Notes on Heat for PHYS1169. Joe Wolfe, UNSW

Example. At atmospheric pressure, water boils at 100 °C or 212 °F, and freezes at 0 °C or 32 °F. At what temperature do the two scales have the same value? When is the Farenheit temperature twice the Centigrade temperature?

Centigrade scale (symbol θ) and the Farenheit scale (symbol ϕ) are linearly related:

$\phi = a\theta + b$ *How do you know this?*

Write down givens using this:

$$212\ °F = a\ 100\ °C + b$$
$$32\ °F = 0 + b$$

Solve: $b = 32\ °F, \quad a = 1.8\ \frac{°F}{°C}$

If $\phi = \theta$: $\theta = a\theta + b$,

so $\phi = \theta = -\frac{b}{a - 1} = -40\ °C = -40\ °F$

If $\phi = 2\theta$: ....

$2\theta = a\theta + b$

so $\theta = \frac{b}{2 - a} = 160\ °C = 320\ °F.$

Thermal Physics

Thermodynamics: laws relating macroscopic variables (P, V, T etc.).

Statistical Mechanics: molecular explanation.

Difference between heat and temperature

Intensive or extensive properties?

Which relates to sense of hotness?

Define temperature:

Thermal equilibrium:

Thermal properties do not change with time

Definition of Temperature (T):

T is equal in any 2 bodies at thermal equilibrium.

Zeroth Law of Thermodynamics:

if $T_A = T_B$ and $T_B = T_C$, then $T_A = T_C$.

What is temperature? How to measure it?

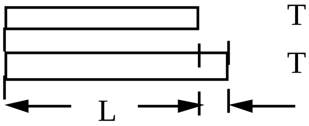
Thermometers: Hg in glass, thermocouple, thermistor, liquid crystal layer, constant volume gas thermometer

Scales. Obvious definition of temperature θ: choose a property X and make X proportional to or linear with θ. This can only be done once for any temp scale θ.

Reference temperature

Melting or freezing? Depends on the pressure.

Thermal Expansion



Usually, $\frac{\Delta L}{L} \propto \Delta T$ for small ΔT

$\therefore$ **Define** $\frac{\Delta L}{L} = \alpha \, \Delta T$

α is **coefficient of linear expansion**

e.g. steel $\alpha_{st} = 1.1 \times 10^{-5}\ K^{-1}$

Al $\alpha_{Al} = 2.3 \times 10^{-5}\ K^{-1}$

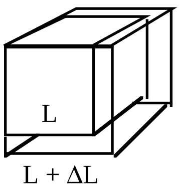
Example Bridge span is 1 km long.
Mid-winter, T = -5˚C summer, T = 45˚C
What is Δl?

$$\Delta L = \alpha \Delta T . L = ... = 55 \text{ cm}$$

Volume Increase

define $\frac{\Delta V}{V} = \beta \Delta T$

β = **coefficient of volume expansion**



$$\Delta V = (L + \Delta L)^3 - L^3$$

$$= L^3 \left(1 + \frac{\Delta L}{L}\right)^3 - L^3$$

$$= L^3 \left(1 + \frac{3\Delta L}{L} + \ldots\ldots\ldots -1\right)$$

$$\approx V\, 3\alpha \Delta T$$

$$\therefore \ \beta \approx 3\,\alpha$$

Note: Water is unusual: 0˚ - 4˚C, $\beta < 0$

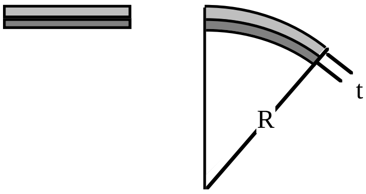
Example What is change in ρ for steel between 0˚ C and 100˚ C?

$$\rho = \frac{M}{V} \qquad \therefore \ d\rho = -\frac{M}{V^2}\, dV$$

$$\therefore \ \Delta\rho \approx -\frac{M}{V^2}\Delta V = -\rho \frac{\Delta V}{V}$$

$$\therefore \ \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} = -\beta \Delta T = ... = -0.33\%$$

Example: Bimetallic Strip,10 cm long, made of 1 mm Al and 1 mm steel. Straight at 0 ˚C, what angle at 50 ˚C?

