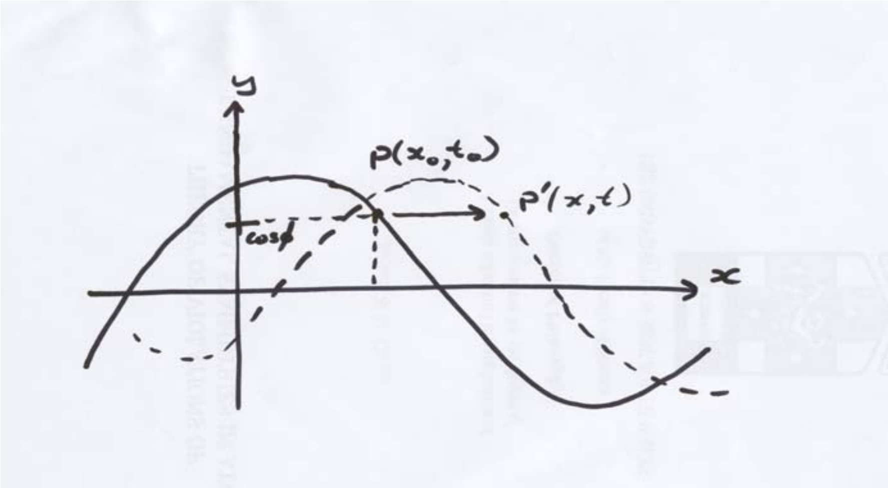
PHYS1169: Tutorial 8 Solutions

Wave Motion

1). Let us consider a point P on the wave with a phase φ, so $y = \cos\varphi = \cos(kx \pm \omega t)$. At t_0, this point has position x_0, so $\varphi = kx_0 \pm \omega t_0$. Now, at some later time *t*, the position *x* of this point on the wave P′ can be found by $\varphi = kx - \omega t$, since the point should have the same displacement *y* and same phase φ. Thus, we have

$$x = \frac{1}{k}(\varphi \mp \omega t) \text{ and } x_0 = \frac{1}{k}(\varphi \mp \omega t_0).$$



The velocity of the wave is the distance travelled over the time taken:

$$v = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}.$$

With the above expressions for *x* and x_0, this gives

$$v = \frac{\frac{1}{k}\left[(\varphi \mp \omega t) - (\varphi \mp \omega t_0)\right]}{t - t_0} = \mp\frac{\omega}{k}.$$

But, expressing the angular frequency ω in terms of the frequency *f*, $\omega = 2\pi f$ and the wave number *k* in terms of the wavelength λ, $k = \frac{2\pi}{\lambda}$, gives $v = \mp f\lambda$. Thus, a point with a given phase will move in the positive *x*-direction with velocity $v = f\lambda$ (corresponding to the lower sign in the above analysis), while the velocity is $v = -f\lambda$, with travel in the negative *x*-direction if the positive sign is used.

2). Use $v = \sqrt{\frac{F}{\mu}}$, where *v* is the velocity of the wave, *F* is the tension in the wire, and μ is the mass per unit length, to find the velocity of the wave in each wire.

For steel,

$$\mu_S = \frac{m_S}{L_S} = \frac{\rho_S V_S}{L_S} = \frac{\rho_S \pi r_S^2 L_S}{L_S} = \rho_S \pi r_S^2$$

