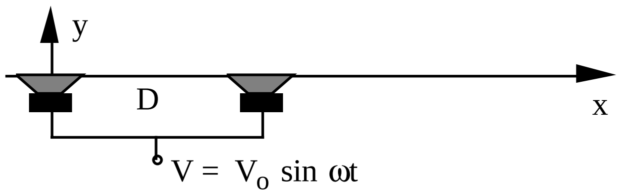
Extra problems for PHYS 1169. Joe Wolfe, UNSW

Example. Sinusoidal signal drives two identical loudspeakers a distance D apart, away from any reflecting surfaces. How does the sound pressure vary along the line between them? Is there any region where one would get a standing wave?



Consider only the range between speakers (0<x<D).

Remember the formulæ for waves travelling l & r?

from left speaker (L) $p_L = A_L \sin(kx - \omega t)$

from right $p_R = A_R \sin(k(x-D) + \omega t)$

where we chose the t to set $\phi = 0$.

but A = A(x). How does amplitude vary with x?

$$I \propto p^2 \text{ and } I \propto \frac{1}{r^2} \qquad \text{so} \qquad p \propto \frac{1}{r}$$

$$p_L = \frac{C_{ext}}{x} \sin(kx - \omega t) \qquad\qquad p_R = \frac{C_{ext}}{x - D} \sin(k(x-D) + \omega t)$$

Where C_{ext} is the constant 'extrapolated' to x = 0. Of course the speaker has finite size, so one never measures A_{ext}. Near the middle point, $A_L(x) \cong A_R(x) \rightarrow$ so we could get a standing wave, although we mightn't notice unless D>>λ. But not near either extreme.

Indeed, very near left speaker, $p_{total} \cong p_L$

(A classic) example A wave travels in a stretched string. Derive an expression for the ratio of the speed of the *string* to the slope of the string at any point.

Wave in +ve x direction: $y = A \sin(kx - \omega t)$ where k = wave number $\equiv \frac{2\pi}{\lambda}$ $\quad \omega = 2\pi f$

Slope of string $= \frac{\partial y}{\partial x}$ *(note the partical derivative: t const, in other words a 'snapshot')*

$$= Ak \cos(kx - \omega t)$$

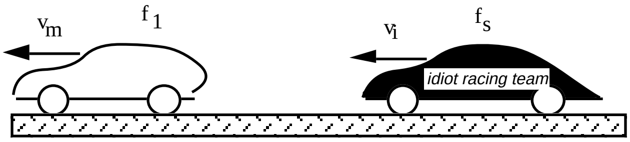
Speed **of a particle** in the string $= \frac{\partial y}{\partial t}$ *(partical derivative: x const. What is happening at* ***a particular*** *position— in fact, it is SHM)*

$$= -A\omega \cos(kx - \omega t)$$

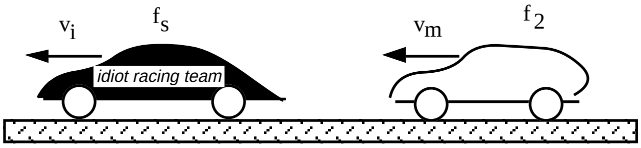
$$\frac{\text{speed}}{\text{slope}} = \frac{-A\omega \cos(kx - \omega t)}{Ak \cos(kx - \omega t)} = -2\pi f.\frac{\lambda}{2\pi} = -f\lambda = -v$$

(which is an elegant result.)

Example While travelling at steady 110 km.hr^{-1} on the freeway, I am overtaken, and hear the other's engine tone fall by a major third (ratio 1.25) as it overtakes. How fast was idiot going?



$$f_1 = f_s\frac{v + v_o}{v - v_s} = f_s\frac{v - v_m}{v - v_i}$$



$$f_2 = f_s\frac{v + v_o}{v - v_s} = f_s\frac{v + v_m}{v + v_i}$$

Diagrams really help!

The only difficulty in this problem is in applying the sign convention, so look carefully and check you understand. +ve velocities indicate the direction of approach.

Now apply the observation:

$$\frac{5}{4} = \frac{f_1}{f_2} = \frac{v - v_m}{v - v_i}\ \frac{v + v_i}{v + v_m} \qquad \text{so}$$

$$\frac{5}{4}(v^2 + vv_m - vv_i - v_iv_m)$$

$$= v^2 + vv_i - vv_m - v_iv_m$$

$$v^2 + 9vv_m = 9vv_i + v_iv_m$$

This looks like a quadratic, but we want v_i: the other terms are constant and given. So it's a linear equation and rearrangment gives

$$v_i = \frac{v + 9v_m}{9v + v_m}\, v$$

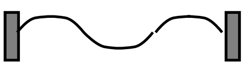
Usually it's best to work in SI units
but here we have some numbers in kph and want the answer in kph

$$v = 340\ \text{ms}^{-1} = 340*\frac{3600}{1000}\ \text{kph} = 1224\ \text{kph}$$

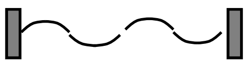
substitute gives $v_i = 240$ kph

Example. A guitarist tunes the A string of her guitar to 110 Hz. She then wants to tune the E string to $\frac{3}{4}$ 110 = 82.5 Hz. How to do this?

A string:

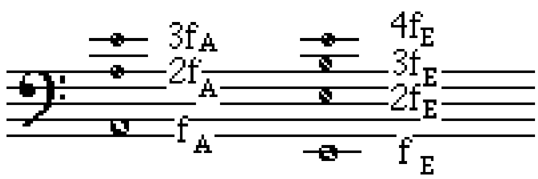


$$f_{3A} = \frac{v}{\lambda} = \frac{3\,v}{2L} = 3f_A$$ (3rd harmonic on A string)



$$f_{4E} = \frac{v}{\lambda} = \frac{4\,v}{2L} = 4f_E$$ (4th harmonic on E string)

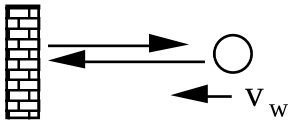
When $3f_A = 4f_E$, $\frac{f_E}{f_A} = \frac{3}{4}$



So get the strings approx in tune
and then **remove the beats**

For guitarists: remember that guitar music is transposed up an octave. This tuning method in general requires modification, depending on the temperament you play. www.phys.unsw.edu.au/~jw/strings.html

Example. You walk towards a wall, blowing a whistle at f = 500 Hz. You hear beats at 5 Hz between your whistle and the reflected sound. How fast are you walking?



Again, a problem to break into small parts. Do not try to 'plug and chug' into some equation. You must think about what the terms mean. Here the sound is emitted and then reflected. We didn't analyse such a problem when deriving the formula, so we need to consider the stages separately.

You hear your own whistle at frequency f, because your lips and ears are attached to the same head.

The wall receives a frequency that we'll call $f' = f\dfrac{v + v_o}{v - v_s} = f\dfrac{v + 0}{v - v_w}$

This is the source of the reflection. But you are moving w.r.t the wall, so you hear a new frequency, which we shall call $f'' = f'\dfrac{v + v_o}{v - v_s} = f'\dfrac{v + v_w}{v - 0}$

From here it's just algebra: substitute to get $f'' = f\dfrac{v + v_w}{v - v_w}$

$$f''(v - v_w) = f\,(v + v_w)$$

$$(f'' - f)v = (f'' + f)v_w$$

$$v_w = v\frac{f'' - f}{f'' + f} = v\,\frac{5}{1005} = 1.7\ \mathrm{ms^{-1}}$$