1169T2/2002

Question 1 (marks)

a) i) A wave with amplitude A, wave number k and angular frequency ω is travelling to the left, in the negative x direction. The displacement y due to this wave is zero at position x = 0 and time t = 0. Write an equation that gives the displacement y_1 due to this wave at any time t and position x.

ii) Derive an expression for the particle velocity v(t) at a position x = L, due to the action of this wave alone.

iii) The wave in part (i) is reflected at x = 0 by a rigid boundary condition, ie total displacement y at x = 0 is zero for all time. Write an equation for the displacement y_2 due to the reflected wave.

iv) Determine the points on the positive x axis for which $y = y_1 + y_2$ is always zero, in terms of the variable given in part (i). Explain your answer briefly.

b) A chimney pipe has a cylindrical cross-section, and is 10 m tall. It is closed at the bottom and open at the top.

i) Determine the three lowest frequencies at which the air in the chimney may resonate. The speed of sound in the hot air in the chimney is 440 ms^{-1}.

ii) Draw sketches to represent the resonances of part (i). State whether your sketch represents sound pressure or particle displacement.

Question 1 (marks)

a) i) $y_1 = A \sin (kx + \omega t)$ (*As required,* $y_1(0,0) = A \sin 0 = 0$.)

ii) $$v_{particle} = \frac{\partial y_1}{\partial t} = A\omega \cos (kx + \omega t)$$

$$v_{particle}(L) = \frac{\partial y_1}{\partial t} = A\omega \cos (kL + \omega t)$$

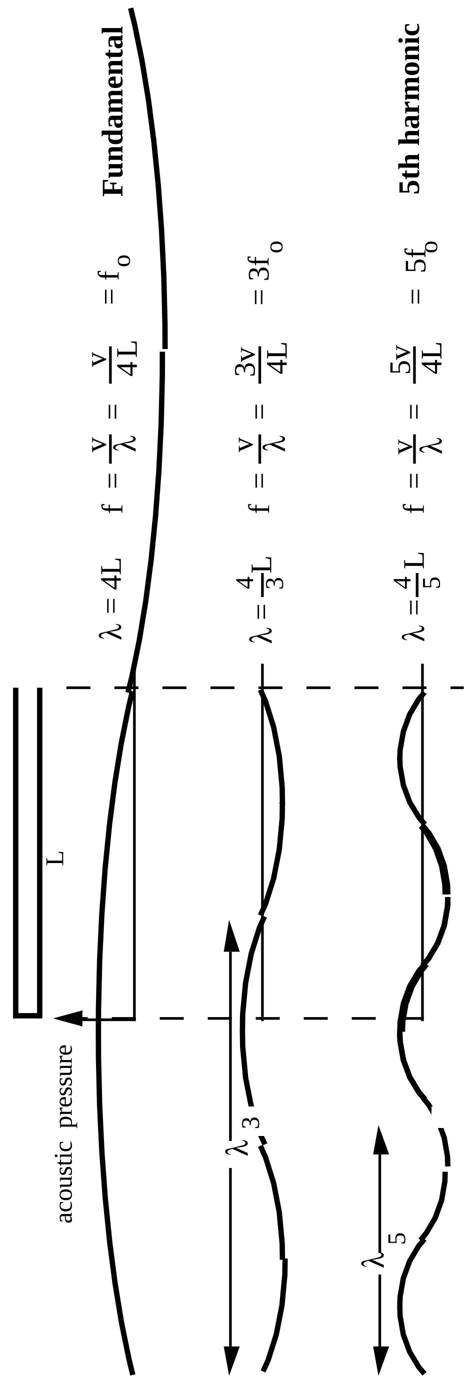
iii) In general for a wave travelling to the right, $y_2 = A \sin (kx - \omega t + \phi)$. We require $y_2(0) = 0$ for all time, so $\phi = 0°$ or more conveniently write:

$y_2 = A \sin (kx - \omega t)$.

