PHYS1169: Tutorial 9 Solutions

Superposition and Standing Waves

1. a) The wavelength λ of the waves can be found by $\lambda = \frac{v}{f} = \frac{343}{1715} = 0.2m$. The wave from source S2 must travel an extra 0.2m to reach the listener, which corresponds to exactly one full wavelength of the waves. The wave from S2 will thus be exactly one full cycle behind the wave from source S1 at the listener, and a phase lag of one full cycle corresponds to a phase lag of 2π radians.
   b) If the two waves are out of phase by one complete cycle, 2π rad, then they are in phase and will constructively interfere. The amplitude of the resulting wave will thus be the sum of the amplitudes of each individual wave, so the resultant wave will have an amplitude of 2A.

2. When two waves travelling in opposite directions interfere, they will produce a standing wave which can be described as follows:
   $$y(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t) = 2A\sin kx\cos\omega t$$
   using the standard trig result $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$.
   For this standing wave, we can see that the wave number $k = 0.25m^{-1}$, so the wavelength is $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.25} = 25.1m$, and that the angular frequency $\omega = 120\pi s^{-1}$, so the frequency is $f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60Hz$.

3. Firstly, we can find the speed of propagation of the waves along the string by
   $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20}{9\times10^{-3}}} = 47.1ms^{-1}.$$
   Then we can form standing waves along the string, assuming both ends of the string to be fixed, by ensuring an integral number of half wavelengths fit along the string. The length of the string must thus satisfy $L = n\frac{\lambda_n}{2}$, and the first four wavelengths to cause standing waves are thus $\lambda_1 = 2L = 60m$, $\lambda_2 = L = 30m$, $\lambda_3 = 2L/3 = 20m$, and $\lambda_4 = L/2 = 15m$. The frequencies corresponding to these wavelengths ($f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$), are the harmonics.
   Thus, the fundamental frequency to cause a standing wave is
   $f_1 = \frac{v}{\lambda_1} = \frac{47.1}{60} = 0.786Hz$, and the first three harmonics are
   $f_2 = \frac{47.1}{30} = 1.57Hz$, $f_3 = \frac{47.1}{20} = 2.36Hz$, and $f_4 = \frac{47.1}{15} = 3.14Hz$.

4. The intensity I of a wave is the power P transmitted through a given area A by the wave, $I = \frac{P}{A}$. Assuming the sound is emitted uniformly in all directions from the speaker, then at a distance 4m from the source was are to have an intensity of $1.2 Wm^{-2}$ over a spherical surface area of $A = 4\pi r^2 = 4\pi \times 4^2 = 64\pi\, m^2$. The power required is
$$P = IA = 1.2 \times 64\pi = 241\, W.$$

5. The power of $P = 50W$ distributed over a spherical surface area $A = 4\pi r^2 = 4\pi \times 5^2 = 100\pi\, m^2$ at distance 5m from the source produces an intensity of
$$I = \frac{P}{A} = \frac{50}{100\pi} = 0.159 Wm^{-2}.$$
Expressing this in decibels with respect to the reference intensity level of $I_0 = 10^{-12} Wm^{-2}$ that corresponds to the threshold level of human hearing,
$$SL(dB) = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{0.159}{10^{-12}}\right) = 112 dB.$$
We assume that the output power level achieved by the speaker is the same at all of the listed frequencies, so the sound level should be 112dB for all of those frequencies.

6. This is an example of the Doppler effect, where the frequency of sound f_o heard by the observer can be related to the frequency of the sound as emitted by the source f_s by
$$f_o = f_s\left(\frac{v + v_o}{v + v_s}\right)$$
where v is the velocity of the sound waves, v_o the velocity of the observer, v_s is the velocity of the source, and the velocities are taken as positive for observer moving toward source and source moving away from the observer. In this question, the stationary observer $v_o = 0$ hears a frequency of $f_o = 560 Hz$ for the sound ($v = 343 ms^{-1}$) emitted by the approaching police car with frequency f_s. The velocity of the police car is thus given by
$$560 = f_s\left(\frac{343}{343 - v_s}\right)$$
When the car is receding, the observed frequency is 480 *Hz*, so
$$480 = f_s\left(\frac{343}{343 + v_s}\right)$$
We can divide the two above equations to eliminate the unknown frequency emitted by the source, f_s:
$$\frac{560}{480} = \frac{343 + v_s}{343 - v_s}.$$

