1969 T2 2003

Question 1 (15 marks)

a) Two surfers, Barbara and Marilyn, are floating in deep water. Each observes that her vertical motion is sinusoidal and the total displacement from bottom to top during a cycle is 2.4 m. The wave speed is 10 ms^{-1}. Marilyn observes that she gets to the top just as Barbara gets to the bottom. They observe that they experience *maximum* accelerations of 0.015 g.

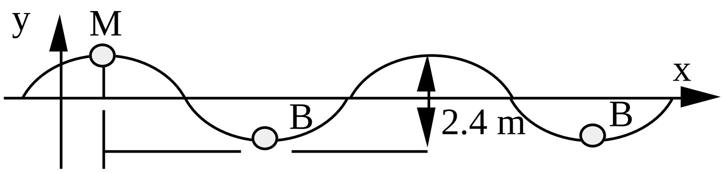
i) Draw a graph of vertical position (y) as a function of displacement (x) in the direction of travel of the wave. On this graph, locate possible positions for Barbara and Marilyn.

ii) Showing your working, calculate two possible values for the horizontal separation between Barbara and Marilyn.

b) The 'solar constant'—a quantity of great interest in solar energy, meteorology, thermal engineering, ecology etc—is the intensity of light from the sun, measured above the Earth's atmosphere, at right angles to a line towards the sun. It has a value of 1.38 kWm^{-2}. The Earth is 150 million km from sun. From these data, compute the rate at which the sun is producing energy in the form of light.

Question 1

a) i)



or any variation, provided that they are $(m+1/2)\lambda$ apart.

Equation for a travelling wave:

$$y = y_m \sin\left(kx - \omega t\right)$$

$$a = \frac{\partial^2 y}{\partial t^2} = -y_m \omega^2 \sin\left(kx - \omega t\right)$$

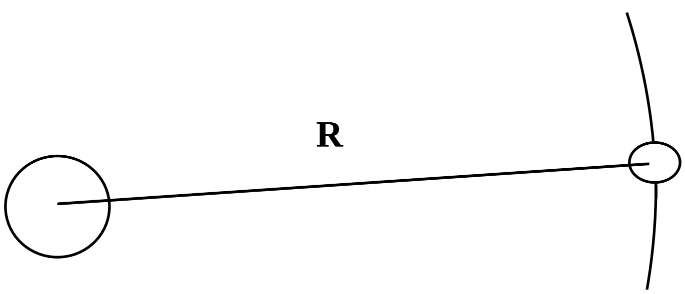
so $a_{max} = y_m \omega^2$

$$.015\ g = 0.015*9.8\ m.s^{-2} = 0.15\ ms^{-2} = \frac{2.4\ m}{2}\ \omega^2$$

$$\omega = 0.35\ s^{-1} \rightarrow f = 0.056\ Hz$$

$$\lambda = \frac{v}{f} = 180\ m.$$

They are 90 or 270 0r 450 ...m apart *any two correct score.*



b)

$$I = \frac{P}{4\pi r^2} = 1.38\ kWm^{-2}$$

$$P = 1.38\ kWm^{-2}\ 4\pi r^2 = 3.9\ 10^{26}\ W.$$

Question 2 (15 marks)

a) A badly designed cooling system of an industrial plant is mounted on the ground (which you may treat as an infinitely large reflecting surface). It radiates an acoustic power of 10 W isotropically into the air.

i) If unprotected workers are not to be exposed to a continuous sound intensity level exceeding 90 dB, (with respect to I_o = 1 pW.m^{-2}) how close may they safely stay near the noise source?

ii) The workers are supplied with hearing protectors giving an insulation of 26 dB (ie the sound level inside the ear enclosures is at least 26 dB lower than that outside). Wearing their protectors, how close to the noise source may the workers safely stay?

b) Two exhaust fans turn at 1000 revolutions per minute. One (fan a) has 3 blades in the fan, the other (fan b) has 30 blades. The individual blades on the second fan are smaller than those on the first, with the result that both fan emit emit exactly the same acoustic power.