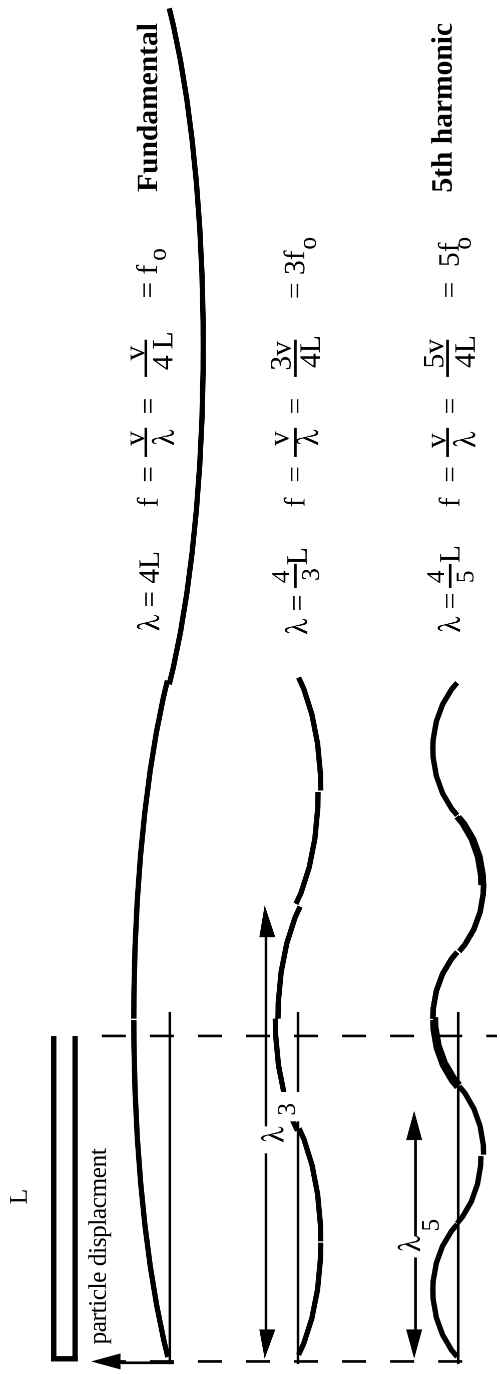
iv) There is a node at x = 0, and nodes occur at every half wavelength, so x = 0, $\lambda/2$, $2\lambda/2$... $n\lambda/2$ ....). But we need λ in terms of ω or k. By definition, $k = 2\pi/\lambda$, so $\lambda = 2\pi/k$.

So y = 0 at $x = 0, \pi/k, 2\pi/k, 3\pi/k, ... n\pi/k$. (n an integer from 0 to ∞)

b) ii) **Either** in terms of pressure



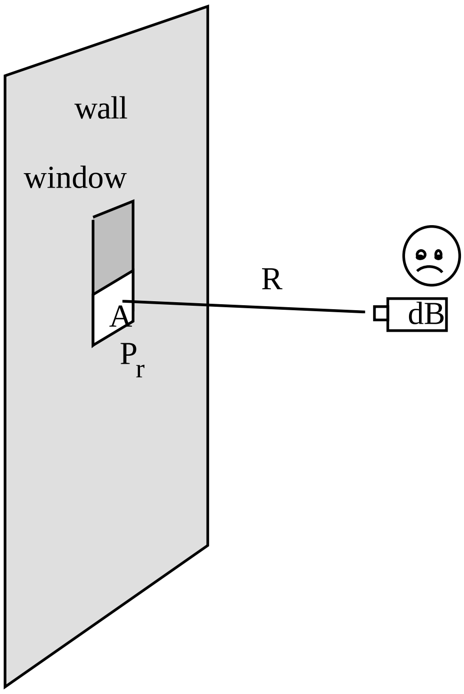
Or in terms of displacement



i) $f_n = \frac{v}{\lambda_n} = v.\frac{n}{4h}$, where n is a positive odd integer

$$f_n = n.\frac{440\ m.s^{-1}}{4*10\ m} = n*11\ Hz = 11\ Hz,\ 33\ Hz,\ 55\ Hz.$$

Question 2 (marks)



During an end-of-exam party, the sound level inside a student's flat, just inside the window, is L_i = 106 dB, with respect to reference intensity I_o = 10^{-12} Wm^{-2}. Because of her passion for rock music, this student has fitted the flat with insulation and double-glazed insulating windows, which may be regarded as perfect sound insulators for the purposes of this question. Hence, the only sound that escapes does so via the open part of the window.

i) During the party, the window is open leaving an area A through which sound escapes into the environment outside. The sound level just inside the window is still L_i = 106 dB, with respect to reference intensity I_o = 10^{-12} Wm^{-2}. Derive an expression for the sound power P_r that is radiated out through this open window, in terms of the open area A.

ii) Assume that this escaping sound is radiated uniformly in all directions outside the building, and that the external wall is large and absorbs no sound. You may also assume that the window is effectively a point source. Derive an expression for the sound sound intensity I_n that is experienced by a neighbour at a distance R from the open window.

iii) The neighbour at R = 10 m from the window has a sound level meter. He has decided to call the police if the sound level L_n at his position reaches 70 dB. Showing your working, determine the area A of open window at which this will happen. (Neglect other sources of sound, and any reflections from surfaces other than the wall.)

