PHYS1169 Light notes. Joe Wolfe, UNSW

Physical optics (→ interference and diffraction)

Electromagnetic radiation

Speed $c = 3.00\ 10^{8}\ ms^{-1}$

$2.997923458\ 10^{-8}\ ms^{-1}$ (This is definition of metre)

λ: for visible light, $400\ nm < \lambda < 800\ nm$

Typical $f = \frac{c}{\lambda} \sim \frac{3\ 10^{8}\ ms^{-1}}{5\ 10^{-7}\ m} = 6\ 10^{14}$ Hz = 600 THz

See http://www.phys.unsw.edu.au/~jw/EMspectrum.html

Geometrical optics only works if size >>λ. Why?

Huygen's principle: each point on a wave front ~ a new source

Is light

a) a particle (Newton) → geometric optics

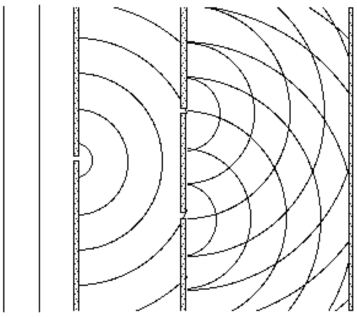
b) a wave (Young) → physical optics

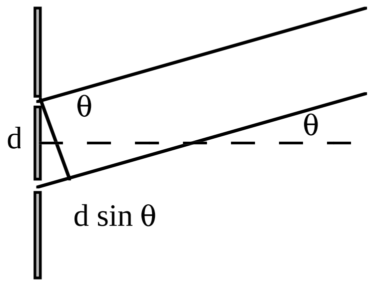
c) both

d) none of the above → quantum physics

Hence: **Young's experiment**

Light through one slit (gives ***coherent source***) then **two slits** gives interference pattern on screen.





This is the basis of interferometry - powerful tools

Constructive interference if

$d \sin\theta = m\lambda$

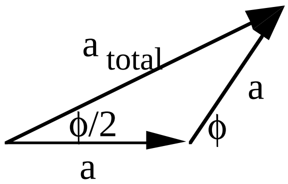
Destructive interference if $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

At an angle θ, the phase difference ϕ is

$$\frac{\phi}{2\pi} = \frac{\Delta\ \text{pathlength}}{\lambda} = \frac{d \sin\theta}{\lambda}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda} d \sin\theta$$

how to add two sin waves?
phasor diagrams



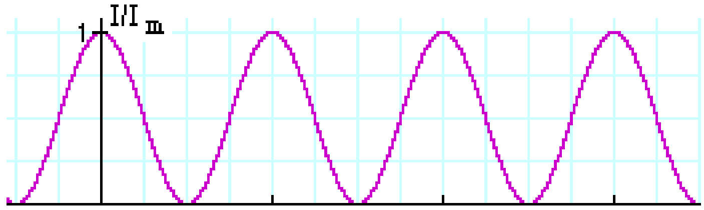
$a_{tot} = 2a \cos \beta$

$\beta = \phi/2 = \frac{\pi}{\lambda} d \sin \theta$

Intensity $\propto$ amplitude^2 $\therefore$ $I \propto 4a^2 \cos^2 \beta$

I_{max} if $\beta = \phi/2 = 0$

$$I = I_{max} \cos^2 \beta \quad \text{where } \beta = \frac{\pi}{\lambda} d \sin \theta$$



0 $\pi/2$ π $3\pi/2$ 2π $\phi = \frac{2\pi}{\lambda} d \sin \theta$

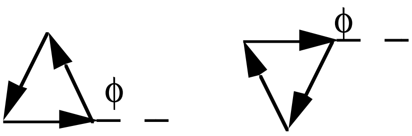
Numerical example:

Example 3 narrow slits radiate uniformly and in phase. Interference pattern on screen. (a) Show phase diagrams for (i) central maximum (ii) 1st order maximum (iii) minima between i and ii.

