PHYS1169 Waves

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Waves are moving pattern of displacements. May transmit energy and signals.

Travelling waves, superposition and interference, velocity, reflection and transmission, harmonic waves, spherical and plane waves.

Sound.

Doppler effect, standing waves in strings and air columns, beats, decibel scale

Light.

Lab Ray approximation & geometric optics:

Lab Reflection and refraction, Huygen's pple, total internal reflection, mirrors, images, lenses, magnifier, compound microscope, telescope

Interference and Diffraction

Conditions for interference, Young's experiment, and interference pattern, phasor addition, reflection, thin films, diffraction

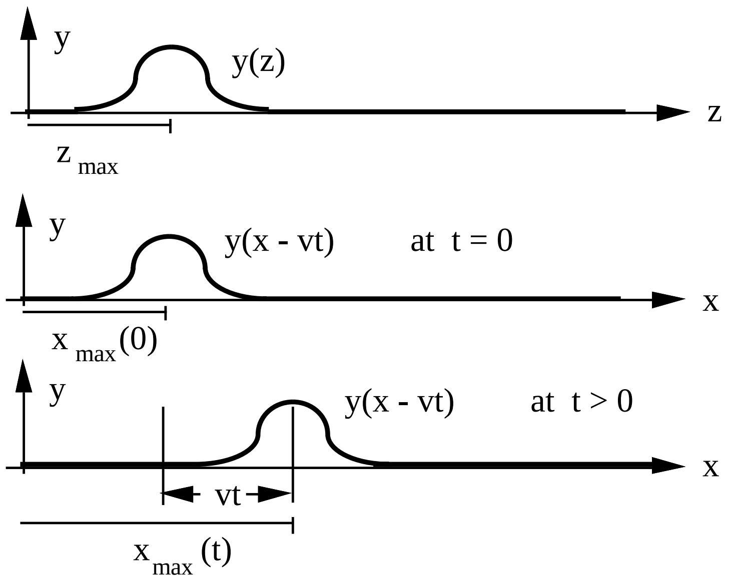
Mechanical waves

example	*type*	*restoring force*
Wave in string	transverse	tension in string
Water wave	transverse	gravity
Sound wave	longitudinal	air pressure

Only pattern travels, not medium.

Displacement motion $\frac{\partial y}{\partial t}$ is usually slower than wave speed

$f(x - vt)$ is a wave travelling at v in +x dir^n:



In time t, the wave has travelled $\Delta x = vt$.

peak is $y(z_{max}) = y(x_{max} - vt)$

$$y(x_{max}(0) - v.0) = y(x_{max}(\Delta t) - v\Delta t)$$

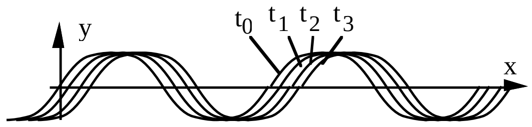
$$y(x_{max}(0) - 0) = y(x_{max}(0) + \Delta x - v\Delta t)$$

$$0 = \Delta x - v\Delta t$$

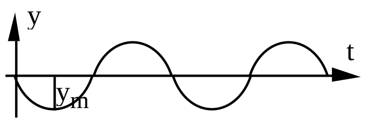
$\therefore$ $y(x - vt)$ is the equation of a wave travelling to the ***right***.

$y(x + vt)$ is a wave travelling to the left

e.g. $y = y_m \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$



At one point
(eg x = 0)



One cycle of SHM takes T, wave travels λ, ∴

$$v = \frac{\lambda}{T} = f\lambda$$

define $\omega = 2\pi f$, wave number $k \equiv \frac{2\pi}{\lambda}$

$$y = y_m \sin\left(kx - \omega t\right)$$

Speed of wave

$$v_{wave} = \sqrt{\frac{\text{springy const}}{\text{inertial const}}} \qquad v_{string} = \sqrt{\frac{T}{\mu}}$$

$$v_{sound} = \sqrt{\frac{\gamma P}{\rho}} \qquad \left(c = \sqrt{\frac{k_{elec}}{k_{mag}}} = \sqrt{\frac{1}{\mu_o \varepsilon_o}}\right)$$

$\gamma = \frac{c_p}{c_v}$ is ratio of specific heats

Example A wave has $y = y_m \sin\left(kx - \omega t\right)$,

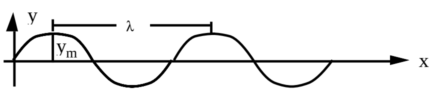
$y_m = 10$ nm, $k = 18.5\ \text{m}^{-1}$, $\omega = 6300\ \text{rad.s}^{-1}$

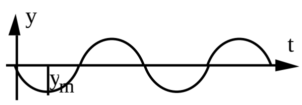
i) what is the speed of the wave?

ii) What is (max) average speed of particles?