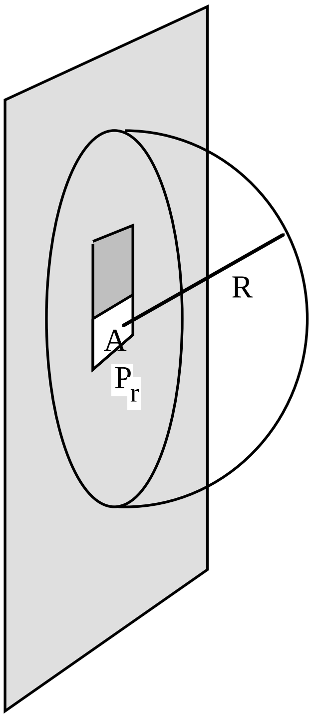
i) For sound power P incident normally on an area A, the intensity I = P/A, so $P_r = AI$

From the definition of sound level,

$$L = 10 \log_{10} \frac{I}{I_o} \quad \text{so} \quad 10^{L/10} = \frac{I}{I_o} \quad \text{so} \quad I = I_o\, 10^{L/10}$$

$$P_r = AI_o\, 10^{L/10} = AI_o\, 10^{106/10} = (0.04\ \mathrm{Wm^{-2}}).A.$$

ii)



For a point source radiating into 2π sterradians (ie into half a sphere with radius R), the intensity is

$$I_n = \frac{P_r}{2\pi R^2} = \frac{(0.04\ \mathrm{Wm^{-2}}).A}{2\pi R^2} \qquad *$$

$$I_n = I_o\, 10^{L_n/10} = I_o\, 10^7 = 1.0\ 10^{-5}\ \mathrm{Wm^{-2}}.$$

Question 3 (marks)

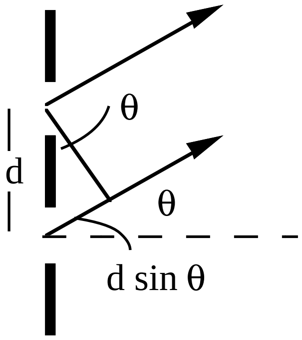
a) i) A diffraction grating has a spacing between adjacent slits of d. For normally incident light of wavelength λ, intensity is of the diffracted light is a maximum at several angles θ from the normal to the grating. Using a sketch, derive a relation between the spacing d and the angles θ at which diffraction maxima occur.

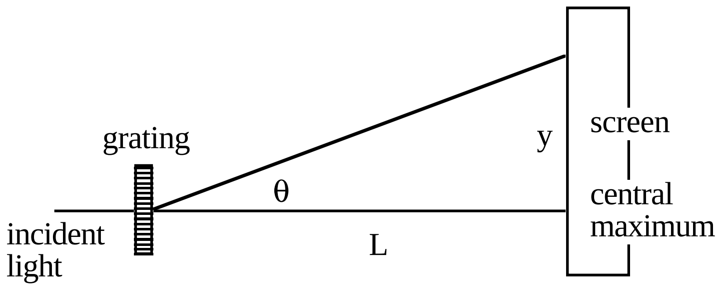
ii) A particular diffraction grating has a spacing between adjacent slits of $d = 2.00\ \mu m$. Normally incident white light is diffracted by the grating and falls on a screen placed at a distance $L = 500$ mm from the grating, parallel to it. What is the distance on the screen between the position of the primary maximum for light with wavelength 630 nm and the central maximum? Show your working.

b) Unpolarised light is normally incident upon two ideal polarisers, placed one on top of the other. One of the sheets is rotated so that no light is transmitted. This orientation is kept constant throughout the problem. A third sheet of ideal polarising material is now placed between the other two, and rotated so that the light transmitted through all three sheets is a maximum. Determine the direction of its polarisation direction with respect to that of one of the other sheets, and use this to derive an expression for ratio of the intensity of the incident unpolarised light to the light transmitted through all three sheets. You may use without proof the relation $\sin 2\theta = 2 \sin \theta \cos \theta$.

