) \text{ down} \quad \cong \text{mg down}$$

where $g \cong \left(\frac{GM_{earth}}{r_{earth}^2}\right) = ... = 9.8\ ms^{-2}$

Electric force q_1 $q_2 \rightarrow \hat{\mathbf{r}}$

$$\mathbf{F}_{elec} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

van der Waals force (electrodynamic force)

$p_1 \downarrow$ $\uparrow p_2$

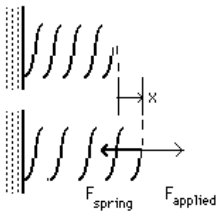
$F_{vdw} \propto \frac{1}{r^6}$ always attractive

Phases of matter

$$KE = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 = \frac{3}{2}kT$$

High T	kT >> |PE|	$\rightarrow$	gas
Intermediate T	kT ~ |PE|	$\rightarrow$	liquid
Low T	kT << |PE|	$\rightarrow$	solid
Very High T	*kT > work function*	$\rightarrow$	*plasma*
	kT > nuclear energies	$\rightarrow$	*exotic matter*

Hooke's Law.



No applied force
(x = 0)

Under tension
x > 0

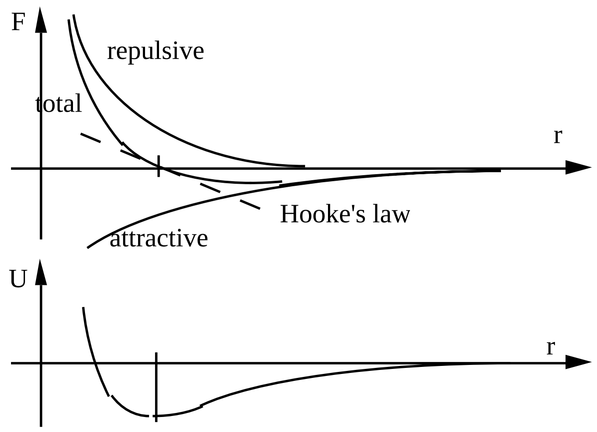
$\mathbf{F}_{spring}$ in opposite direction to x.

Experimentally, $|\mathbf{F}_s| \propto |x|$ over small range of x

$F = -kx$ **Hooke's Law.**

Properties of condensed phases

Inter-atomic & intermolecular forces amd energies



$\rightarrow$ **Linear elasticity** (*parabolic minimum in U(r)*)

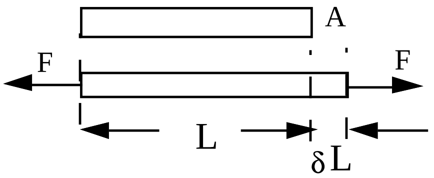
Linear approximation to inter-molecular forces

Stress $\sigma \equiv F/A$

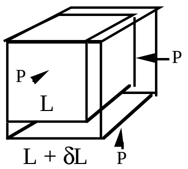
Strain $\varepsilon \equiv$ dimensionless change, e.g. $\frac{\delta L}{L}$

Hooke's Law: $\frac{\sigma}{\varepsilon}$ = **elastic modulus**

Longitudinal stress:

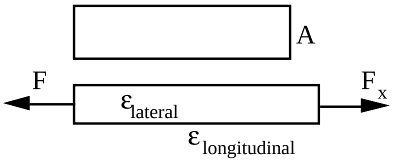


Young's modulus $Y = \frac{F/A}{\delta L/L} = \frac{FL}{A\delta L}$



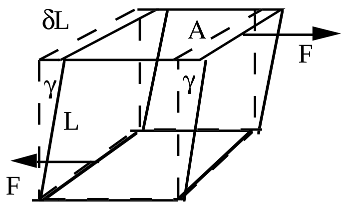
Bulk modulus

$\kappa = -\frac{P}{\delta V/V}$



Poisson's ratio $\nu \equiv -\frac{\varepsilon_{lat.}}{\varepsilon_{long.}} = -\frac{\delta d/d}{\delta L/L}$