i) What fundamental frequencies do they emit? (Hint: how many times per second do fan blades pass a given point?)

ii) At the same distance, which one is louder? Explain your answer in one or two clear sentences.

c) In still air, a motorist travels at an unchanging speed of v_m = 110 km.hr^{-1} along the freeway. A police car, its siren turned on, is travelling in the same direction, is first approaching from behind, and then overtakes the motorist. The motorist, who is a musician, hears the police siren fall in pitch as the police car overtakes. From the fall in pitch, he deduces that the frequency he heard before being overtaken (f_1) was 1.20 times higher than that he heard afterwards (f_2). The speed of sound is 340 ms^{-1}. Determine the speed of the police car, km.hr^{-1}. (Hint: you may wish to relate f_1 and f_2 to f_s, the frequency emitted by the police siren. Diagrams may be helpful.)

Question 2 (marks)

a) i) Because the ground is an infinite baffle, sound radiates into half a sphere (2π sterradians), so

$$I = \frac{P}{2\pi r^2} \qquad r^2 = \frac{P}{2\pi I} \qquad r = \sqrt{\frac{P}{2\pi I}}$$

$$L_I \equiv 10 \log_{10} \frac{I}{I_o} \qquad \frac{I}{I_o} = 10^{L_I/10} \qquad I = I_o 10^{L_I/10}$$

Here, $I_{safe} = I_o 10^{L_I/10} = 1\ \mathrm{mW.m^{-2}}$.

$$r = \sqrt{\frac{P}{2\pi I}} = \sqrt{\frac{10\ \mathrm{W}}{2\pi * 1\ \mathrm{mW.m^{-2}}}} = 40\ \mathrm{m}$$

ii) 20 dB $\rightarrow$ factor 10^2 intensity, 6 dB $\rightarrow$ factor 4 intensity,

Intensity may be 400 times higher, $I \propto 1/r^2$, so factor of 20 in distance

$\rightarrow$ r = 2 m

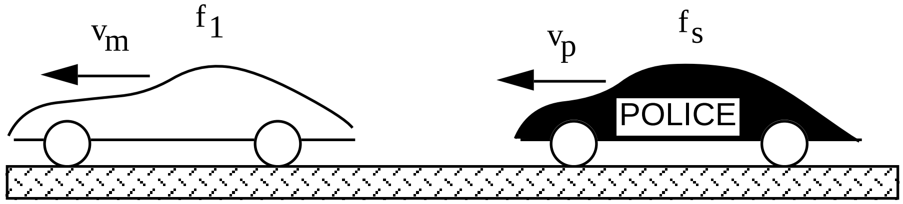
(or by direct calculation:

$$r = \sqrt{\frac{P}{2\pi I'}} = \ldots\ldots.. = 2\ \mathrm{m}$$

b) i) $f_a = 3*1000$ rpm $= 50$ Hz. $f_b = 500$ Hz

ii) (b) is (much) louder. The human ear is much less sensitive at 50 Hz than at 500 Hz. (*'much' not required in answer*) c)