$$v_{wave} = f\lambda = \frac{\omega}{2\pi} \cdot \frac{2\pi}{k} = \frac{\omega}{k} = \ldots = 340\ \text{ms}^{-1}$$

what is v_{part}?





$$v_{part} = \frac{\partial y}{\partial t}$$

$$= \frac{\partial}{\partial t} y_m \sin\left(kx - \omega t\right)$$

$$= \omega y_m \cos\left(kx - \omega t\right)$$

$$v_{max} = \omega y_m$$

$$= 6300\ \text{s}^{-1} \times 10\ \text{nm} = 63\ \mu\text{m.s}^{-1}$$

average *speed, not individual speed, cf wind*

Reflection:

Going from less dense to more dense, waves are reflected with a phase change of π.

e.g. reflection at a 'fixed' end
thin string to thick string,
air to water

From more dense to less dense, no phase change

e.g. reflection at 'free' end, etc

Superposition

In a linear medium, waves superpose linearly, i.e. their displacements simply add.

Most media linear for ***small*** amplitude waves.

Superpose incident & reflected waves → standing waves

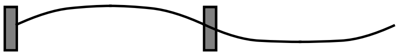
Standing waves

$y_1 = y_m \sin(kx - \omega t)$

$y_2 = y_m \sin(kx + \omega t)$

$y_T = \ldots$*(algebra)*$\ldots = 2y_m \sin kx \cos \omega t$

stationary wave / simple harmonic motion



$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$

$f_2 = \frac{v}{\lambda} = \frac{2v}{2L} = 2f_1$

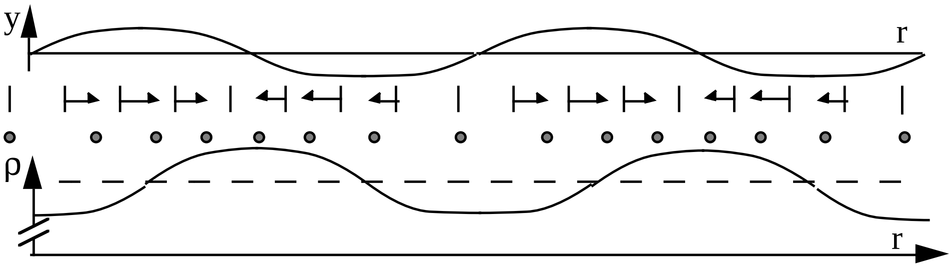
(2nd harmonic)

$f_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_1$

(3rd harmonic)

http://www.phys.unsw.edu.au/~jw/strings.html

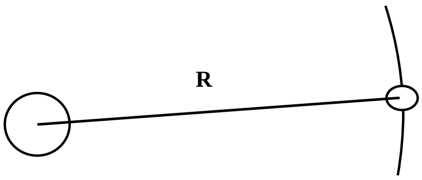
Sound is a compression wave - longitudinal

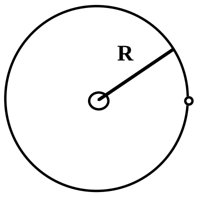


Radiation

Intensity $\qquad I \equiv \frac{\text{power}}{\text{area}}$

Example: What is the intensity of solar radiation? $P_{sun} = 3.9\ 10^{26}$ W.
Earth is 150 million km from sun.

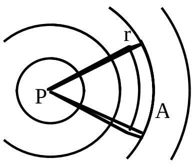




$I = \frac{P}{4\pi r^2} = \ldots. = 1.38\ \text{kWm}^{-2}$

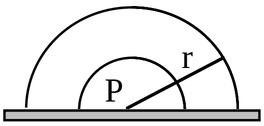
above atmosphere, $\perp$ *radiation*

Isotropic radiation:



$I = \frac{P}{4\pi r^2}$

Radiation with plane reflector



$I = \frac{P}{2\pi r^2}$

Intensity $\propto$ Power $\propto$ amplitude^2 $\propto$ pressure^2

Sound intensity level:

$L_I \equiv 10 \log_{10} \frac{I}{I_o}$ where $I_o = 10^{-12}$ W.m^{-2}

(L_I in decibels)

$$L_2 - L_1 = 10\left(\log_{10} \frac{I_2}{I_o} - \log_{10} \frac{I_1}{I_o}\right) = 10 \log_{10} \frac{I_2}{I_1}$$

p_2/p_1	ΔL_p	I_2/I_1	ΔL_I
$\sqrt{2}$	3 dB	2	3 dB
$\sqrt{10}$	10 dB	10	10 dB

http://www.phys.unsw.edu.au/music/dB.html
www.phys.unsw.edu.au/PHYSICS_!/SPEECH_HELIUM/speech.html

Example. If sound level L_I = 3 dB at 10 cm from a source radiating uniformly, what is the acoustic power of the source?

$$3 \text{ dB} = L_I \equiv 10 \log \frac{I}{I_o}$$

$$0.3 = \log \frac{I}{I_o}$$

I/I_o = antilog 0.3 = $10^{0.3}$ = 2

$I = 2\, I_o = 2\; 10^{-12}$ Wm^{-2}

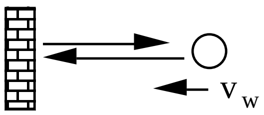
$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

P = ... = 0.25 pW

Doppler effect. $f' = f\left(\frac{v + v_o}{v - v_s}\right)$

v_o and v_s are positive for approaching
measure all velocities with respect to medium

Example. You walk towards a wall, blowing a whistle at f = 500 Hz. You hear beats at 5 Hz between your whistle and the reflected sound. How fast are you walking?