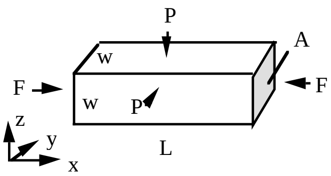
Rigidity modulus (shear modulus)



Rigidity modulus

$G = \frac{F/A}{\delta L/L} = \frac{\tau}{\gamma}$

Example



F applied to the ends. P applied to other 4 sides.
What is P so that cross section A is unchanged?

$$\varepsilon_y = -\nu\sigma_x/Y + \sigma_y/Y - \nu\sigma_z/Y$$

$$0 = +\nu F/YA - P/Y + \nu P/Y$$

$$\nu F/A = P(1-\nu)$$

$$P = \frac{F}{A}\frac{\nu}{(1-\nu)}$$

Example

For the potential $U(r) = -\frac{A}{r^n} + \frac{B}{r^m}$, what is F(r)?

What is the r_o for mechanical equilibrium?

$$dU = -dW \equiv \mathbf{F}.d\mathbf{s} \rightarrow$$

$$F = -\frac{dU}{dr}$$

$$F(r) = n\frac{A}{r^{n+1}} - m\frac{B}{r^{m+1}}$$

F = 0 at minimum of energy (dU = 0)

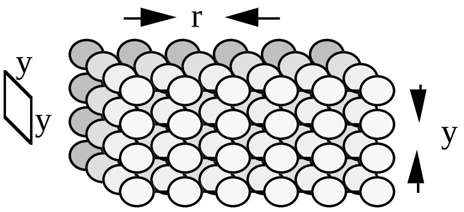
$$n\frac{A}{r_o^{n+1}} = m\frac{B}{r_o^{m+1}}$$

$$\frac{r_o^{m+1}}{r_o^{n+1}} = r_o^{m-n} = \frac{mB}{nA}$$

$$r_o = \sqrt[m-n]{\frac{mB}{nA}}$$

$$U(r_o) = -\frac{A}{r^n} + \frac{B}{r^m} = ... = \text{binding energy} \quad \textit{cf} \quad \textit{max force}$$

Example Using the polynomial model for U(r), determine the Young's modulus (*C&L p43*)



$$Y = \frac{F/A}{\delta x/x}$$

$$= \frac{F/y^2}{\delta x/r_o}$$

$$\cong \frac{dF}{dr}\frac{r_o}{y^2}$$

As before $U(r) = -\frac{A}{r^n} + \frac{B}{r^m}$

$$F(r) = n\frac{A}{r^{n+1}} - m\frac{B}{r^{m+1}}$$

$$r_o = \left(\frac{mB}{nA}\right)^{1/m-n}$$

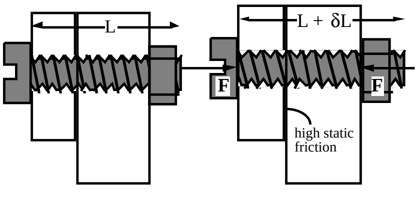
$$\frac{d}{dr}F(r) = \frac{d}{dr}\left(n\frac{A}{r^{n+1}} - m\frac{B}{r^{m+1}}\right)$$

$$= -n(n+1)\frac{A}{r^{n+2}} + m(m+1)\frac{B}{r^{m+2}}$$

at $r = r_o$, $\frac{d}{dr}F(r) = ... = \frac{nA}{r_o^{n+2}}(m-n)$

$$Y = \frac{dF}{dr}\frac{r_o}{y^2} = \frac{(m-n)nA}{r_o^{n+1}\,y^2}$$

Example. A steel set screw (Y = 206 GPa) has 50 turns, diameter 3 mm, and is just long enough. From contact, it is tightened one complete turn. Estimate the axial force it exerts.



$$Y \equiv \frac{\sigma}{e} = \frac{\text{stress}}{\text{strain}}$$

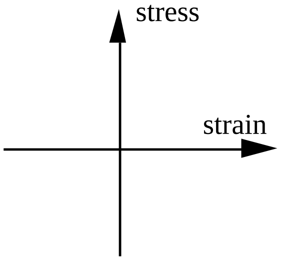
e here is $\delta L/L \cong 1/50$.