Question 3 (marks)

a)





i) From the right-angled triangle shown, the path difference $\delta l = d \sin \theta$. For $\theta = 0$, we get the central bright maximum (no pathlength difference). For other interference maxima, this pathlength must be an integral number of wavelengths, so

$d \sin \theta = m \lambda$ where m is an integer.

ii) $\tan \theta = y/L$

$y = L \tan \theta$

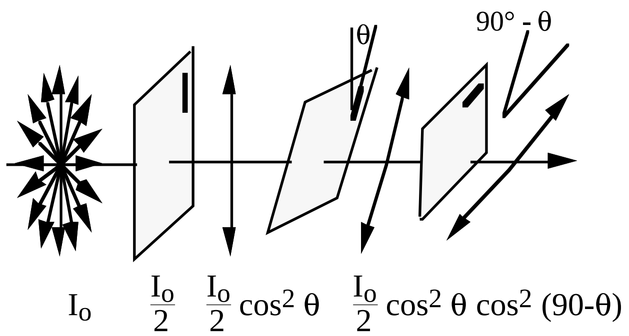
$$= L \tan\left(\sin^{-1} \frac{\lambda}{d}\right)$$

$$= 0.500 \tan\left(\sin^{-1} \frac{630 \text{ nm}}{2.00\ \mu\text{m}}\right)$$

$= 166$ mm

(*small angle approximation gives 156 mm and loses a mark*)

b)



The two outer sheets (alone) give zero transmission when their polarisation directions are at 90°. Let the central sheet be positioned at angle θ as shown. Let the intensity of the unpolarised incident light be I_o. The intensity of the polarised light transmitted from this sheet is then

$I_1 = I_o/2$.

From the law of Malus, that transmitted by the second sheet is

$$I_2 = \frac{I_o}{2} \cos^2 \theta$$

and that by the third sheet $\quad \frac{I_o}{2} \cos^2 \theta \cos^2 (90-\theta)$

This may be written $\quad I = \frac{I_o}{2} \cos^2 \theta \sin^2 \theta \quad = \frac{I_o}{8} \sin^2 2\theta$

Now this has a maximum when $2\theta = 90°$ ie when $\theta = 45°$, so $\quad I/I_o = 1/8$

Question 4 (marks)

a) Using the equation of state for an ideal gas, derive a simple expression for the thermal coefficient of volume expansion for an ideal gas at constant pressure.

b) For a fancy-dress party, you are dressed as an atstronaut. The suit covers you completely, has an area of 2.0 m^2, and a uniform thickness of 2.0 mm. During the dancing, you find that your both your skin temperature and the inside temperature of the suit has risen to 35°C and remains steady at that temperature, while the constant outside temperature of the suit is 30°C. You estimate that you producing heat at 800 W. Neglect the effects of thermal radiation. Showing your working, estimate the thermal conductivity of the material from which the suit is made.

Question 4 (marks)

a) $\alpha \equiv \frac{1}{V}\frac{\partial V}{\partial T}$ $\quad V = \frac{nRT}{P}$, so at constant P, $\frac{\partial V}{\partial T} = \frac{nR}{P}$ so

$$\alpha \equiv \frac{1}{V}\frac{\partial V}{\partial T} = \frac{P}{nRT}\cdot\frac{nR}{P} = \frac{1}{T}$$

b) $H \equiv k\,\frac{A.\Delta T}{d}$

$$\therefore\ k = \frac{d.H}{A.\Delta T} = \frac{.0020\ \text{m x } 800\ \text{W}}{2.0\ \text{m}^2\ 5\ \text{K}} = 0.16\ \text{Wm}^{-1}\text{K}^{-1} \sim 0.2\ \text{Wm}^{-1}\text{K}^{-1}$$

Question 5 (marks)

a) What is an amorphous solid?

b) Some liquids can form either a crystal or an amorphous solid when they cool. Explain briefly how such a liquid can be cooled to form an amorphous solid, and why this procedure results in such a solid.

c) The attractive and repulsive forces between atoms and molecules are in general rather complicated functions of the inter-atomic or inter-molecular spacing. Nevertheless, Hooke's law of elasticity works well for many such materials.

   i) What limitation on the strain is necessary for Hooke's law to be true?

   ii) Subject to this limitation, and ***with the aid of a clearly labelled sketch***, explain briefly how Hooke's law arises.

d) Draw a stress-strain curve that exhibits (i) linear elastic response, (ii) yield and (iii) hysteresis. Indicate which region or feature corresponds to each of these phenomena.

