

## 1169T2/2004

**PHYS 1169 Test 2, 2004****Question 1 (19 marks)**

- a) i) A sinusoidal wave travelling along a stretched string in the positive  $x$  direction has speed  $v$ , frequency  $f$  and amplitude  $A$ . Write an equation for such a wave, in terms of these quantities.
- ii) For the wave in part (i), derive an expression for the velocity of a particle at  $x = 0$  as a function of time.
- iii) For the wave in part (i), derive an expression for the slope of the string at  $x = 0$ , as a function of time.
- b) A steel guitar string has a diameter of 0.33 mm, a vibrating length of 650 mm and a density of  $5,600 \text{ kg}\cdot\text{m}^{-3}$ .
- i) Calculate the tension required to tune it so that its fundamental frequency of vibration is 330 Hz.
- ii) State two of the other frequencies at which the string can vibrate easily, and illustrate the corresponding modes of vibration with a sketch.

**Question 1** (19 marks)

a) i)  $y = A \sin(kx - \omega t + \phi) = A \sin(2\pi(x/\lambda - ft + \phi))$  (derivation and  $\phi$  not needed)

$$= A \sin(2\pi f(x/v - t) + \phi) \quad \text{OR} \quad = A \sin\left(\frac{2\pi f}{v}(x - vt) + \phi\right) \quad (2 \text{ marks})$$

ii)  $v_y = \frac{\partial y}{\partial t}$

$$= -2\pi f A \cos(-2\pi ft + \phi)$$

(May be written  $-2\pi f A \cos(2\pi ft + \phi)$ ,  $\phi$  not needed) (3 marks)

iii) slope =  $\frac{\partial y}{\partial x}$

$$= \frac{2\pi f}{v} A \cos(-2\pi ft + \phi)$$

(May be written  $\frac{2\pi f}{v} A \cos(2\pi ft + \phi)$ ,  $\phi$  not needed) (3 marks)

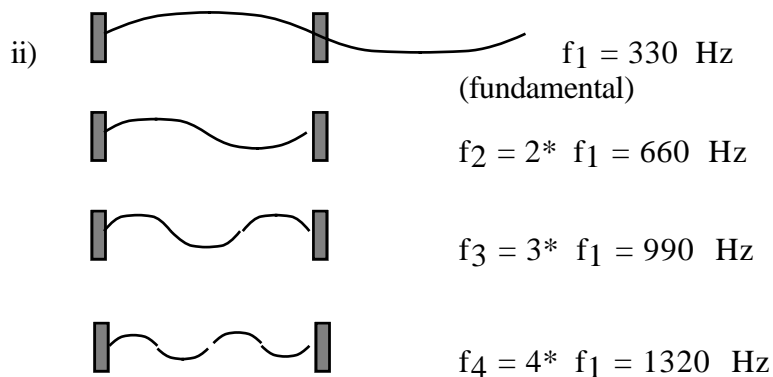
- b) i) For an ideal stretched string, fixed at the ends, the longest wavelength standing wave (the fundamental) has

$$\lambda = 2L.$$

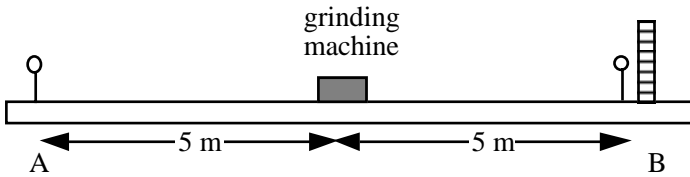
$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad \therefore T = (2Lf_1)^2 \mu$$

$$\mu = \frac{m}{L} = \frac{\pi r^2 L \cdot \rho}{L} = \rho \pi (d/2)^2.$$

$$T = (2Lf_1)^2 \rho \pi (d/2)^2 = \rho \pi (dLf_1)^2 = 88 \text{ N} \quad (7 \text{ marks})$$



(Any three harmonics (including fundamental) will suffice to get the marks) (4 marks)

**Question 2** (11 marks)

a) A grinding machine is placed on the ground. Its sound output is  $P = 11 \text{ W}$  and it radiates isotropically. The ground may be considered as a perfect reflector of sound. Two people are standing at distances of 5 m from the machine. Person A is standing on clear ground, person B is standing very close to a brick wall.