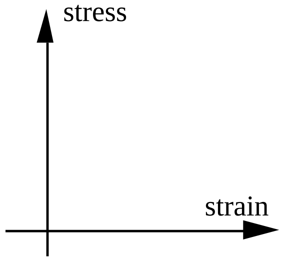
$\sigma = F/A = F/\pi r^2$

$$F = \pi r^2\sigma = \pi\left(\frac{d}{2}\right)^2 Ye$$

$$= ..... = 29\ \text{kN}$$

Hysteresis **Other non-elasticity**





Inter-atomic & intermolecular forces

Ionic solids: electrostatic force

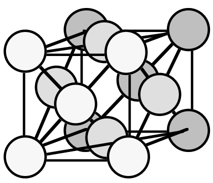
Hydrogen bonds H has $+\delta$ charge

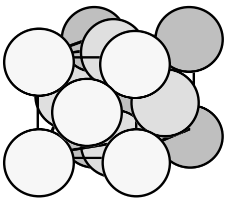
van der Waals attraction attraction between transient dipoles $\propto r^{-6}$ at short range

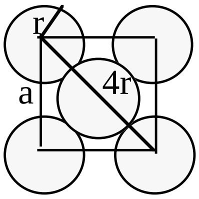
Crystalline solids

Packing factor fraction of space occupied by touching hard spheres

Example Calculate packing factor and ρ of FCC







Face diagonal = 4r
Side of unit cube = 4r/sin 45°
$= 2\sqrt{2}r$

$$\text{p.f.} \equiv \frac{\text{Volume of spheres in unit cube}}{\text{Volume of unit cube}}$$

$$= \frac{\text{8 corners + 6 faces}}{(2\sqrt{2}r)^3}$$

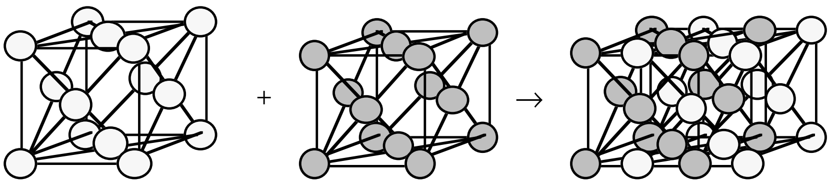
$$= \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2})\ \frac{4}{3}\pi r^3}{(2\sqrt{2}r)^3} = 74\%$$

$$\rho = \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2})\ \text{atomic mass}}{a^3} = \frac{4m}{a^3}$$

Ionic crystals

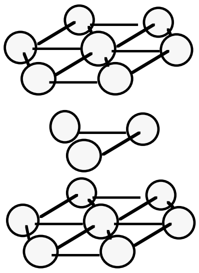
e.g. NaCl: ions of similar size, each ion has six neighbours of opposite charge (**coordination number** six)

(It's like two interlaced FCCs)



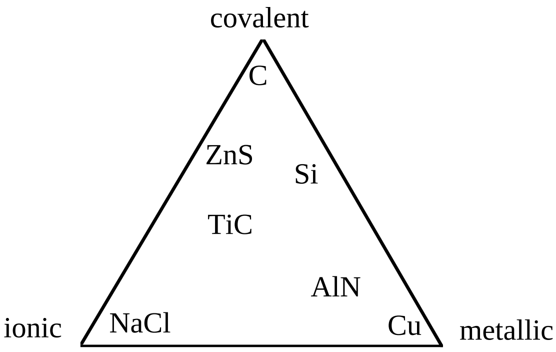
Covalent crystals: share outer electrons. Depends on the angles of the electron orbitals

Metallic crystal: 'ions' in a sea of shared electrons. Often close packed in FCC or HCP. This, gives high ρ, especially if atomic number is large (e.g. Au, Pt)



Hexagonal close packing
(*vertical axis expanded here*)

Intermediate bond types:



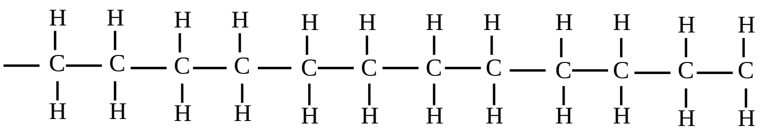
Amorphous solids (a.k.a. glass, vitreous phase)

Metallic glass. Cool the metal very quickly, e.g. small drops in liquid N_2.