You hear your own whistle at frequency f.

The wall receives

$$f' = f\frac{v + v_o}{v - v_s} = f\frac{v + 0}{v - v_w}$$

This is the source of the reflection. You hear

$$f'' = f'\frac{v + v_o}{v - v_s} = f'\frac{v + v_w}{v - 0}$$

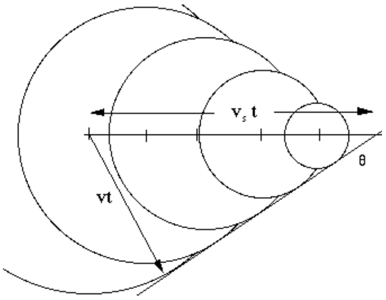
$$f'' = f\frac{v + v_w}{v - v_w}$$

$$f''(v - v_w) = f\,(v + v_w)$$

$$(f'' - f)v = (f'' + f)v_w$$

$$v_w = v\frac{f'' - f}{f'' + f} = v\,\frac{5}{1005} = 1.7 \text{ ms}^{-1}$$

$\mathbf{v_s > v} \quad \rightarrow$ **Shock wave**



Crests combine to form a shock wave

Cone has $\sin\theta = \dfrac{v}{v_s}$

$\dfrac{v_s}{v} \equiv$ Mach number

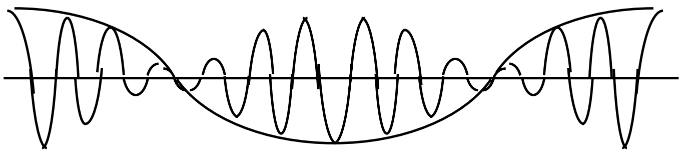
Beats

$y_1 = A\cos 2\pi f_1 t$ $\qquad y_2 = A\cos 2\pi f_2 t$

$$y_1 + y_2 = 2A\cos 2\pi\frac{f_1+f_2}{2}t\,\cos 2\pi\frac{f_2-f_1}{2}t$$

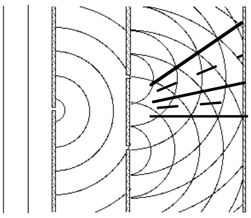
/ average frequency

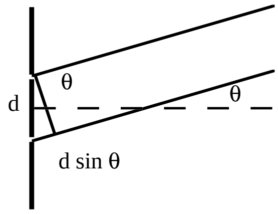
\ Beat frequency is $f_2 - f_1$



One cycle (diagram) of $\cos 2\pi\dfrac{f_2-f_1}{2}t$ has two beats

Young's experiment Coherent source $\rightarrow$ **two slits** gives interference pattern on screen.

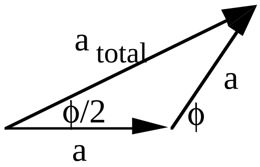




Constructive interference (max) if $d \sin\theta = m\lambda$

Destructive interference (min) if $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

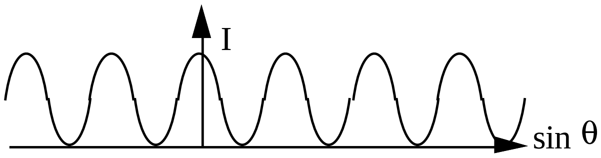
$$\frac{\phi}{2\pi} = \frac{\Delta \text{ path}}{\lambda} = \frac{d \sin\theta}{\lambda} \quad \therefore \quad \phi = \frac{2\pi}{\lambda} d \sin\theta$$



$$a_{tot} = 2a \cos\beta$$

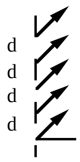
$$\beta = \phi/2 = \frac{\pi}{\lambda} d \sin\theta$$

Intensity $\propto$ amplitude^2 $\therefore$ $I \propto 4a^2 \cos^2\beta$



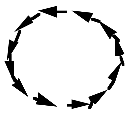
$$I = I_{max}\cos^2\beta \quad \text{where } \beta = \frac{\pi}{\lambda} d \sin\theta$$

$$\frac{\phi}{2\pi} = \frac{\Delta \text{ path}}{\lambda} = \frac{d \sin\theta}{\lambda} \quad \therefore \quad \phi = \frac{2\pi}{\lambda} d \sin\theta$$



Diffraction grating has very many slits.
Used to measure λ very accurately.
If there are N slits per unit lenth, $d = 1/N$.