For copper,

$$\mu_C = \rho_C \pi r_C^2$$

$\mu_S = 7.86 \times 10^3 \times \pi \times \left(0.5 \times 10^{-3}\right)^2$

$\mu_S = 6.17 \times 10^{-3}\, kgm^{-1}$

$v_S = \sqrt{\dfrac{F}{\mu_S}} = \sqrt{\dfrac{150}{6.17 \times 10^{-3}}}$

$v_S = 156 ms^{-1}$

$\mu_C = 8.92 \times 10^3 \times \pi \times \left(0.5 \times 10^{-3}\right)^2$

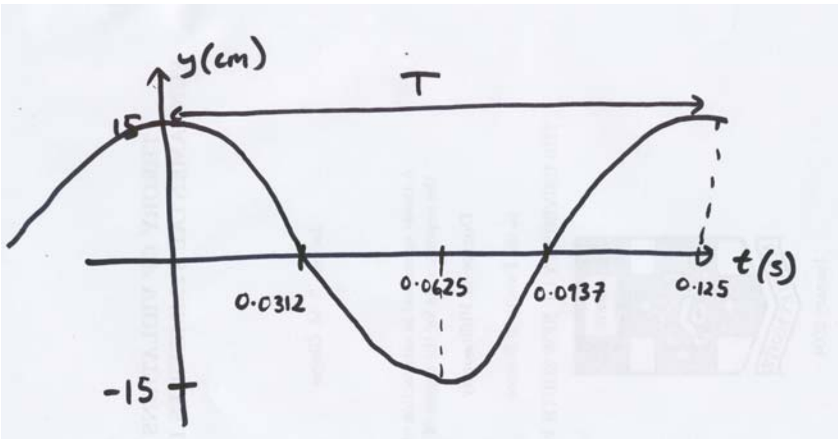
$\mu_S = 7.01 \times 10^{-3}\, kgm^{-1}$

$v_C = \sqrt{\dfrac{F}{\mu_C}} = \sqrt{\dfrac{150}{7.01 \times 10^{-3}}}$

$v_C = 146 ms^{-1}$

The time for the wave to travel the 30m of steel wire is thus $t_S = \dfrac{d_S}{v_S} = \dfrac{30}{156} = 0.192s$, and similarly for the copper wire $t_C = \dfrac{d_C}{v_C} = \dfrac{20}{146} = 0.137s$. The total time to travel along the two wires is $t = t_S + t_C = 0.329s$.

3). At $x = 0$, $y = (15cm)\cos(-50.3t) = (15cm)\cos(50.3t)$, since $\cos x$ is an even function (ie. $\cos(-x) = \cos x$). (Not to scale)



The period of the wave is $T = 0.125$s. This agrees with $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50.3} = 0.125s$.

4). a) Start with $y = A\sin(kx - \omega t)$ and use $\omega = kv$ to get $y = A\sin k(x - vt)$.

b) Then use $k = \dfrac{2\pi}{\lambda}$ in a) to get $y = A\sin\left[\dfrac{2\pi}{\lambda}(x - vt)\right]$.

c) Then use $v = f\lambda$ in b) to give $y = A\sin\left[2\pi(\dfrac{x}{\lambda} - ft)\right]$.

d) Finally, use $\lambda = \dfrac{v}{f}$ in c) to get $y = A\sin\left[2\pi f(\dfrac{x}{v} - t)\right]$.

5). The wave $y = (0.25m)\sin(0.3x - 40t)$ is in the form $y = A\sin(kx - \omega t)$, so by inspection we can determine that:

a) amplitude A = 0.25m.

b) angular frequency $\omega = 40s^{-1}$.

c) wave number $k = 0.3m^{-1}$.

d) wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.3} = 20.9m$.

e) wave speed $v = \frac{\omega}{k} = \frac{40}{0.3} = 133.3ms^{-1}$.

f) from question 1, the negative sign in the phase relationship $kx - \omega t$ implies that the wave is travelling in the positive x-direction.

6). The wave $y = (0.08m)\sin(0.24x - 30t)$ is in the form $y = A\sin(kx - \omega t)$, so by inspection $k = 0.24m^{-1}$ and $\omega = 30s^{-1}$.

a) Velocity of the wave is $v = \frac{\omega}{k} = \frac{30}{0.24} = 125ms^{-1}$.

b) The transverse velocity is the velocity perpendicular to the direction of propagation of the wave, that is, the velocity in the y-direction. The velocity in the y-direction is just the change in y position with respect to time, for a given x point in the waves path, $v_t = \left(\frac{dy}{dt}\right)_{x,constant}$ (that is, the derivative of y with respect to t, treating x as a constant to consider the transverse displacement at a fixed x point in the wave's path). Thus,

$$v_t = \left(\frac{dy}{dt}\right)_{x,constant} = 0.08 \times (-30)\cos(0.24x - 30t),$$

using the standard result $\frac{d}{dt}\left(A\sin(at - b)\right) = Aa\cos(at - b)$.