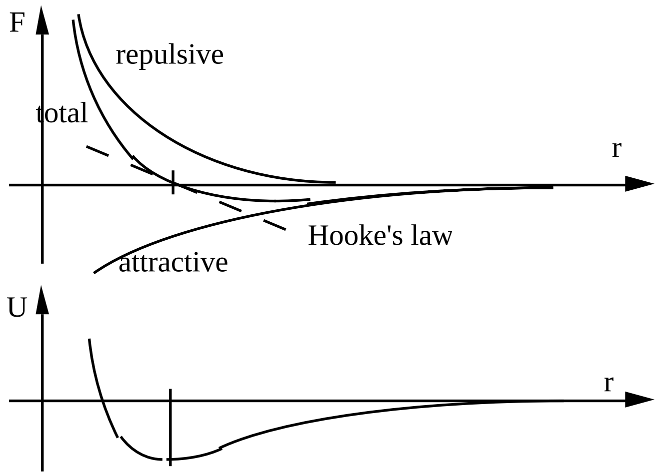
e) During the cycle you have described in (d), is mechanical energy conserved? Explain your answer briefly.

Question 5 (marks)

a) An amorphous solid has no crystalline structure, ie no long range molecular or atomic order.

b) If the viscosity is high, molecules diffuse and rotate slowly, so the rate of formation of crystal nuclei is low, and the rate of crystal growth is low. If the viscosity can be raised (eg by adding solutes) and/or if it is cooled very rapidly, then there may be negligible crystal formation before the molecular diffusion and rotation become so slow that the material may be regarded as a solid.

c)



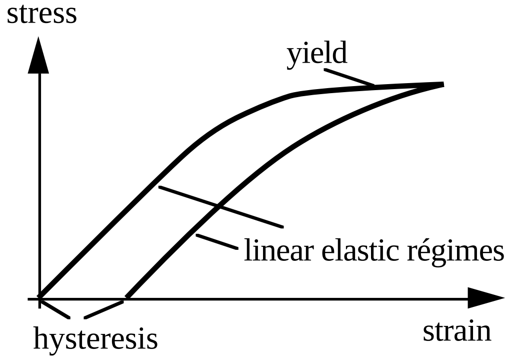
i) Hooke's law only works for strains << 1.

ii) The attractive and repulsive forces are both curvilinear functions of intermolecular separation r, as shown. The mechanical equilibrium of the material, at zero stress, requires that the attractive and repulsive forces add to zero.

For e << 1, the resultant curve may be linearised.

(Alternatively, the energy curve, the integral of the above, has a locally parabolic minimum for e<<1.)

d)



The hysteresis is shown by the fact that, when the stress is returned to zero, the strain has changed, so that the shape of the material is history-dependent.

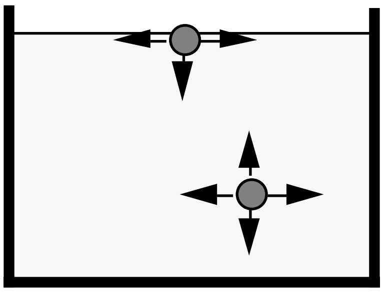
e) No. The integral of the stress strain curve gives the work per unit volume done upon it. This is the area inside the curve, which is non-zero. Going around this curve, mechanical work done on the material is converted into internal potential energy or heat or a combination.

Question 6 (marks)

a) With the aid of simple sketch, explain why molecules at the surface of a liquid have a higher energy than those in the interior of the liquid.

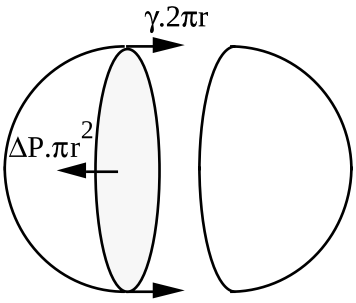
b) Derive the Young-Laplace equation that relates the pressure excess ΔP inside a droplet to its radius r and the surface tension γ.