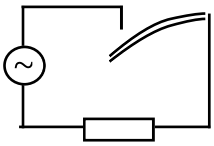


$$L + \Delta L_{st} = 2\pi R \,.\, \frac{\theta}{360^\circ}$$

$$L + \Delta L_{A1} = 2\pi(R + t)\,\frac{\theta}{360^\circ}$$ subtract $\rightarrow$

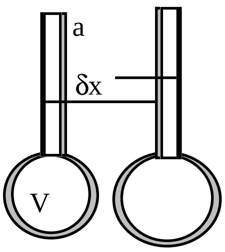
$$2\pi t\,\frac{\theta}{360^\circ} = \Delta L_{A1} - \Delta L_{st} = L\Delta T(\alpha_{A1} - \alpha_{st})$$

$$\theta = \qquad\qquad = 3.4\ ^\circ\mathrm{C}$$

e.g. oven switch: 

Example: thermometer, $\mathrm{Vol}_{cylinder} << \mathrm{Vol}_{sphere}$

What is its calibration slope $\frac{\partial x}{\partial T}$?



$$\frac{\Delta V}{V} \equiv \beta\Delta T$$

$$a\,\delta x \approx \Delta V_{fl} - \Delta V_{sphere}$$

$$= \beta_{f1} V\Delta T - \beta_{g1} V\Delta T$$

$$\frac{\partial x}{\partial T} = \frac{V}{a}\left(\beta_{f1} - \beta_{g1}\right)$$

Ideal gas temperature scale

Uses reference temp:
Triple point - co-existance of ice, water, steam

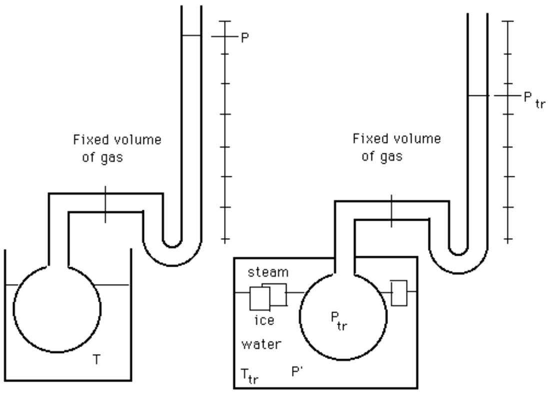
call it θ_{tr}.

defines $\mathbf{P} \propto \theta$ for constant volume of gas
But gases are not (quite) ideal

e.g. consider boiling temp θ_s at some P:

$\dfrac{\theta_s}{\theta_{tr}}$ is different for different gases and at different densities.
At very low density or pressure,

all gases $\to$ ideal, $\therefore \dfrac{\theta_s}{\theta_{tr}} \to$ same limit



$$T = T_{tr}.\lim_{\rho_{tr}\to 0}\left(\frac{P}{P_{tr}}\right)_V$$

where $T_{tr} = 273.16$ K

Why 273.16? This defines the Kelvin so that

$\Delta T = 1$ K $\Leftrightarrow$ $\Delta T = 1$ °C

(Working defn. is more complicated)

Celsius Scale: $T_C = T - 273.15^\circ$

$\left.\begin{array}{l} T_C = 0\ °C \text{ water freezes} \\ T_C = 100\ °C \text{ water boils}\end{array}\right\}$ at P_A

Farenheit Scale

Heat

Definition: that which is transferred between a system and its surroundings as result of Δ T only.
Joule showed:

mechanical energy $\to$ heat *(by friction etc.)*.

(**Carnot** showed
heat at high T $\to$ heat at low T + work)

$\therefore$ measure heat as energy; i.e. S.I.unit. Joule (J)

Heat Capacity: (for a body) $C = \dfrac{\Delta Q}{\Delta T}$ *extensive quantity*

Specific Heat: (of a substance) $c = \dfrac{\Delta Q}{M\Delta T}$ *intensive quantity*

e.g. $c_{H_20} = 4.2\ \text{kJ.kg}^{-1}\text{K}^{-1}$, $c_{A1} = 900\ \text{J.kg}^{-1}\text{K}^{-1}$

Latent Heat: heat required for change of phase (at constant T).

Example. A 240 V kettle has a working resistance of 50 Ω. Put in 500 ml of water at 20 °C and turn on. How long before it boils dry? (Specific heat of c_w = water = 4.2 J.kg.K^{-1}, Latent heat of vaporisation L_{vap} = 2.3 MJ.kg^{-1}.)