The transverse velocity as a function of time is then $v_t = (-2.4ms^{-1})\cos(0.24x - 30t)$. The expression will have its maximum velocity when the cosine factor is -1, so the maximum transverse velocity reached is $(v_t)_{\max} = 2.4ms^{-1}$.

Sound Waves

1). Let the source of the lightening strike be a distance d away from the observer. The time taken for the sound at $v_s = 343ms^{-1}$ to reach the observer is $t_s = \frac{d}{v_s}$. Similarly, the time taken for the light to reach the observer at $c = 3.0 \times 10^8 ms^{-1}$ is $t_L = \frac{d}{c}$. The time difference between the light and the sound arrival times is then

$$\Delta t = t_s - t_L = \frac{d}{v_s} - \frac{d}{c}.$$

Because the light travels much, much faster than the sound, the time for the light to reach the observer t_L is so much smaller than the time for the sound to reach the observer t_s that we can neglect it (if your unsure, plug in the numbers and see!).

$$\Delta t = 16.2s \approx \frac{d}{v_s} \text{ so that } d = v_s \Delta t = 16.2 \times 343 = 5560m .$$

Thus, the source of the lightning is 5.56km away.

2). Use $v = \sqrt{\frac{B}{\rho}}$, where *v* is the velocity of the sound wave, *B* is the bulk modulus of the medium, and ρ is the density of the medium.

$$v = \sqrt{\frac{2.8 \times 10^{10}}{13600}} = 1430 ms^{-1}.$$

3). The intensity sound level of sound wave is expressed relative to the threshold level of human hearing, which is taken as $I_0 = 10^{-12} Wm^{-2}$. The sound level in decibels is

$$SL(dB) = 10\log_{10}\left(\frac{I_1}{I_0}\right) = 10\log_{10}\left(\frac{4x10^{-6}}{10^{-12}}\right) = 66dB.$$

4). Firstly, find the intensity in Wm^{-2} for the sound level of 120dB, via

$$SL(dB) = 120dB = 10\log_{10}\left(\frac{I_1}{I_0}\right) = 10\log_{10}\left(\frac{I_1}{10^{-12}}\right)$$

$$\therefore \quad I_1 = I_0 \times 10^{12} = 1Wm^{-2}.$$

This intensity can then be related to the pressure amplitude Δp of the sound wave via the relation $I = \frac{(\Delta p)^2}{2\rho v}$, where ρ is the density of the medium and *v* is the velocity of the wave. Then,

$$\Delta p = \sqrt{I \times 2\rho v} = \sqrt{1 \times 2 \times 1.2 \times 343} = 28.7 Pa.$$

Extra Problems

E1). a) At $x = 5cm$ and $t = 2s$, the phase of the first wave and second wave are:

$\phi_1 = 20x - 30t = 20 \times 5 - 30 \times 2 = 40rad \qquad \phi_2 = 25x - 40t = 25 \times 5 - 40 \times 2 = 45rad$

Thus, there is a phase difference $\Delta\phi = \phi_2 - \phi_1 = 5rad$.

b) At $t = 2s$, the addition of the two waves is

$$y_T = y_1 + y_2 = 2\sin(20x - 60) + 2\sin(25x - 80)$$

We can use the trig relation to express this as the product of a sine and a cosine factor,

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

that gives

$$y_T = 4\sin(22.5x - 70)\cos(2.5x - 10)).$$

Thus y_T will be zero whenever the sine and the cosine terms are zero. We seek the smallest positive *x* such that y_T is zero, so we may take the smallest positive *x* so that the sine term is zero and the smallest positive *x* that the cosine term is zero, then the smaller of the two will be our desired value of *x*.