- Showing your working, estimate the sound level to which A is exposed. (Reference level  $1 \text{ pW}\cdot\text{m}^{-2}$ )
  - What is the sound level to which B is exposed? Explain your answer.
  - Person B has hearing protectors that are rated at 26 dB insulation. Calculate the factor by which these protectors reduce the sound pressure?
  - Which would be louder: sound with intensity  $1 \text{ mW}\cdot\text{m}^{-2}$  at 50 Hz or sound with intensity  $1 \text{ mW}\cdot\text{m}^{-2}$  at 1 kHz? Explain your answer in one sentence.
- b) The speed of sound in water is faster than the speed of sound in air. In one or two sentences, and with one equation, explain why this is so.

**Question 2** (11 marks)

- Because the ground reflects, the sound is radiated through a hemisphere, so  

$$I = P/2\pi R^2 \quad (2)$$

$$L_I \equiv 10 \log I/I_0 = 10 \log \frac{P}{2\pi R^2 I_0} = 109 \text{ dB} \quad (2)$$
  - (For the frequencies important to human hearing) the brick wall is a good reflector, so the sound intensity will be approximately 3 dB higher. So  $L = 112 \text{ dB}$ . (2)  
 (Anyone how does the more subtle calculation: reflection doubles  $p$ , so 4 times the intensity at a max and zero at a minimum therefore average of double intensity gets marks as well.)
  - $L_p \equiv 20 \log p/p_0 \therefore$  (this line not need for marks)  

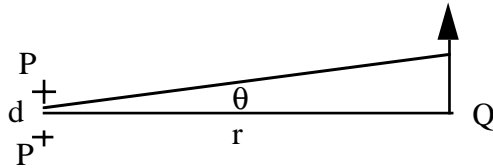
$$\Delta L = 20 \log p_2/p_1$$

$$10\Delta L/20 = p_2/p_1$$

$$p_2/p_1 = 20 \quad (2)$$
  - The sound at 1 kHz would be louder as this is in the range over which the ear is most sensitive. (OR, the ear is not very sensitive at 50 Hz.) (1)
- b)  $v = \sqrt{\frac{\text{elastic modulus}}{\text{density}}}$ . Although the density of water is much higher than that of air (about  $1000 \text{ kg}\cdot\text{m}^{-3}$ /  $1.3 \text{ kg}\cdot\text{m}^{-3}$ ), the modulus of elasticity of water exceeds that of air by an even larger factor (about 2 GPa/140 kPa). (values not required in answer) (2)

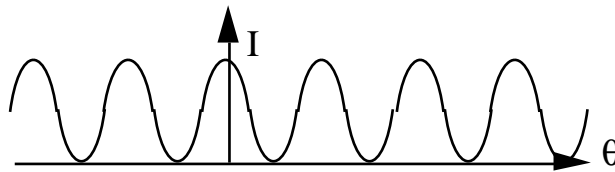
**Question 3 (15 marks)**

a)



Two radio antennæ, separated by distance  $d$ , radiate the same signal, with the wavelength  $\lambda$  ( $\ll d$ ) and same power  $P$ . They are in phase. At a distance  $r$  away ( $r \gg d$ ), the intensity  $I$  is measured as a function of  $\theta$ .

- i) Sketch the dependence of the measured intensity as a function of  $\theta$ , i.e.  $I(\theta)$ .
  - ii) At what value of  $\theta$  does the first minimum occur?
  - iii) When one of the antennæ is turned off, what is the change in the value of the intensity  $I$  at the point  $Q$  (i.e. at  $\theta = 0$ )? Explain how you reached your answer.
- b)
- i) With the aid of a diagram, explain how an anti-reflective coating on a lens works.
  - ii) Estimate the thickness of the coating if the coating material has a refractive index of 1.4 and the antireflective property is optimised for light of wavelength  $\lambda = 550$  nm in air.

**Question 3 (15 marks)**

3a i)

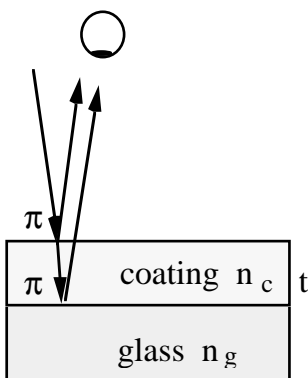
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ii) 1st minimum  $\frac{d \sin \theta}{\lambda} = \frac{1}{2} \quad \therefore \theta = \sin^{-1} \frac{\lambda}{2d} \quad \left( \cong \frac{\lambda}{2d} \right)$  2