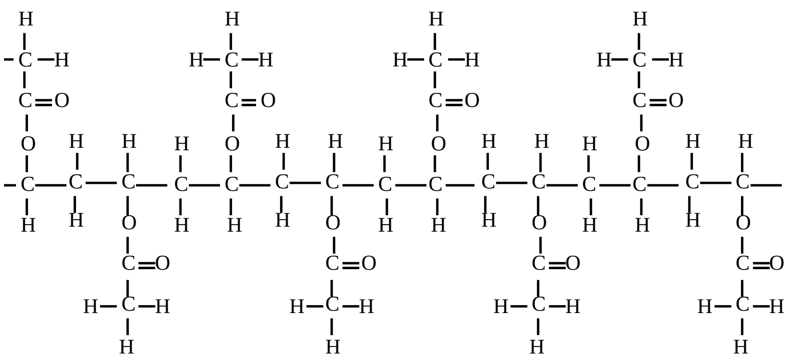
Polymers

Long chains repeating one unit e.g. poly(ethylene) PE

(what sort of hydrocarbon is this?)



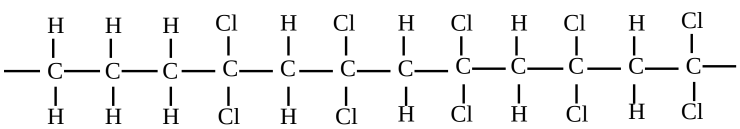
poly(vinyl acetate) PVA



Long *flexible* chains: usually tangle rather than crystallise, especially if they have side groups. Attractive force is vdW (and tangling)

Amorphous (partly) **Crystalline**

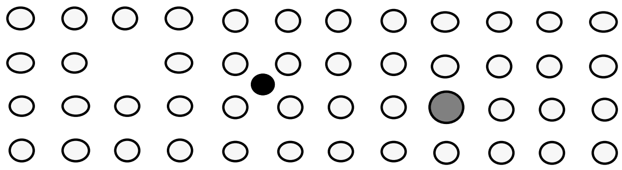
Crystalline polymers are only partly crystallised. Need uniform cross section, e.g. poly(vinylidene chloride)



Cross linking. Chemical bonds rigidify 3D structure. e.g. resins, vulcanisation in rubber (S bonds).

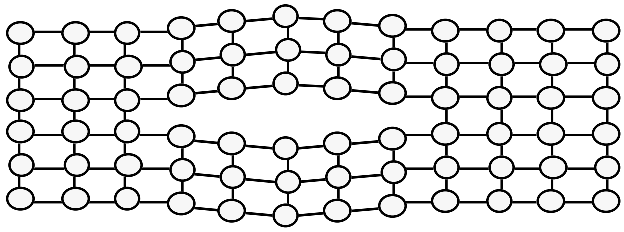
Point defect

vacancy, interstice, substitution

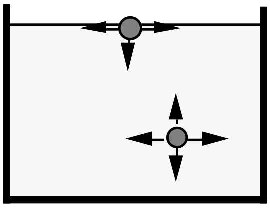


Line defects Edge and screw

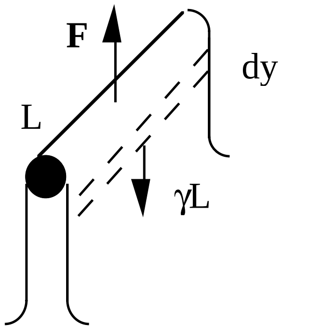
Plane defects, espec. microcracks



Surface tension and surface energy



molecule in bulk is uniformly attracted in each direction. Molecule at surface has ~ no attraction to atmosphere, ∴ work done against the nett force in order to make a surface.



Work to make new surface is done against **surface tension** γ.