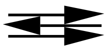
(b) Sketch $I(\phi)$

i)  $\phi = 0$, central max

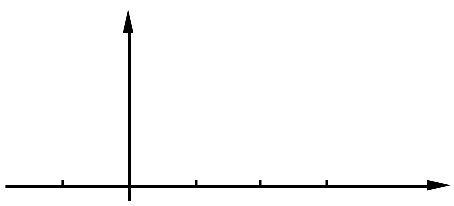
ii)  $\phi = 2\pi$, 1st order max

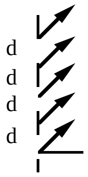
iii) 

$\phi = 2\pi/3$ $\phi = 4\pi/3$ both $\rightarrow$ 0 resultant

b) if $\phi = \pi$,  amplitude = 1/3 central max

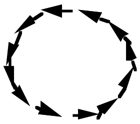
$\therefore$ intensity = $I_{max}/9$





Diffraction grating has very many slits. Used to measure λ very accurately.
If there are N slits per unit lenth, $d = 1/N$.

The first minimum is *very* close (small ϕ to close polygon), ie very narrow maxima

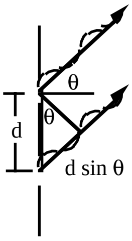


For constructive interference

$$d \sin\theta = m\lambda$$

$m = 1 \rightarrow$ 1st order spectrum

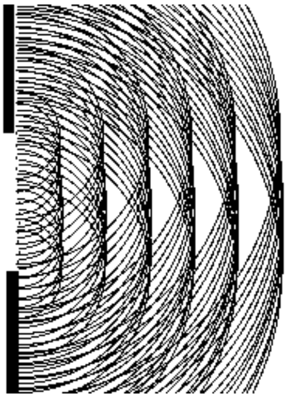
$m = 2 \rightarrow$ 2nd order spectrum



$$\theta_{red} = \sin^{-1}\frac{m\lambda_{red}}{d}$$

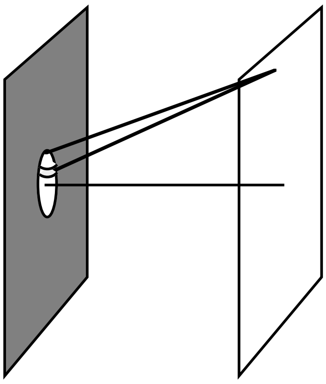
$$\theta_{blue} = \sin^{-1}\frac{m\lambda_{blue}}{d}$$

Diffraction from a slit (Later from a circle)

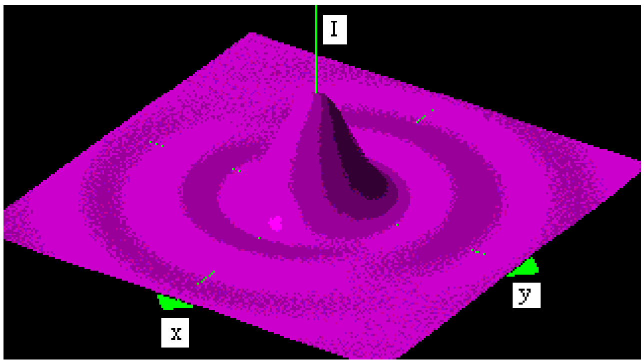


Use Huygen's construction: beam of finite width, interference if $d \sim \lambda$

Circular aperture:

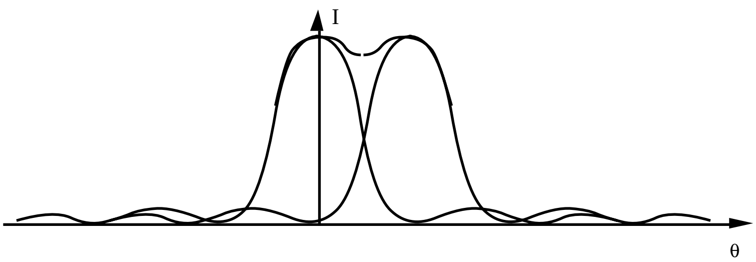


messy integral $\rightarrow$



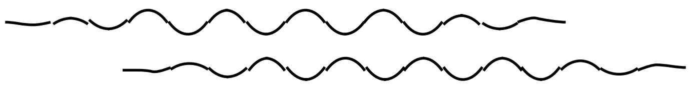
First minimum at $\sin\theta = 1.22\frac{\lambda}{d}$ d is diameter

can resolve ~ θ with lens diam d. Rayleigh's criterion



Coherence length

Wave trains have finite length - coherence length



Only interfere if Δ pathlength < l

Examples

Radio transmitter $E = E_m \sin(kx - \omega t)$ *E_m, ω, k vary slowly*

Hot body (e.g. lamp) l ~ 1 m, but different regions have different, random phase. ∴, to get interference, use a pin hole and keep Δ path << 1 m

LASER (**L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) l >> km

Interference in thin films

Light in a medium travels at c/n.

$$\lambda_{medium} = \lambda/n$$

$$\Delta\phi = 2\pi \frac{\Delta\ \text{path}}{\lambda_{med}} = 2\pi \frac{n\ \Delta\ \text{path}}{\lambda}$$

Define **Optical path length** ≡ n . pathlength

$$\Delta\phi = 2\pi \frac{\Delta\ \text{optical pathlength}}{\lambda}$$

Reflections *remember reflections in strings*

From less dense to more dense

$$\Delta\phi = \pi$$

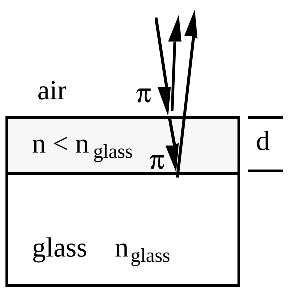
From more dense to less dense

$$\Delta\phi = 0$$

transmitted wave has no phase change

Example: Non reflective coating

useful in camera lens etc



Coating has $1 < n < n_{glass}$

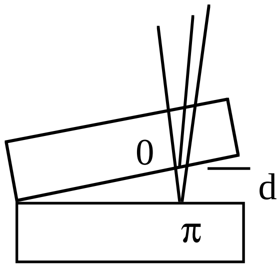
How thick should it be to give minimum reflection?

Both reflections have π phase change. As d → 0, → constructive interference.

For destructive, we want Δ optical pathlength = λ/2

2nd = λ/2 $\quad d \sim \frac{\lambda}{4n} = \frac{550\text{ nm}}{4\ \ 1.2} \sim 100\text{ nm}$

Air wedge



Destructive interference if

$2d = m\lambda$

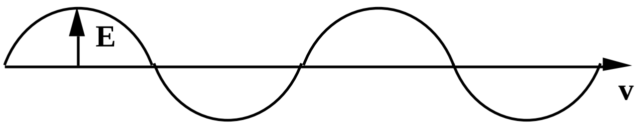
Constructive interference if

$2d = \left(m + \frac{1}{2}\right)\lambda$

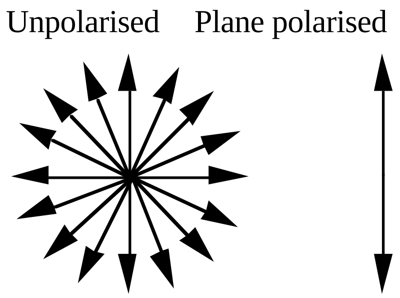
At L.H. end, destructive (dark), then count fringes to get thickness. *(see corridor display)*

Also: Newton's rings, oil slicks

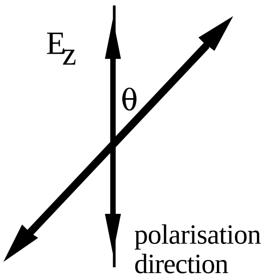
Polarisation.