OR minima at  $d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \quad \therefore \theta = \sin^{-1} \frac{\lambda}{2d} \quad \left( \cong \frac{\lambda}{2d} \right)$

- iii) For one antenna,  $I = KA^2$  where  $A$  is amplitude on axis, the signals are in phase so the amplitudes add,  
 $I_T = K(2A)^2 = 4I$ .  
 Intensity is reduced by a factor of 4. 3

b)



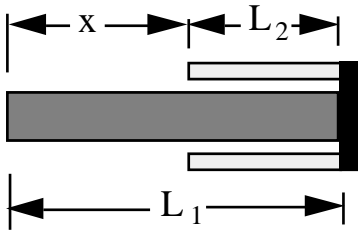
If the film has a value of refractive index intermediate between that of the glass and that of the external medium, then there will be a phase change of  $\pi$  at the two reflections shown. For normal incidence, these two rays will have a phase difference of  $\pi$  if

$$\text{optical pathlength difference} = \lambda/2$$

$$2tn = \lambda/2 \quad \therefore \quad t = \lambda/4n = 100 \text{ nm} \quad 6$$

**Question 4** (10 marks)

a)



The figure shows a low expansion mounting, designed so that the distance  $x$  has minimal change with temperature. The inner and outer shafts are made of materials with linear thermal expansivity coefficients  $\alpha_1$  and  $\alpha_2$ , respectively.

Derive an equation that relates the two lengths  $L_1$  and  $L_2$  to the coefficients  $\alpha_1$  and  $\alpha_2$ .

b) Using the equation of state for an ideal gas, derive an expression for the density  $\rho$  of an ideal gas in terms of the pressure  $P$ , temperature  $T$ , molecular mass  $m$  and constants.

**Question 4** ( marks)

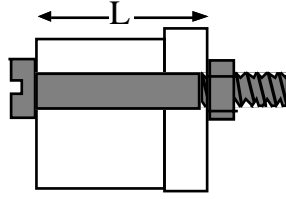
a)  $x = L_1 - L_2 = L_{10}(1 + \alpha_1\Delta T) - L_{20}(1 + \alpha_2\Delta T) = L_{10} - L_{20} + L_{10}\alpha_1\Delta T - L_{20}\alpha_2\Delta T$   
 $x = x_0 + (L_{10}\alpha_1 - L_{20}\alpha_2)\Delta T$  If  $x = x_0$  independent of  $T$ , then second term is zero so  
 $(L_{10}\alpha_1 - L_{20}\alpha_2) = 0$  so  $L_{10}/L_{20} = \alpha_2/\alpha_1$  or  $L_1/L_2 = \alpha_2/\alpha_1$  (6)

b)  $PV = nRT$        $\rho \equiv \frac{\text{mass}}{\text{volume}} = \frac{nN_A m}{V} = \frac{PN_A m}{RT}$  (4)

**Question 5** (17 marks)

- a) A bolt has a diameter of 6.0 mm and is made of steel, for which the Young's modulus is = 206 GPa. Its thread has 12 turns per cm. It is passed through a hole drilled at right angles through two large, prism shaped metal components. The hole is 120 mm long. A nut is put on the bolt and turned until all of the components just make contact, with minimal force, as shown in the sketch. The nut is then tightened one half turn. Determine the normal force exerted between the two components as the result of tightening this one nut and bolt. (Neglect deformation of the bolt head, the nut, or the prism shaped objects.)
- b) The attractive and repulsive forces between atoms and molecules are in general rather complicated functions of the inter-atomic or inter-molecular spacing. Nevertheless, Hooke's law of elasticity works well for many such materials.
- What limitation on the strain is necessary for Hooke's law to be true?
  - Subject to this limitation, and *with the aid of a clearly labelled sketch*, explain briefly how Hooke's law arises.

**Question 5** (17 marks)



a)

$$Y \equiv \frac{\sigma}{e} = \frac{\text{stress}}{\text{strain}}$$

e here is  $\frac{\delta L}{L} = \frac{\frac{1}{2} \frac{1}{12} \text{ cm}}{120 \text{ mm}} = \frac{1}{288}$