Solving for the source velocity:

$$v_s = 343\left(\frac{\frac{560}{480}-1}{\frac{560}{480}+1}\right) = 26.4\,ms^{-1}.$$

7. a) For the listener in the car, just before passing, the motion of the car is toward the source, the train, so the velocity of the observer $v_o = +40ms^{-1}$ is taken as positive in the Doppler effect formula, while the motion of the train is away from the observer, so $v_s = +20ms^{-1}$ is taken as positive too. Thus, the emitted frequency $f_s = 320Hz$ will be heard at a frequency of

$$f_s = 320\left(\frac{343+40}{343+20}\right) = 338\,Hz.$$

b) Just after the car passes the train, the source velocity (car) is away from the observer (train) and the observer velocity is toward the source, so both velocities may be taken as positive again. Thus, the observed frequency is

$$f_s = 510\left(\frac{343+20}{343+40}\right) = 483\,Hz.$$

8. Firstly, the velocity of the wave along the string can be found with

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50}{0.1\times10^{-3}}} = 707\,ms^{-1}.$$

Then, we can find the fundamental frequency of the string as the frequency for which one half a wavelength fits along the string, $L = \lambda/2$, so

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \frac{707}{2\times0.6} = 589\,Hz.$$

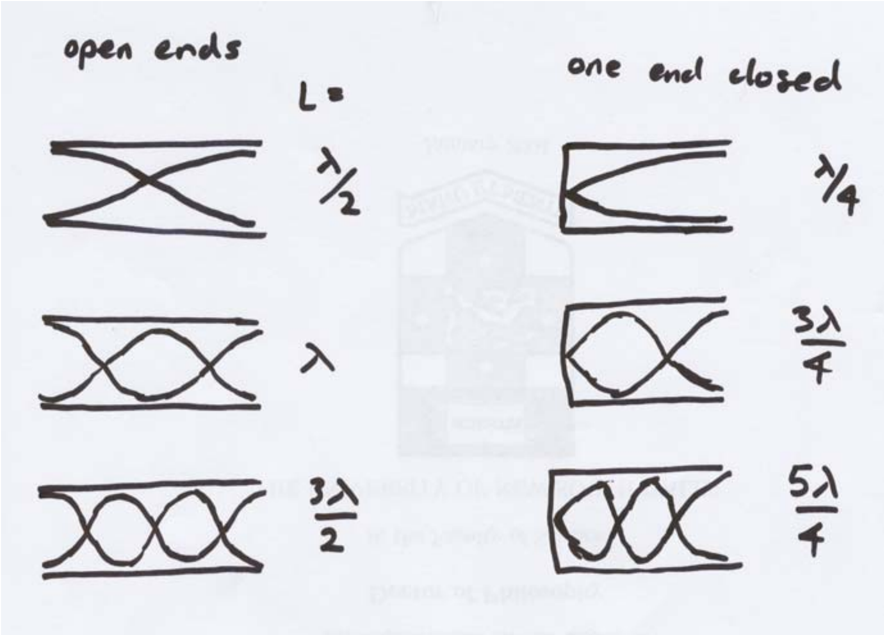
The harmonics are all the integer multiples of the fundamental frequency, so $f_n = nf_1$ for $n = 1,2,3,\ldots$. The harmonic closest to 20 kHz is

$n = \frac{20.0\,Hz}{589Hz} \approx 34$. This frequency is $f_{34} = 34\times589 = 20.0\,kHz$.

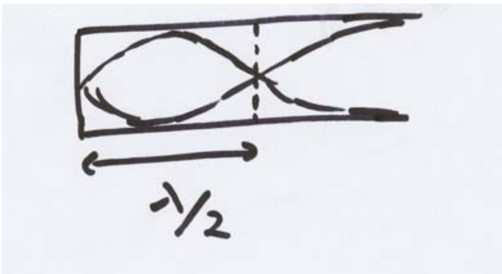
9. a) If the pipe is open at both ends, then there must be an anti-node at each end of the pipe. The first configuration to achieve this has one-half wavelength in the pipe, and then each successive half wavelength will also produce a resonance, so the condition for resonance has $L = n\frac{\lambda}{2}$. In terms of the speed and frequency of the wave, this is $L = \frac{nv}{2f}$.