Energy in = power.t = $\dfrac{V^2}{R}\,t$ = (1.15 kW).t

= Q to raise T of water + Q to evaporate water

$= m_w c_w (T_f - T_i) + m_w l_{vap}$

$= (0.5\ \text{kg})\,(4.2\ 10^3\ \text{J.kg}^{-1}\text{K}^{-1})\,(100 - 20)°\text{C}$

$\quad + (0.5\ \text{kg})\,(2.3\ 10^6\ \text{J.kg}^{-1})$

= 168 kJ + 1.15 MJ = 1.32 MJ

$\therefore \quad t = \dfrac{1.32\ \text{MJ}}{1.15\ \text{kW}}$ = ... = 1150 s = 19 minutes

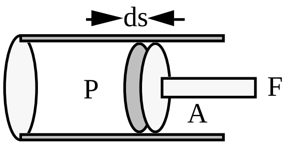
(it boils after $\dfrac{168\ \text{kJ}}{1.15\ \text{kW}}$ = 2.5 mins)

Work: energy transmitted from one system to another without ΔT or transfer of Q.

e.g. work done by force F

$dW = \mathbf{F.ds}$

e.g. work done against pressure P



$dW = \mathbf{F.ds} = PA\,ds = PdV$

Internal Energy

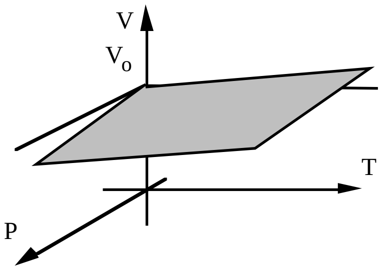
Heat dQ added to a system increases its internal energy U.

Work dW done **by** the system lowers its internal energy.

1st Law $dU = dQ - dW$

where U is a state function

Digression Equation of state of condensed phase

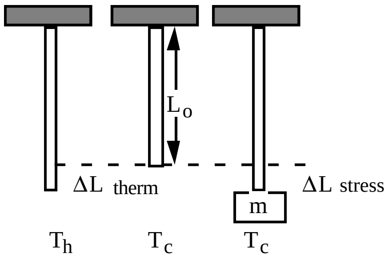


$$V = V_o\left(1 - \frac{P}{B} + \beta(T - T_o)\right)$$

Volumetric coefficient of thermal expansion $\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

Bulk modulus $\frac{1}{B} \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

Tut prob 3:



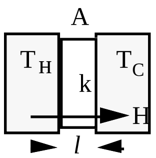
$$\text{strain} \equiv \frac{\Delta L}{L_o} \equiv \frac{\text{stress}}{\text{modulus}} \equiv \frac{\text{force/area}}{Y}$$

Phase diagram in one dimension:

$$L = L_o\left(1 + \frac{\Delta L_{therm}}{L_o} + \frac{\Delta L_{stress}}{L_o}\right)$$

$$= L_o\left(1 + \alpha(T - T_o) + \frac{m\,g}{AY}\right)$$

Heat conduction



Reservoirs at T_H and at T_C.
H is **rate of heat transfer** through a material in steady state.

$$H \equiv kA\frac{T_H - T_C}{l}$$

defines the **thermal conductivity k**

H in W, so k in $W.m^{-1}.K^{-1}$.

Copper	401	$W.m^{-1}.K^{-1}$
Stainless Steel	14	
Glass	1	
Water	0.5	
Pine (wood)	0.11	
Dry air	0.026	

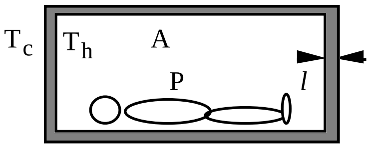
Thermal resistance or R-value sometimes used for building materials

$$R \equiv \frac{l}{k} \qquad \text{so} \quad H = A\frac{\Delta T}{R}$$

(High conductivity, low R value and *vice versa*.)

Example. Wht is the R value of 1 cm pine?

Whe his Auststudy is cut off, a student lives in a pine packing crate, area 8 m^2, thickness 1.0 cm. If the shivering student produces 300 W, which is lost by conduction through the crate, how much warmer is it inside the crate?



$$R \equiv \frac{l}{k} = 0.09 \text{ K.m}^2\text{W}^{-1}$$

$$H \equiv kA\frac{T_H - T_C}{l} \qquad \textit{or} = A\frac{\Delta T}{R}$$

$$T_H - T_C = \frac{l\,H}{kA}$$

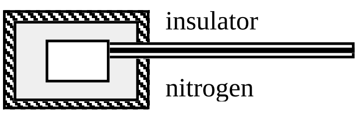
$$= \frac{0.010 \text{ m} * 300 \text{ W}}{0.11 \text{ W.m}^{-1}.\text{K}^{-1} * 8 \text{ m}^2}$$

$= 3$ K. *Other benefits: wind, rain, radiation*

What if he has a friend?

2 students at 300 W $\rightarrow$ 600 W $\rightarrow$ 7 K.