EM waves are **transverse** waves, ∴ can be polarised. Usually light has waves with **E** in all directions



Polaroid materials allow **E** in only one dirn



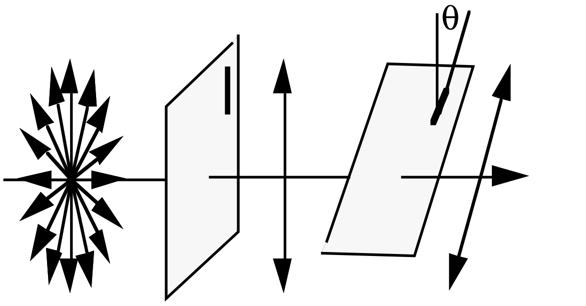
$E_{transmitted} = E\cos\theta$

Malus' Law:

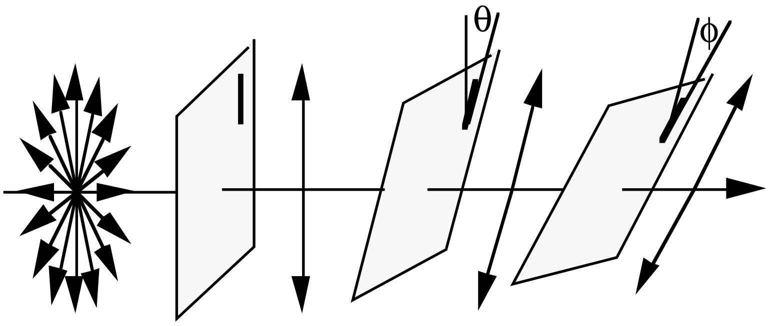
$I_{trans} = I_{in}\cos^2\theta$

Average of $\cos^2\theta$ over all angles is 1/2 ∴

Unpolarised plane polarised plane polarised



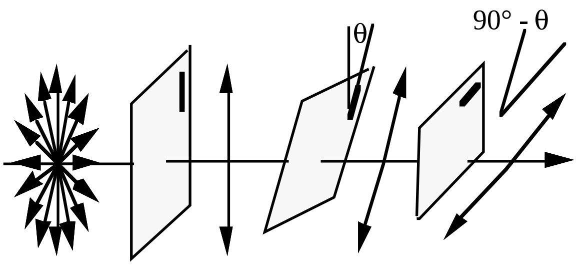
I_o $\qquad \frac{I_o}{2}$ $\qquad \frac{I_o}{2}\cos^2\theta$



I_o $\qquad \frac{I_o}{2}$ $\qquad \frac{I_o}{2}\cos^2\theta$ $\qquad \frac{I_o}{2}\cos^2\theta\cos^2\phi$

Note: even if $\theta + \phi = 90°$, $I \neq 0$

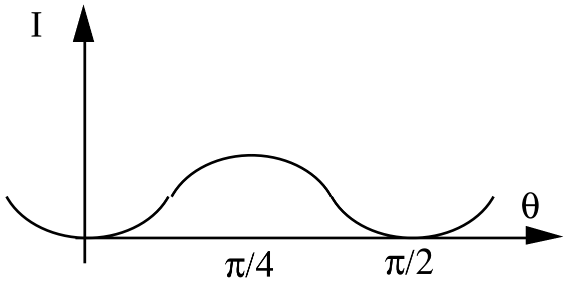
Example. Two crossed polarisers (polar^n dir^n are at right angles) have a third polariser between them. What is the transmitted intensity? What angle for the middel polariser gives maximum transmission?