b) If the pipe is closed at one end, then for resonance there must be a node at the closed end and an anti-node at the open end. This can be satisfied by having a quarter wavelength in the pipe, then with each successive half wavelength more. So the resonance conditions are

$$L = \frac{\lambda}{4} + (n-1)\frac{\lambda}{2} = \frac{(2n-1)\lambda}{4} = \frac{(2n-1)v}{4f}.$$



10. The difference in distance between the two resonances should correspond to half a wavelength of the waves. That is $\frac{\lambda}{2} = 0.683 - 0.228 = 0.455\,m$, and the wavelength of the waves is then $\lambda = 0.91\,m$. The speed of the sound is then $v = f\lambda = 384 \times 0.91 = 350\,ms^{-1}$. The next resonance will occur a further half wavelength along, at $68.3 + 45.5 = 114\,cm$.



11. a) Firstly, the frequency reflected by the wall can be found using the Doppler effect, with a source moving with velocity $v_s = -1.33\,ms^{-1}$ toward a stationary observer. Thus, the frequency reflected by the wall is $f_w = 256\left(\frac{343}{343 - 1.33}\right)$.
The wall can then be treated as the source, so that the frequency of this echo as heard by the observer moving toward the wall with velocity $v_o = 1.33\,ms^{-1}$ is
$$f = f_w\left(\frac{343 + 1.33}{343}\right) = 256\left(\frac{343 + 1.33}{343 - 1.33}\right) = 257.99\,Hz.$$
The beat frequency is then just the difference in the frequency of the fork and the frequency of the echo, so the beat frequency should be 1.99 Hz.
b). Replacing the $1.33\,ms^{-1}$ in the above analysis with the unknown speed of the walker –*v*, with the negative sign to account for the change in direction, to

produce an echo frequency of 5 Hz lower than the tuning fork frequency, and so a beat frequency of 5 Hz,

$$256 - 5 = 251 = 256\left(\frac{343 - v}{343 + v}\right).$$

Solving for *v* gives:

$$v = 343\left(\frac{1 - \frac{251}{256}}{1 + \frac{251}{256}}\right) = 3.38\,ms^{-1}.$$

Extra Problems

E1. The two waves can be added using the trig result

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$$

Therefore,

$$y(x,t) = y_1 + y_2 = 5\sin(\pi(4x - 1200t)) + 5\sin(\pi(4x - 1200t - 0.25))$$

$$y(x,t) = 10\sin(\pi(4x - 1200t - 0.125))\cos(0.125\pi)$$

a) The amplitude of the wave is thus $10\cos(0.125\pi) = 9.24\,m$.

b) The angular frequency of the wave can be seen by inspection to be $\omega = 1200\pi\,s^{-1}$, so the frequency is $f = \dfrac{\omega}{2\pi} = 600\,Hz$.

E2. Adding the two waves together as above,

$$y(x,t) = y_1 + y_2 = 3\sin(\pi(x + 0.6t)) + 3\sin(\pi(x - 0.6t))$$

$$y(x,t) = 6\sin(\pi x)\cos(0.6\pi t)$$

a) At x = 0.25cm, the amplitude of the wave is $6\sin(0.25\pi) = 4.24\,cm$. Thus the maximum displacement at this position is 4.24cm.

b) At x = 0.5cm, we have $6\sin(0.5\pi) = 6.00\,cm$.

c) At x = 1.5cm, we get $6\sin(1.5\pi) = 6.00\,cm$.

d) The anti-node are the points where the displacement reaches it maximum, which for the above wave is 6cm, since the sine and cosine factors can be at most one. These anti-nodes occur at 0.5cm, 1.5cm, and also 2.5cm, as anti-nodes are evenly spaced by one half-wavelength.

E3. a) Each segment of vibration corresponds to half a wavelength. Thus, there are two wavelengths in the 120cm of string, so the wavelength is 60cm.

b) The speed of propagation of the waves are $v = f\lambda = 120 \times 0.6 = 72\,ms^{-1}$. The fundamental frequency would have half a wavelength along the string, so its wavelength would be 240cm and the frequency is $f = \dfrac{v}{\lambda} = \dfrac{72}{2.4} = 30\,Hz$.