The first minimum is *very* close (small ϕ to close polygon), ie very narrow maxima



For constructive interference

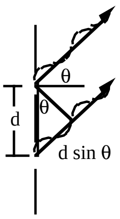
$d \sin\theta = m\lambda$

$m = 1 \rightarrow$ 1st order spectrum

$m = 2 \rightarrow$ 2nd order spectrum

$$\theta_{red} = \sin^{-1} \frac{m\lambda_{red}}{d}$$

$$\theta_{blue} = \sin^{-1} \frac{m\lambda_{blue}}{d}$$

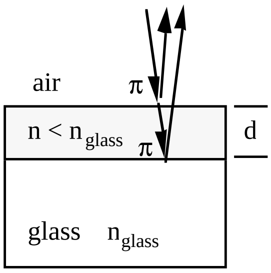


Thin film interference

Optical path length OPL $\equiv$ n . pathlength

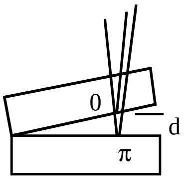
$$\Delta\phi = 2\pi \frac{\Delta \text{ optical pathlength}}{\lambda}$$

e.g. Newton's rings & non reflective coating



$1 < n < n_{glass}$
For destructive
$\Delta OPL = \lambda/2$
$2nd = \lambda/2$
$d \sim \dfrac{\lambda}{4n}$

Air wedge



Denstructive interference if
$2d = m\,\lambda$
Constructive interference if
$2d = \left(m + \frac{1}{2}\right)\lambda$

Diffraction from a slit

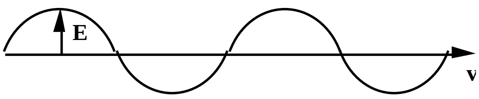
$$I = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{where } \alpha = \frac{\pi a \sin\theta}{\lambda}$$

Resolution from circular aperture:

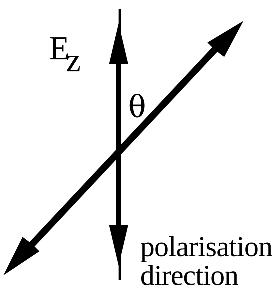
First minimum at $\sin\theta = 1.22\,\dfrac{\lambda}{d}$

can resolve ~ θ with lens diam d. Rayleigh's criterion

Polarisation.



EM waves are **transverse** waves: ($\mathbf{E} \perp \mathbf{v}$) ∴ can be polarised. Usually light has waves with **E** in all directions



Polaroid materials

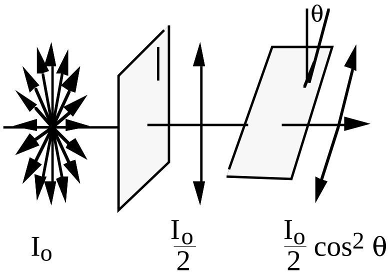
allow **E** in only one dir^n

$E_{transmitted} = E\cos\theta$

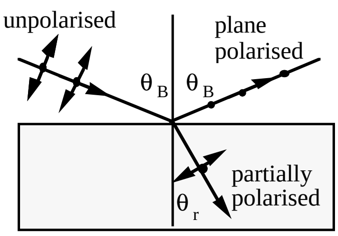
Malus' Law:

$I_{trans} = I_{in}\cos^2\theta$

Average of $\cos^2\theta$ over all angles is 1/2 ∴



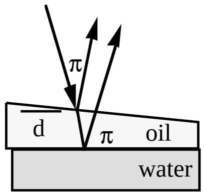
Polarisation by reflection



When refracted ray $\perp$ reflected $\rightarrow$ plane polarised reflected wave (Brewster's angle θ_B). If $\theta_B + \theta_r = 90°$

$$\sin\theta_r = \cos\theta_B \quad \Rightarrow \quad n = \frac{\sin\theta_B}{\cos\theta_B} \quad \Rightarrow \quad \theta_B = \tan^{-1} n$$

Example. An oil slick (n = 1.20) floats on water. What are the thicknesses for which red light ($\lambda \cong 700$ nm) is reflected weakly? What does the slick look like at its thinnest point?



$n_{water} > n_{oil}$

Constructive interference if

Δ OPL = m λ

Destructive interference if

$\Delta \text{ OPL} = \left(m + \frac{1}{2}\right)\lambda$

i) If red has destructive interference,

$$\Delta \text{ OPL} = 2nd = \left(m + \tfrac{1}{2}\right)\lambda_{red}$$

$$d = \frac{\lambda_{red}}{2n}\left(m + \tfrac{1}{2}\right)$$

m = 0, m = 1, m = 2 .....

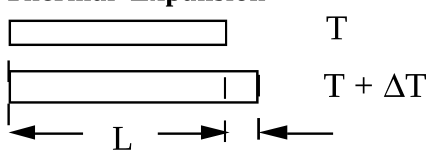
= 150 nm, 440 nm, 730 nm etc

ii) If d << λ, π phase difference on both paths so constructive interference for all λ, so it looks bright and 'white'.