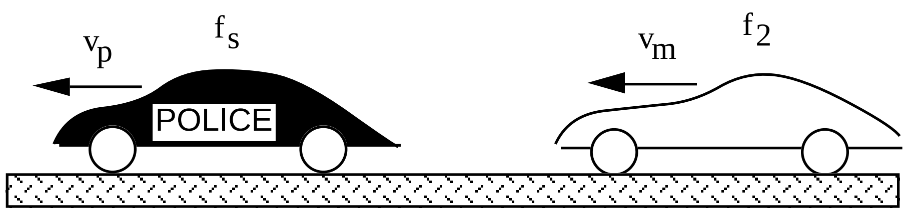
BEFORE



Motorist is moving away from police, police car is moving towards motorist, so

$$f_1 = f_s \frac{v + v_o}{v - v_s} = f_s \frac{v - v_m}{v - v_p}$$

AFTER



Motorist is moving towards police, police car is moving away from motorist, so

$$f_2 = f_s \frac{v + v_o}{v - v_s} = f_s \frac{v + v_m}{v + v_p}$$

$$1.20 = \frac{f_1}{f_2} = \frac{v - v_m}{v - v_p} \frac{v + v_p}{v + v_m}$$

$$1.20(v^2 + vv_m - vv_p - v_pv_m) = v^2 + vv_p - vv_m - v_pv_m$$

$$0.20v^2 + 2.20vv_m - 2.20vv_p - 0.20v_pv_m = 0$$

$$v^2 + 11vv_m - 11vv_p - v_pv_m = 0$$

$$v_p(11v + v_m) = v^2 + 11vv_m$$

$$v_p = \frac{v + 11v_m}{11v + v_m} v$$

$$v = 340 \text{ ms}^{-1} = 340*\frac{3600}{1000} \text{ kph} = 1224 \text{ kph}$$

substitute gives $v_p = 220$ kph

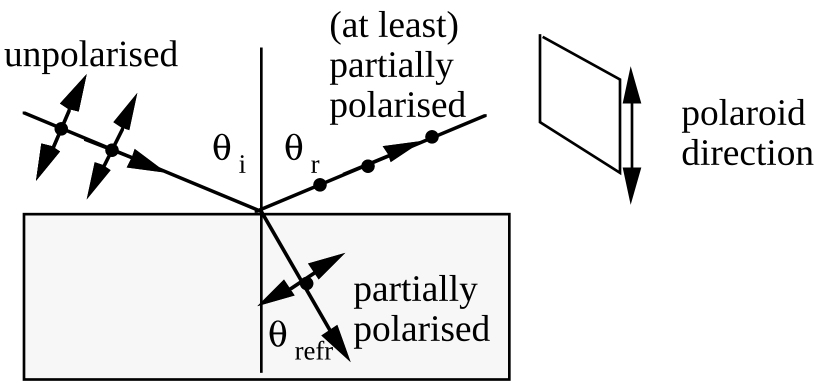
Question 3 (15 marks)

a) Explain, with the aid of a clearly labelled diagram, how polaroid filters (or polaroid sunglasses) reduce the glare from horizontal surfaces. (Several clear sentences will do. Your diagram should clearly show dominant polarisation directions of the light and the appropriate polarisation direction of the filter.)

b) i) A diffraction grating has a spacing between adjacent slits of d. For normally incident light of wavelength λ, intensity is of the diffracted light is a maximum at several angles θ from the normal to the grating. Using a sketch, derive a relation between the spacing d and the angles θ at which diffraction maxima occur.

ii) A laser beam (wavelength 630 nm) is at normal incidence to a grating with 500 lines/mm. The diffraction pattern falls on a wall, 8.0 metres distant. What is the distance on the screen between the position of the primary maximum for light with wavelength 630 nm and the central maximum? Show your working.

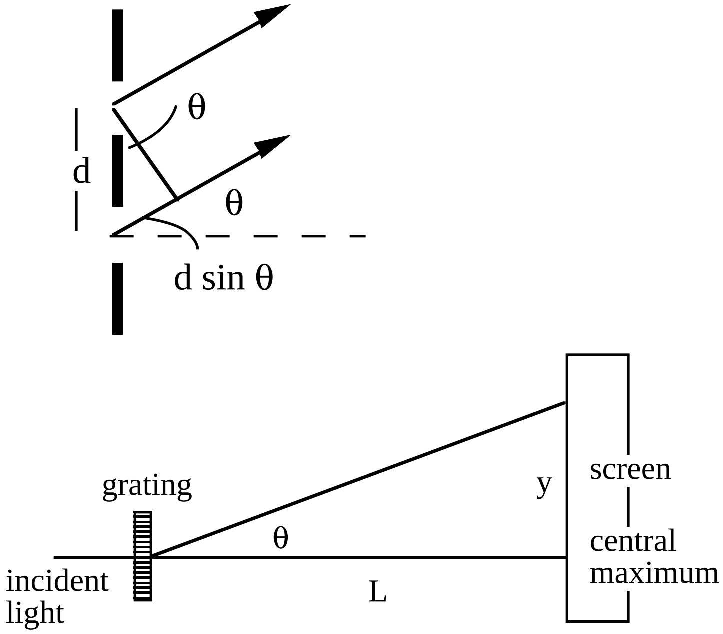
a)



the lower half of the diagram is there for completeness, but is not required by the question