Example. To reduce thermal noise, a low temperature circuit is immersed in liquid nitrogen (77 K, L = 199 kJ.kg^{-1}). It is connected to the outside circuitry by 3 well-insulated copper wires, length l = 100 mm, diameter 0.3 mm. What is the rate of N_2 evaporation due to the heat conducted down the wires?



Power to evaporate N_2 = heat transfer

$$L.\frac{dm}{dt} = \quad H \equiv kA\frac{T_H - T_C}{l}$$

$$\frac{dm}{dt} = \frac{kA}{L}\frac{T_H - T_C}{l}$$

= .....

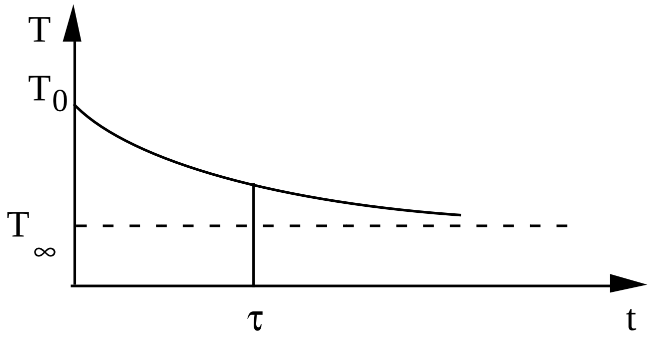
$= 90\ \mu\text{gs}^{-1} = 340 \text{ mg.hr}^{-1}$

Example. A flask of coffee, initially at 90 °C, cools to 81 °C in one hour in 20 °C atmosphere. How long will it take to cool to 60 °C?

$$H = kA\frac{T - T_C}{L}$$

$$H = -\frac{dQ}{dt} \equiv -\frac{mcdT}{dt}$$

$$\frac{dT}{dt} = -\frac{kA}{mc}\frac{T - T_C}{L}$$



Characteristic time: $\tau = \frac{mcL}{kA}$ Solve DE $\rightarrow$

$$T = T_C + (T_o - T_C)\, e^{-t/\tau}$$

$$81\ °C = 20\ °C + (70\ °C)e^{-(1\ h)/\tau}$$

$\therefore\ \tau = 7.3$ h ...$\rightarrow$ 4.1 h to cool to 60 °C

Kinetic Theory of Gases

The Ideal Gas - postulates
equation of state
r.m.s. velocity
Temperature
Internal energy
specific heats

Ideal gas equation of state

($\cong$ limit for all gases at low ρ):

$PV = nRT = NkT$

no. of moles — no. of molecules

gas constant — Boltzmann's Constant

$R = 8.31\ JK^{-1}$ $\quad k = \frac{R}{N_A} = 1.38\ 10^{-23}\ JK^{-1}$

NB not the same k and R as in heat conduction!

Kinetic theory: Ideal Gas Postulates

i) gas made of (identical) molecules

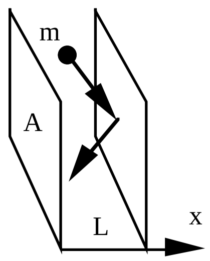
ii) these obey Newton's laws, with random motion

iii) no. of molecules is large *(~ Avagadro's number)*

iv) total volume molecules is negligible fraction *(~ 10^{-3})*

v) no interaction except during collision *(average $U_{interaction}$ < 10^{-4} K.E.)*

vi) collisions elastic, negligible duration. *(~ 10^{-3} of time)*



parallel plates, area A. Volume $V = AL$.

N molecules (mass m) of an ideal gas.

Each collision →

Δ momentum = $2mv_x$

time between collisions $t = 2L/v_x$.

$$\overline{F} = \frac{\Delta \text{ momentum}}{\Delta \text{ time}} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

F on all N molecules is

$$F_{\text{all molecules}} = \frac{Nm\overline{v_x^2}}{L} = PA$$

$v^2 = v_x^2 + v_y^2 + v_z^2$;

random motion $\Rightarrow \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} \quad \Rightarrow \quad \overline{v_x^2} = \frac{1}{3}\overline{v^2}$, so:

$$PAL = PV = Nm\overline{v_x^2} = \frac{N}{3}m\overline{v^2} \qquad\qquad P = \frac{N\,m}{3V}\overline{v^2} = \frac{1}{3}\rho\,\overline{v^2}$$

Molecular speeds:

$\mathbf{v_{rms}}$ root mean square velocity

$$v_{r.m.s.} \equiv \sqrt{\overline{v^2}}$$

c) What is v_{rms} in atmosphere? (approximate it as an ideal gas at P_A, with $\rho_A = 1.3\ \text{kg.m}^{-3}$)

$$\rightarrow v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.3}} = 480\ \text{ms}^{-1}$$