Temperature (T):

T is equal in any 2 bodies at thermal equilibrium

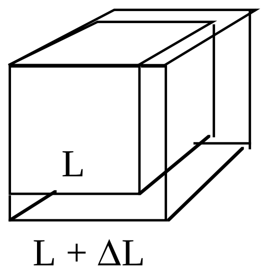
Thermal Expansion



$\therefore$ **Define** $\frac{\Delta L}{L} = \alpha\,\Delta T$ $\qquad \frac{\Delta V}{V} = \beta\Delta T$

α is **coefficient of linear expansion**

β is **coefficient of volume expansion**



$\Delta V = (L + \Delta L)^3 - L^3$

$= .....$

$\approx V.3\alpha\Delta T$

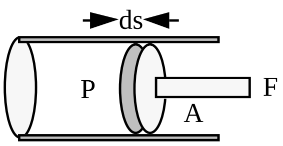
$\therefore \ \beta \approx 3\alpha$

Heat Capacity: (for a body) $C = \frac{\Delta Q}{\Delta T}$ *extensive*

Specific Heat: (of substance) $c = \frac{\Delta Q}{M\Delta T}$ *intensive*

Latent Heat: heat required per unit mass for change of phase (at constant T).

Work done against pressure P



$$dW = \mathbf{F.ds} = PA\,ds = PdV$$

Work done against pressure P

1st Law $dU = dQ - dW$
where U is a state function

Isobaric P const

Adiabatic Process: $\Delta Q = 0$ (fast or insulated)

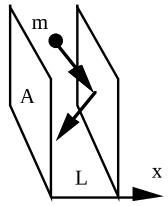
Calculate work:

Isobaric: $W = \int P\,dV = P\Delta V$

Isothermal: $W = \int P\,dV = \int \frac{nRT}{V}\,dV = nRT\,ln\,\frac{V_f}{V_i}$

Adiabatic: $PV^\gamma = \text{constant}$

Kinetic Theory



Δ momentum = $2mv_x$
collide every $2L/v_x$

$$|\overline{F}| = \left|\frac{\Delta p}{\Delta t}\right| = \frac{2mv_x}{2L/v_x} = \frac{mv_x{}^2}{L}$$

$$F_{all} = PA = \frac{Nm\overline{v_x^2}}{L}$$

$v^2 = v_x{}^2 + v_y{}^2 + v_z{}^2$; random motion $\Rightarrow$ $\overline{v_x{}^2} = \frac{1}{3}\overline{v^2}$, so:

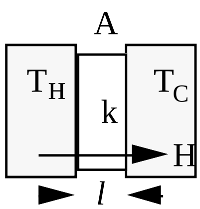
$$PAL = PV = \frac{N}{3} m \overline{v^2} \qquad P = \frac{1}{3}\rho\,\overline{v^2}$$

$$\tfrac{1}{2} m \overline{v^2} = \overline{\varepsilon} \text{ and } PV = NkT$$

$$\therefore \quad \overline{\varepsilon} = \frac{1}{2} m \overline{v^2} = \frac{3}{2}\frac{PV}{N} = \frac{3}{2} kT$$

$$v_{r.m.s.} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

Heat conduction



$$H \equiv kA\frac{T_H - T_C}{l}$$

k is thermal conductivity

Thermal resistance or R-value sometimes used for building materials

$$R \equiv \frac{l}{k} \qquad \text{so} \quad H = A\frac{\Delta T}{R}$$

(High conductivity, low R value and *vice versa*.)

Mechanics and forces

Electric force $\quad q_1 \qquad\qquad q_2 \rightarrow \hat{\mathbf{r}}$

$$\mathbf{F}_{elec} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

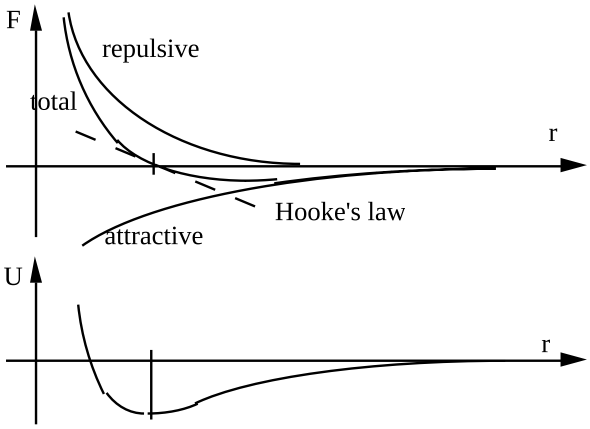
van der Waals force (electrodynamic force)

$p_1 \downarrow \qquad \uparrow p_2$

$$F_{vdw} \propto \frac{1}{r^6}$$ always attractive

Properties of condensed phases

Inter-atomic & intermolecular forces amd energies



$\rightarrow$ **Linear elasticity** (*parabolic minimum in U(r)*)