Unpolarised light reflected from a horizontal surface is at least partially polarised in the horizontal direction. A polaroid filter, whose polarisation direction is vertical, will not pass this horizontally polarised component. So the reflected light (glare) from the surface will be largely absorbed.

b)



i) From the right-angled triangle shown, the path difference

$\delta l = d \sin \theta$.

For $\theta = 0$, we get the central bright maximum (no pathlength difference). For other interference maxima, this pathlength must be an integral number of wavelengths, so

$$d \sin \theta = m \lambda$$

where m is an integer.

ii) $\tan \theta = y/L$

$$y = L \tan \theta$$

$$= L \tan \left(\sin^{-1} \frac{\lambda}{d} \right)$$

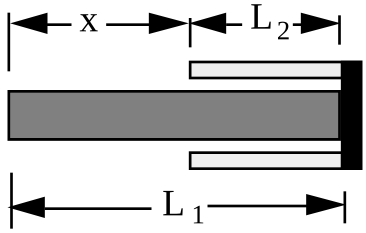
$$= 8 \text{ m} \tan \left(\sin^{-1} \frac{630 \text{ nm}}{2.00\ \mu\text{m}} \right)$$

$$= 2.6 \text{ m}$$

(*small angle approximation gives 2.5 m and loses a mark*)

Question 4 (5 marks)

a)



The figure shows a low expansion mounting, designed so that the distance x has minimal change with temperature. The inner and outer shafts are made of materials with linear thermal expansivity coefficients α_1 and α_2, respectively. Their lengths L_1 and L_2 are shown on the diagram.

For this device to work, what should be the ratio L_1/L_2? (Hint: derive an equation that relates L_1/L_2 to the coefficients α_1 and α_2.

Q4) $x = L_1 - L_2 = L_{1o}(1 + \alpha_1\Delta T) - L_{2o}(1 + \alpha_2\Delta T)$

$= L_{1o} - L_{2o} + L_{1o}\alpha_1\Delta T - L_{2o}\alpha_2\Delta T$

$x = x_o + (L_{1o}\alpha_1 - L_{2o}\alpha_2)\Delta T$

If $x = x_o$ independent of T, then

$(L_{1o}\alpha_1 - L_{2o}\alpha_2) = 0$

so $L_{1o}/L_{2o} = \alpha_2/\alpha_1$ or $L_1/L_2 = \alpha_2/\alpha_1$

Question 5 (17 marks)

a) i) What is mechanical hysteresis? (one sentence will do)

ii) Sketch a stress-strain graph that shows both an elastic response and mechanical hysteresis. Use arrows to indicate how the deformation progresses in time. Indicate on the sketch, or explain in a sentence or two, what part(s) or the sketch represent hysteresis.

iii) State clearly how a part of your graph relates to an elastic modulus.

iv) Continue your graph so that it shows an intercept on the strain axis. State clearly the meaning of this intercept.

b) For a particular type of steel, the Young's modulus Y is 206 GPa and the bulk modulus κ is 181 GPa

i) A solid bridge cable made of this material has a diameter of 10 mm. What weight can be supported by the cable with a 1% increase in length?

ii) What pressure is required to increase the density of steel by 3%?

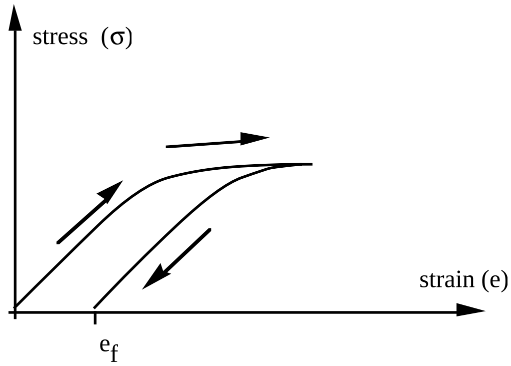
iii) Eccentric inventor Josef Lupus has patented *skweezit*™, an amorphous material with the rare and interesting property that its Poisson's ratio ν has the value 0.9.