Meaning of temperature:

We had $\qquad PV = \frac{N}{3}m\overline{v^2}$

both sides are familiar

$$\frac{1}{2}m\overline{v^2} = \overline{\varepsilon} \equiv \text{average K.E. per molecule}$$

But T defined by (1 and 5): $\quad PV = NkT$

$$\therefore \quad \overline{\varepsilon} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{PV}{N} = \frac{3}{2}kT \quad (7)$$

For ideal gas **all energy E is kinetic** so:

$$E = N\overline{\varepsilon} = \frac{3}{2}NkT \qquad (8)$$

$T \propto$ average K.E. of molecules in an ideal gas.

3 degrees of motional freedom (x, y, z)

i.e. $\quad \frac{1}{2}kT$ per **degree of freedom**

(At ordinary temperatures, $\quad kT \cong 4\ 10^{-21}$ J)

molecular speeds again:

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

$$v_{r.m.s.} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

Example: What is v_{rms} of O_2, N_2. and H_2 at T = 293K?

(7) $$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT\ N_A}{mol\ wt}}$$

for O_2 : $$= \sqrt{\frac{3 \times 1.38\ 10^{-23} \times 293 \times 6.02\ 10^{23}}{0.032}}$$

$$= 478\ ms^{-1}$$

for $N_2 \rightarrow$ 511 ms^{-1} for H_2 1.91 kms^{-1}

c.f. v_{escape} = 11 kms^{-1} So what?

note that for air $v_{rms} > v_{sound}$
but recall from waves:

$$v_s = \sqrt{\frac{K_{ad}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}$$

so $$\frac{v_{rms}}{v_s} = \sqrt{\frac{3}{\gamma}} \sim 1.5$$

Example What is the v_{rms} due to thermal motion (Brownian motion) of: pollen grain (m ~ 10^{-15} kg) and apple (m ~ 0.2 kg)

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$ pollen $\Rightarrow$ 2 mm s^{-1}

apple $\Rightarrow$ 2.5×10^{-10} ms^{-1}

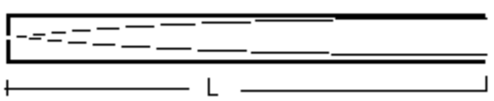
(Brownian motion 1st analysed by Einstein, 1904)

Example: What is the ratio of the speed of sound in He to that in air at the same temperature? How will this affect the pitch of a human voice when the lungs and vocal tract are (temporarily) filled with He?

$$v_s = \sqrt{\frac{\gamma kT}{m}} \quad \therefore \quad \frac{v_{He}}{v_{air}} = \sqrt{\frac{\gamma_{He} m_{air}}{\gamma_{air} m_{He}}}$$

$$\sqrt{\frac{\gamma_{He}}{\gamma_{air}}} = 1.1 \qquad \sqrt{\frac{m_{air}}{m_{He}}} = \sqrt{\frac{30}{4}} \cong 2.7$$

Think carefully: does v_{sound} affect pitch?



$$f_{air} = \frac{c}{\lambda} = \frac{c}{4L} \cong \frac{340\ ms^{-1}}{4 \times 0.17\ m} = 500\ Hz$$

$f_{He} \cong 1350$ Hz

Example. Spherical balloon. Skin (total) has mass $\sigma = 10 \text{g.m}^{-2}$. How big does it need to be to lift 200 kg load if (i) it contains hot air at 100 C? (ii) Helium at STP?

Archimedes: $W_{displaced} = W_{balloon}$

$$\frac{4}{3}\pi r^3 \rho_{air} g = \frac{4}{3}\pi r^3 \rho_{gas} g + 4\pi r^2 \sigma g + mg$$

$$r^3(\rho_{air} - \rho_{gas}) - 3\sigma r^2 = \frac{3}{4\pi} m \qquad \textit{(or solve cubic)}$$

$$r \cong \frac{3m}{4\pi\rho_{air}(1 - \rho_{gas}/\rho_{air})}$$

He: $\rho_{gas}/\rho_{air} = 4/30 \qquad \rightarrow r \cong 3.6 \text{ m}$

Hot air: $\rho = \dfrac{Nm}{V} = \dfrac{Pm}{kT}$

$$\therefore \quad \frac{\rho_{hot}}{\rho_{cold}} = \frac{T_{cold}}{T_{hot}} = \frac{273 \text{ K}}{373 \text{ K}} \qquad \rightarrow r \cong 5.3 \text{ m}$$

check approx^n