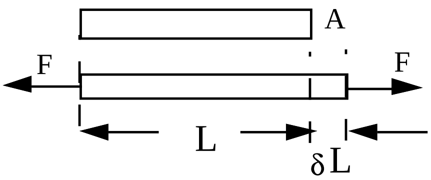
Linear approximation to inter-molecular forces

Stress $\quad \sigma \equiv F/A$

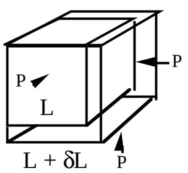
Strain $\quad \varepsilon \equiv$ dimensionless change, e.g. $\frac{\delta L}{L}$

Hooke's Law: $\quad \frac{\sigma}{\varepsilon} =$ **elastic modulus**

Longitudinal stress:

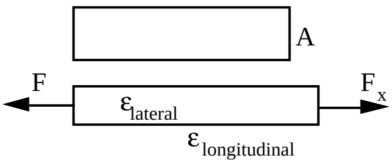


Young's modulus $\quad Y = \frac{F/A}{\delta L/L} = \frac{FL}{A\delta L}$



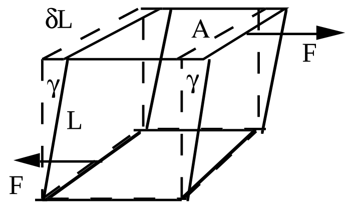
Bulk modulus

$$\kappa = -\frac{P}{\delta V/V}$$



Poisson's ratio $\quad \nu \equiv -\frac{\varepsilon_{lat.}}{\varepsilon_{long.}} = -\frac{\delta d/d}{\delta L/L}$

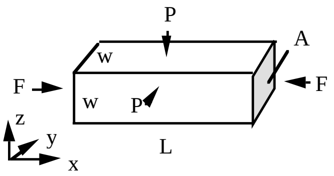
Rigidity modulus (shear modulus)



Rigidity modulus

$$G = \frac{F/A}{\delta L/L} = \frac{\tau}{\gamma}$$

Example



F applied to the ends. P applied to other 4 sides.
What is P so that cross section A is unchanged?

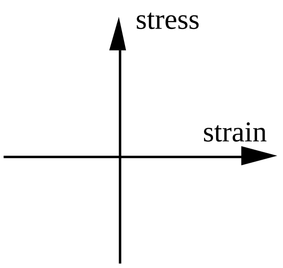
$$\varepsilon_y = -\nu\sigma_x/Y + \sigma_y/Y - \nu\sigma_z/Y$$

$$0 = -\nu F/YA - P/Y + \nu P/Y$$

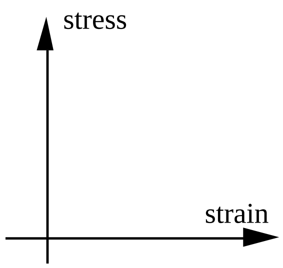
$$\nu F/A = P(1-\nu)$$

$$P = \frac{F}{A}\frac{\nu}{(1-\nu)}$$

Hysteresis



Other non-elasticity



Inter-atomic & intermolecular forces

Ionic solids: electrostatic force

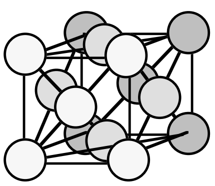
Hydrogen bonds H has $+\delta$ charge

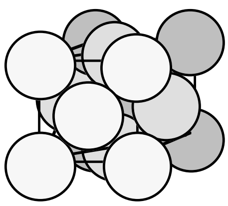
van der Waals attraction attraction between transient dipoles $\propto r^{-6}$ at short range

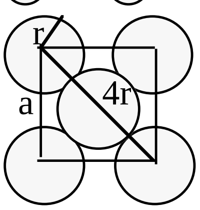
Crystalline solids

Packing factor fraction of space occupied by touching hard spheres

Example Calculate packing factor and ρ of FCC







Face diagonal = 4r
Side of unit cube = 4r/sin 45°
$= 2\sqrt{2}r$

$$\text{p.f.} \equiv \frac{\text{Volume of spheres in unit cube}}{\text{Volume of unit cube}}$$

$$= \frac{\text{8 corners + 6 faces}}{(2\sqrt{2}r)^3}$$

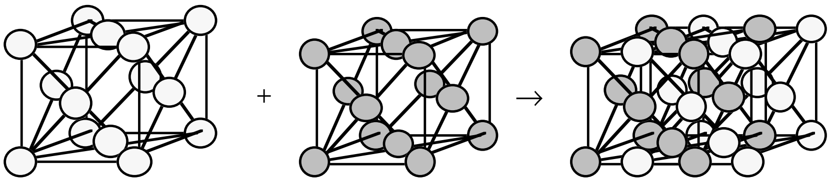
$$= \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2})\ \frac{4}{3}\pi r^3}{(2\sqrt{2}r)^3} = 74\%$$

$$\rho = \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2})\ \text{atomic mass}}{a^3} = \frac{4m}{a^3}$$