I_o $\qquad \frac{I_o}{2}$ $\qquad \frac{I_o}{2}\cos^2\theta$ $\qquad \frac{I_o}{2}\cos^2\theta\cos^2(90-\theta)$

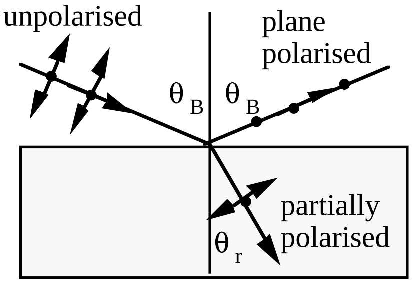
$$I = \frac{I_o}{2}\cos^2\theta\sin^2\theta$$

$$= \frac{I_o}{8}\sin^2 2\theta$$



Polarisation by reflection

For wave in medium, **E** of light causes oscillation // **E**. This oscillation can produce (only) transverse waves, hence polarisation of reflected wave.



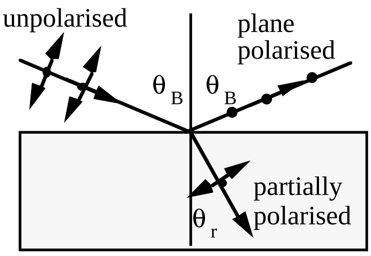
When refracted ray ⊥ reflected → plane polarised reflected wave (Brewster's angle θ_B).

→ polaroid sunglasses (*see your optics kit*)

Scattering of light also polarises

More effective for short λ, → blue sky

Example. What is Brewster's angle for a medium with n = 1.40?



Refraction: $$n = \frac{\sin \theta_B}{\sin \theta_r}$$

If refracted and reflected are at 90°,

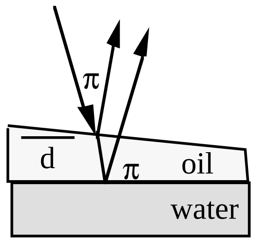
$$\theta_B + \theta_r = 90°$$

so $$\sin \theta_r = \cos \theta_B$$

$$n = \frac{\sin \theta_B}{\cos \theta_B} = \tan \theta_B$$

$$\theta_B = \tan^{-1} 1.4 = 54°$$

Example. An oil slick (n = 1.20) floats on water. What are the thicknesses for which red light ($\lambda \cong 700$ nm) is reflected weakly? What does the slick look like at its thinnest point?



$n_{water} > n_{oil}$

Constructive interference if

Δ OPL = m λ

Destructive interference if

$\Delta \text{ OPL} = \left(m + \frac{1}{2}\right)\lambda$

i) If red has destructive interference,

$$\Delta \text{ OPL} = 2nd = \left(m + \frac{1}{2}\right)\lambda_{red}$$

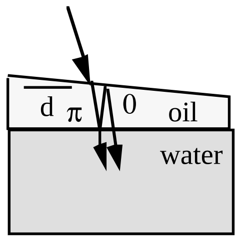
$$d = \frac{\lambda_{red}}{2n}\left(m + \frac{1}{2}\right)$$

m = 0, m = 1, m = 2 .....

= 150 nm, 440 nm, 730 nm etc

ii) If d << λ, π phase difference on both paths so constructive interference for all λ, so it looks bright and 'white'.

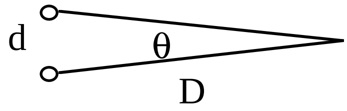
Example. Same problem, but for scuba diver!



Destructive interference if Δ OPL = m λ

Contructive interference if $\Delta \text{ OPL} = \left(m + \frac{1}{2}\right)\lambda$

Example. A binary star has an orbital radius of 100 light seconds, and is 10 light years from us. How big must an optical telescope be to resolve them? (take λ = 550 nm). A radio telescope (take λ = 21 cm).



100 s < 10 yr $\rightarrow$ θ is small

Rayleigh criterion: $\sin \theta_R = 1.22 \frac{\lambda}{a}$

$$\sin \theta_R \cong \theta_R \cong \frac{100 \text{ light seconds}}{10 \text{ light years}}$$

$$1.22 \frac{\lambda}{a} = \frac{c\ 100 \text{ seconds}}{c\ 10 \text{ years}}$$

λ = 550 nm $\rightarrow$ a = 2 m $\qquad$ λ = 21 cm $\rightarrow$ a = 800 km