$\gamma \equiv$ force per unit length in the plane of the surface

e.g. raise wire dy

$F = 2\gamma L$ (*two sides*)

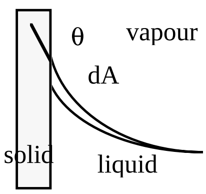
$$\text{Work} \qquad dW = F.dy = 2\gamma L.dy$$

$$dA = 2Ldy \qquad (\textit{two sides})$$

$$\therefore \qquad \gamma = \frac{dW}{dA}$$

So surface tension = surface free energy per unit area

Contact angles and menisci



The surface energy of solid-vapour is S_S,
Solid-liquid is S_{LS}

Surface tension of liquid is γ

Contact angle θ

Suppose that the meniscus rises as shown. Work done by tension

$$dW = -\gamma Ldx \cos\theta = -\gamma dA \cos\theta$$

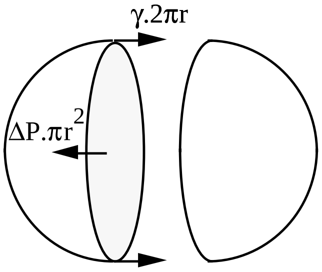
$$= \text{new solid-liquid energy} - \text{solid-vapour energy}$$

$$= S_{LS}dA - S_S dA$$

$$\gamma \cos\theta = S_S - S_{LS}$$

Some surface free energies

Class	Material	$\frac{S}{J.m^{-2}} \left(= \frac{\gamma}{N.m^{-1}} \right)$
Liquid	water	0.073
	Hg	0.051
Glass	SiO_2	4.4
Ionic	NaCl	0.5
solid	KCl	0.11
Mica	in air	0.38
	in vacuum	5
Covalent	Al_2O_3 (sapphire)	6-32
solid	C (diamond)	5.24
Metal	Zn	0.105
	Pb	0.763
	Si	1.24
Polycrystalline	SiC	32
	Graphite	68
	Granite	200
	Cast Iron	1,520



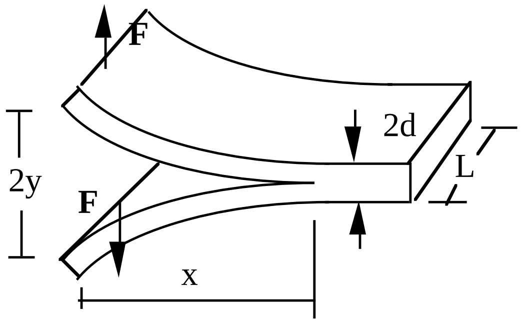
Pressure excess inside balanced by surface tension

$\Delta P.\pi r^2 = \gamma.2\pi r$

$\Delta P = \gamma \frac{2}{r}$

Young-Laplace Equation

Surface energy of solids



Complicated by mechanical strength of materials. Work to cleave $= U_{surf} + U_{bend}$.
Put in expression for deflection of cantilever spring:

$\frac{F}{L} = \frac{3Yd^3y^2}{8x^4}$

Diverse comments about various types of materials and behaviour

Be careful talking of 'strength of materials'

Sometimes you want high E ($\therefore$ small e)

Other times you want high σ_{max}

Yet other times you want low E ($\therefore$ large e)

Macroscopic σ_{max} usually << microscopic σ_{max}

Composite materials aim to minimise the difference by limiting propagation of dislocations

In composites, $\sigma_{micro} \neq \sigma_{macro}$

Ductility refers to the ease of plastic deformation without rupture.

Sometimes 'good', sometimes 'bad'

Work hardening: 'dislocation tangle'. Dislocation stops at slip plane,

fewer moveable dislocations $\therefore$ less ductility

Ductile rupture: break bonds, eventually form macroscopic rupture

For some materials, mechanical properties may depend on time scale (rheology)

For some materials, mechanical properties depend on temperature. Plastic deformation is easier at high T, brittle fracture is more likely at low T

Material properties may also be changed by

Chemical reactions

Radiation (UV or even light from some polymers, γ and X for other materials)

Hydration (for hydrophilic, fibrous materials)