Now, sin*θ* is zero for any multiple of π, so we can find the integer *k* so that $22.5x - 70 = k\pi$ has the smallest positive *x* value. By trial and error, this is *k* = -22, for which $x = \frac{70 - 22 \times \pi}{22.5} = \frac{0.885}{22.5} = 0.0393cm$. Similarly, cos*θ* is zero for any odd

multiple of π/2, so we can find the value of m that give the smallest positive x with $2.5x - 10 = \frac{(2m+1)\pi}{2}$. This happens when m = -3 and then x = 0.858cm.
Thus the first positive value of x at which the two waves add to zero near the origin is 0.0393cm.

E2). Use $v = \sqrt{\frac{T}{\mu}}$ with velocity v = 50 m/s and mass per unit length $\mu = \frac{0.06kg}{5m} = 0.012kgm^{-1}$. The tension in the string is then

$$T = \mu v^2 = 0.012 \times (50)^2 = 30N$$

E3). The velocity of the waves are $v = \frac{dis\tan ce}{time} = \frac{4.25m}{10s} = 0.425ms^{-1}$. The frequency of the wave is the number of vibrations per second, $f = \frac{40vib}{30s} = \frac{4}{3}Hz$.

Then the wavelength is simply $\lambda = \frac{v}{f} = 0.319m$.

E4). a) We can use the general form for a wave travelling in the negative x-direction,

$$y = A\sin(kx + \omega t - \phi)$$

where the amplitude A = 0.08m, wave number $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8} = 7.85m^{-1}$, and angular frequency $\omega = 2\pi f = 2\pi \times 3 = 6\pi\, s^{-1}$. Thus,

$$y = (0.08m)\sin(7.85x + 6\pi t - \phi)$$

We can find the phase ϕ by $y(x=0, t=0) = 0$, so $0 = 0.08\sin(-\phi)$ which allows us to take ϕ = 0. The expression for the wave is then

$$y = (0.08m)\sin(7.85x + 6\pi t)$$

b) As above, except now $y(x = 0.1, t = 0) = 0$, so $0 = 0.08\sin(7.85 - \phi)$ and we may take $\phi = 7.85$ so the wave is described as $y = (0.08m)\sin(7.85x + 6\pi t - 7.85)$.

E5). a) The wavelength is simply $\lambda = \frac{4}{5}m = 0.8m$, so the wave number can be found from $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8} = 7.85m^{-1}$. The angular frequency is just $\omega = vk = 128 \times 7.85 = 1005s^{-1}$. The wave is then described using $y = A\sin(kx - \omega t)$ to give $y = (0.02m)\sin(7.85x - 1005t)$.

b) The frequency of the wave is $f = \frac{v}{\lambda} = \frac{128}{0.8} = 160Hz$. Expressing the wave explicitly showing the wavelength and the frequency is

$$y = A\sin\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right) = (0.02m)\sin\left(2\pi\left(\frac{x}{0.8} - 160t\right)\right).$$

E6). The pressure amplitude Δp_m is related to the displacement amplitude s_m via

$$\Delta p_m = k\rho v^2 s_m.$$

The velocity of the wave is v = 343 m/s, the wave length $\lambda = \dfrac{v}{f} = \dfrac{343}{10 \times 10^3} = 0.0343m$,

so the wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.0343} = 183m^{-1}$, and the density is $\rho = 1.2kgm^{-3}$.

Thus, the displacement amplitude is

$$s_m = \frac{\Delta p_m}{k\rho v^2} = \frac{4 \times 10^{-3}}{183 \times 1.2 \times (343)^2} = 1.55 \times 10^{-10}\, m .$$

E7). The pressure variation of the wave is just a sinusoidal wave as

$$\Delta p = \Delta p_m \sin(kx - \omega t).$$

The wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.1} = 62.8m^{-1}$, and the angular frequency is

$\omega = vk = 343 \times 62.8 = 2.16 \times 10^4 s^{-1}$. Therefore,

$$\Delta p = (0.2Pa)\sin\left(62.8x - 2.16 \times 10^4 t\right).$$