Ionic crystals

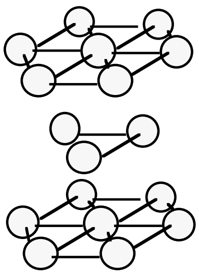
e.g. NaCl: ions of similar size, each ion has six neighbours of opposite charge (**coordination number** six)

(It's like two interlaced FCCs)



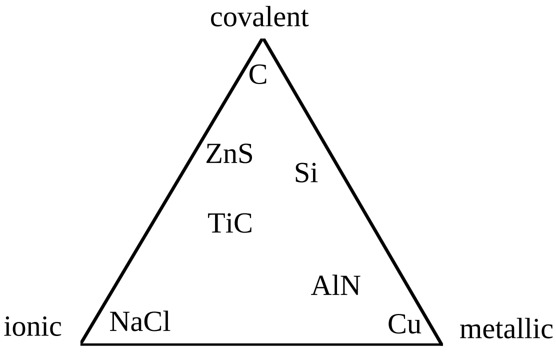
Covalent crystals: share outer electrons. Depends on the angles of the electron orbitals

Metallic crystal: 'ions' in a sea of shared electrons. Often close packed in FCC or HCP. This, gives high ρ, especially if atomic number is large (e.g. Au, Pt)



Hexagonal close packing
(*vertical axis expanded here*)

Intermediate bond types:



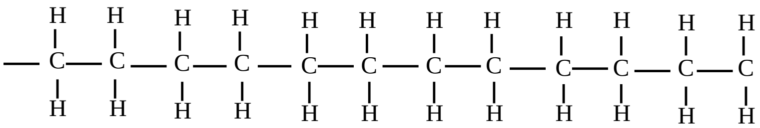
Amorphous solids (a.k.a. glass, vitreous phase)

Metallic glass. Cool the metal very quickly, e.g. small drops in liquid N_2.

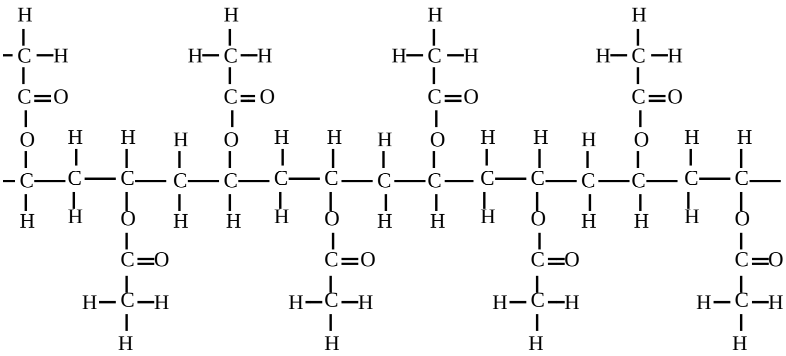
Polymers

Long chains repeating one unit e.g. poly(ethylene) PE

(what sort of hydrocarbon is this?)



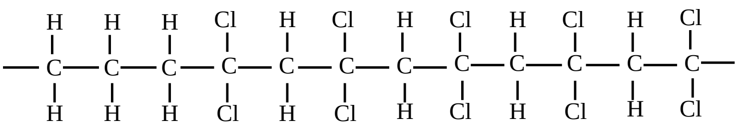
poly(vinyl acetate) PVA



Long *flexible* chains: usually tangle rather than crystallise, especially if they have side groups. Attractive force is vdW (and tangling)

Amorphous (partly) **Crystalline**

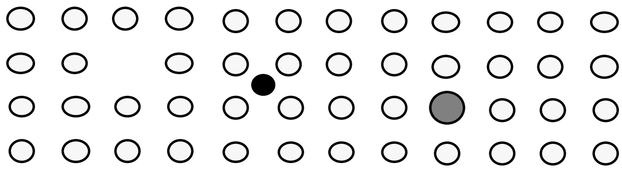
Crystalline polymers are only partly crystallised. Need uniform cross section, e.g. poly(vinylidene chloride)



Cross linking. Chemical bonds rigidify 3D structure. e.g. resins, vulcanisation in rubber (S bonds).