Question 6 (marks)



A molecule in the bulk is uniformly attracted in each direction. Molecule at surface has ~ no attraction to atmosphere, $\therefore$ work done against the nett force in order to make a surface.

b)



Pressure excess inside balanced by surface tension.
The force to the left on the LH hemisphere is due to pressure, that to the right is due to surface tension, so

$\Delta P.\pi r^2 = \gamma.2\pi r$ whence:

$\Delta P = \gamma \frac{2}{r}$

Not used

Q1

v) What is the maximum speed of a particle subject to the combination of both waves? Explain your answer briefly.

v) The two waves produce a standing wave with amplitude 2A. Each point undergoes sinusoidal motion with angular frequency ω, ie $y_T = 2A\cos(\omega t + \phi)$. So maximum speed is $\max\left(\frac{\partial y_T}{\partial t}\right) = \max\big(2A\omega \sin(\omega t + \phi)\big) = 2A\omega$.

a) A sound source moves at speed v_s in a straight line. It emits a periodic sound signal: a short pressure pulse once every period, T, ie one pulse at t = 0, one at t = T, one at t = 2T, etc. The pressure pulses are radiated uniformly in all directions at the speed of sound, v .

Draw a sketch showing

i) the position (0,1,2,3,4) of the source at times t = 0, T, 2T, 3T and 4T.

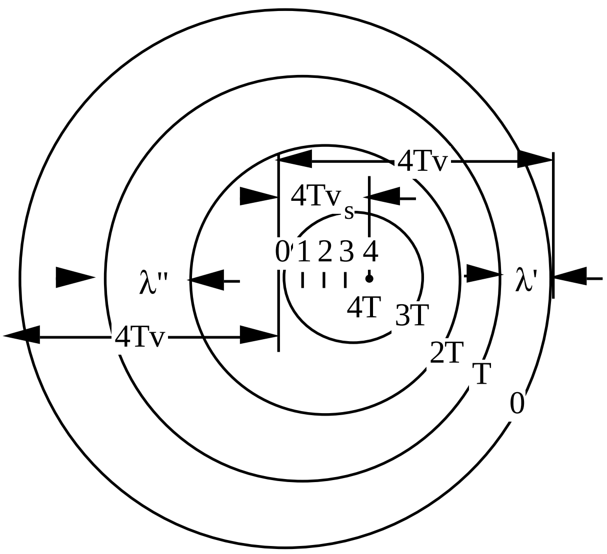
ii) the position of ***each*** of the pulse wavefronts at time t = 4T.

iii) ***Using your sketch***, and other information in the question, derive an expression for the wavelength λ' of sound waves heard by an observer standing directly in front of the source.

iv) Hence state an expression for the frequency f' of sound waves heard by an observer standing directly in front of the source.

iv) Using your sketch, and other information in the question, derive an expression for the wavelength of sound waves heard by an observer standing directly ***behind*** the source.

b)



i, ii)

iii) Wavefront 0 emitted from position 0. The distance between the zeroth and fourth crest is $4Tv - 4Tv_s$, as shown. For the observer in front of the source, the wavelength is

$$\lambda' = \frac{4Tv - 4Tv_s}{4}$$

$$\lambda' = T(v - v_s)$$

iv) $$f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - v_s}\right)$$

v) The distance between the zeroth and fourth crest is $4Tv + 4Tv_s$, as shown. For the observer behind the source, the wavelength is

$$\lambda'' = \frac{4Tv + 4Tv_s}{4}$$

$$\lambda'' = T(v + v_s)$$

Q2

iii) From (*), $I_n = 10^{-5}\ \mathrm{Wm^{-2}} = \dfrac{(0.04\ \mathrm{Wm^{-2}}).A}{2\pi R^2}$

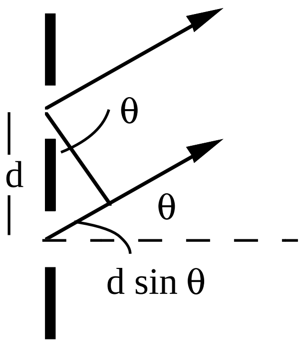
$$A = 2\pi R^2 \frac{10^{-5}\ \mathrm{Wm^{-2}}}{0.04\ \mathrm{Wm^{-2}}}$$

$$= 2\pi(10\ \mathrm{m})^2 \frac{10^{-5}\ \mathrm{Wm^{-2}}}{0.04\ \mathrm{Wm^{-2}}}$$

$$= 0.16\ \mathrm{m^2}$$

Q3

a)



i) From the right-angled triangle shown, the path difference $\delta l = d \sin\theta$. For $\theta = 0$, we get the central bright maximum (no pathlength difference). For other interference maxima, this pathlength must be an integral number of wavelengths, so

$$d \sin\theta = m\lambda$$ where m is an integer.