Why is this property so rare and interesting? (Hint: what would happen if you applied an isotropic pressure to a sample of *skweezit*™?) Your answer should include at least one or two equations.

(**Caution**: if you have brought a sample of *skweezit* into the exam room in anticipation of this question, do not handle or ingest it.)

Question 5

a) i) Mechanical properties that depend on time. (For example, a return to a different shape after transient application of a stress -- *example not required by the question*)



The relaxation part of the $\sigma(e)$ curve, during which the stress decreases, does not follow the stress application $\sigma(e)$ curve.

iii) The slopes of the two straight line regions equal the elastic modulus for this particular stress.

iv) The intercept e_f is the final strain at zero applied stress. This is a plastic deformation: the material has changed shape after the application and removal of this (sufficiently large) stress. (*any reasonable explanation gets full marks.*)

b) i) $$\frac{F/A}{Y} = \frac{\delta L}{L} = 1\%$$

$F = AY \times 1\% = \pi r^2 Y = 162$ kN (*weight of 16.5 t*)

ii) $$\frac{P}{\kappa} = -\frac{\Delta V}{V} = \frac{\Delta\rho}{\rho} = 3\%$$

$P = 5.4$ Gpa (= *54,000 atmospheres--not required in question*)

iii) For any stress component, $e_{transverse} \equiv -\nu e_{longitudinal}$

Either For small strains, the volume strain due to a single linear stress

$$e_{vol} = -(e_x + e_y + e_z) = -(e_{longitudinal} + 2e_{transverse})$$

$$= -(e_{longitudinal} - 2\nu e_{transverse}) \equiv -(1 - 2\nu)\, e_{longitudinal}$$

For *skweezit*, a compressive longitudinal strain therefore produces positive volume strain: it expands when you squeeze it.

Or In any direction (call it x), an isotropic stress compressive stress P produces a strain

$$e_x = -\frac{P}{Y} + \nu\frac{P}{Y} + \nu\frac{P}{Y} = (2\nu - 1)\frac{P}{Y}$$

If $\nu > 0.5$, a material expands when you squeeze it.

(*This is a potentially dangerous property. Put into a rigid container and heated slightly, it expands, increasing stress and strain indefinitely or until the container ceases to be rigid. Not required by question.*)

Question 6 (8 marks)

a) i) What is an amorphous solid?

ii) How can one form an amorphous solid? (One technique will suffice)

iii) When certain types of amorphous solid are heated substantially, but not enough to produce chemical changes, and then allowed to cool, their mechanical and other properties are greatly changed. Explain in a few clear sentences why this is so.

b) Define surface energy (also called specific surface free energy) ***and*** surface tension.

c) State a relation between surface energy and surface tension that applies to a clean interface between two liquids.

d) Why is surface energy important in the cracking of materials? One or two clear sentences will suffice.

a) i) An amorphous solid has no (large scale) crystalline structure *or* no long range order.

ii) By rapid cooling of the liquid phase (*so rapid that there is insufficient time for crystal formation: crystallisation is hindered by the high viscosity that occurs at sufficiently low temperatures---not required by the question)*

iii) Such an amorphous solid is not in equilibrium. When it is heated, the viscosity falls. When it is *slowly* cooled, there is time for crystals to form and so it becomes a crystalline solid.

b) i) The surface energy is the energy per unit area of new surface required to make that new surface.

The surface tension is the force per unit length acting in a surface *or* required to increase the area of a surface *or* required to stop a surface contracting.

c) They are equal.

d) When a crack propagates, new surface area is created, and work must be done (by the imposed stress) to do this.

(An bonus mark for anyone who says that a liquid, usually water, may condense in the crack, lower the local surface energy and thus facilitate cracking.)