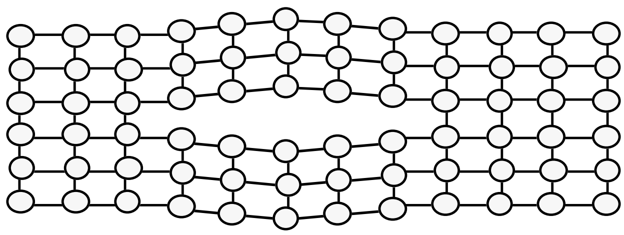
Point defect

vacancy, interstice, substitution

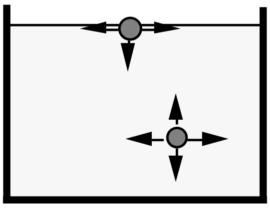


Line defects Edge and screw

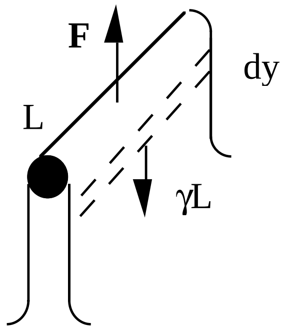
Plane defects, espec. microcracks



Surface tension and surface energy



molecule in bulk is uniformly attracted in each direction. Molecule at surface has ~ no attraction to atmosphere, ∴ work done against the nett force in order to make a surface.



Work to make new surface is done against

surface tension γ.

$\gamma \equiv$ force per unit length in the plane of the surface

e.g. raise wire dy

$F = 2\gamma L$ (*two sides*)

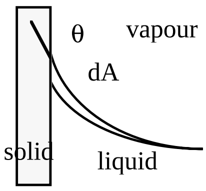
Work $\quad dW = F.dy = 2\gamma L.dy$

$\quad dA = 2Ldy \quad$ (*two sides*)

$\therefore \quad \gamma = \dfrac{dW}{dA}$

So surface tension = surface free energy per unit area

Contact angles and menisci



The surface energy of solid-vapour is S_S,
Solid-liquid is S_{LS}

Surface tension of liquid is γ

Contact angle θ

Suppose that the meniscus rises as shown. Work done by tension

$$dW = -\gamma L dx \cos\theta = -\gamma dA \cos\theta$$

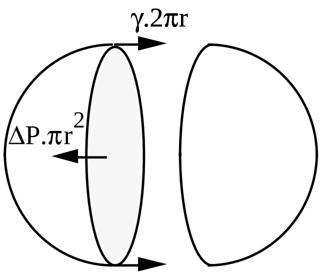
$$= \text{new solid-liquid energy} - \text{solid-vapour energy}$$

$$= S_{LS}dA - S_S dA$$

$$\gamma\cos\theta = S_S - S_{LS}$$

Some surface free energies

Class	Material	$\dfrac{S}{mJ.m^{-2}} \left(= \dfrac{\gamma}{mN.m^{-1}}\right)$
Liquid	water	0.073
	Hg	0.051
Glass	SiO_2	4.4
Ionic	NaCl	0.5
solid	KCl	0.11
Mica	in air	0.38
	in vacuum	5
Covalent	Al_2O_3 (sapphire)	6-32
solid	C (diamond)	5.24
Metal	Zn	0.105
	Pb	0.763
	Si	1.24
Polycrystalline	SiC	32
	Graphite	68
	Granite	200
	Cast Iron	1,520



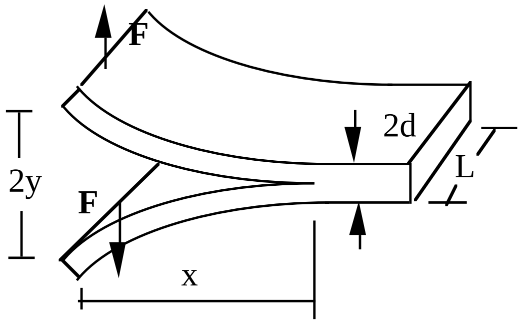
Pressure excess inside balanced by surface tension

$\Delta P.\pi r^2 = \gamma.2\pi r$

$\Delta P = \gamma \dfrac{2}{r}$

Young-Laplace Equation

Surface energy of solids



Complicated by mechanical strength of materials. Work to cleave $= U_{surf} + U_{bend}$.

Put in expression for deflection of cantilever spring:

$$\frac{F}{L} = \frac{3Yd^3y^2}{8x^4}$$

Diverse comments about various types of materials and behaviour

Be careful talking of 'strength of materials'

Sometimes you want high E ($\therefore$ small e)

Other times you want high σ_{max}

Yet other times you want low E ($\therefore$ large e)

Macroscopic σ_{max} usually << microscopic σ_{max}

Composite materials aim to minimise the difference by limiting propagation of dislocations

In composites, $\sigma_{micro} \neq \sigma_{macro}$

Ductility refers to the ease of plastic deformation without rupture.

Sometimes 'good', sometimes 'bad'

Work hardening: 'dislocation tangle'. Dislocation stops at slip plane,

fewer moveable dislocations $\therefore$ less ductility

Ductile rupture: break bonds, eventually form macroscopic rupture

For some materials, mechanical properties may depend on time scale (rheology)

For some materials, mechanical properties depend on temperature. Plastic deformation is easier at high T, brittle fracture is more likely at low T

Material properties may also be changed by

Chemical reactions

Radiation (UV or even light from some polymers, γ and X for other materials)

Hydration (for hydrophilic, fibrous materials)