$$\sigma = F/A = F/\pi r^2$$

$$F = \pi r^2 \sigma$$

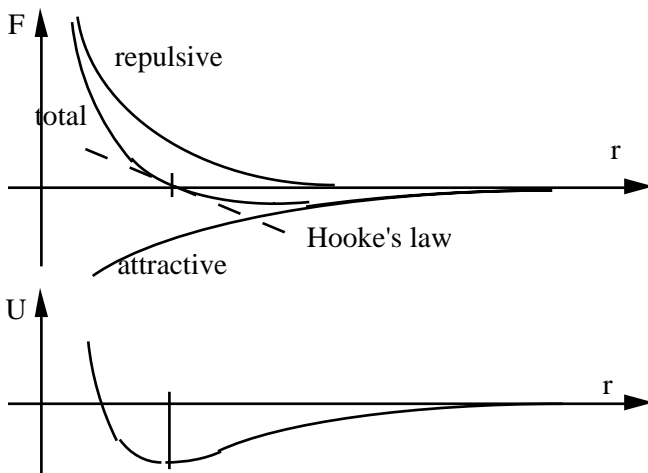
$$= \pi \left(\frac{d}{2}\right)^2 Y e$$

$$= \pi \left(\frac{6 \cdot 10^{-3} \text{ m}}{2}\right)^2 206 \text{ GPa} \frac{1}{288}$$

$$= 20 \text{ kN} \quad (7)$$

(6 for curves)

b)



i) Hooke's law only works for strains  $\ll 1$ . (1)

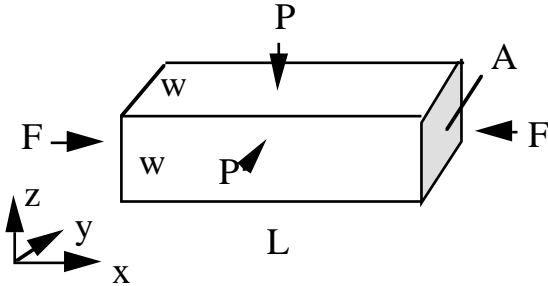
ii) The attractive and repulsive forces are both curvilinear functions of intermolecular separation  $r$ , as shown. The mechanical equilibrium of the material, at zero stress, requires that the attractive and repulsive forces add to zero.

For  $e \ll 1$ , the resultant curve may be linearised. (3)

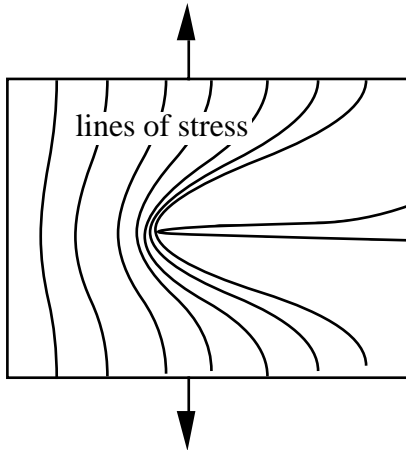
(Alternatively, the energy curve, the integral of the above, has a locally parabolic minimum for  $e \ll 1$ .)

**Question 6** (8 marks)

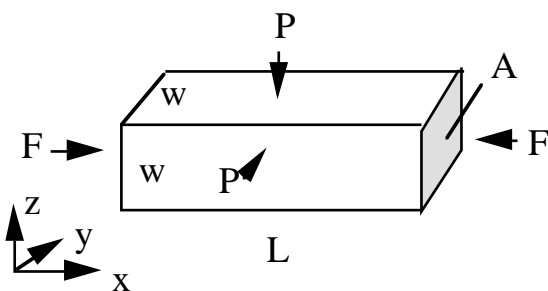
- i) Why does a crack propagate under tensile strength? (A sentence or two and a sketch.)
- ii) A prism shaped block, length  $L$ , and square cross section with side  $w$ , is loaded on the square faces with a compressive force  $F$ . In terms of the geometry and that elastic constants of the material, derive an expression for the pressure  $P$  that must be applied on the rectangular sides of the material so that the area of the square cross section is unchanged when the load  $F$  and the pressure  $P$  on the four sides are applied simultaneously.

**Question 6** (8 marks)

- i) Lines of tensile stress are concentrated near the end of the crack, producing a tensile strength that locally may exceed the yield stress of the material. (4 including sketch)



ii)



From the definitions of Young's modulus  $Y$  and Poisson's ratio  $\nu$ , the lateral strain is:

$$\epsilon_y = -\nu\sigma_x/Y + \sigma_y/Y - \nu\sigma_z/Y$$

If the lateral strain is zero, then

$$0 = -\nu F/YA - P/Y + \nu P/Y$$

$$\nu F/A = P(1 - \nu)$$



$$P = \frac{F}{A} \frac{v}{(1 - v)} \quad (4)$$