1169T2/2001

Question 1 (marks)

a) Write the equations of two travelling waves, $y_1(x,t)$ and $y_2(x,t)$, which, when they superpose, produce a standing wave.
State the amplitude, wavelength and frequency of the resultant standing wave in terms of constants in your equations.

b) A steel guitar string has a diameter of 0.33 mm, a vibrating length of 650 mm and a density of 5,600 kg.m^{-3}.
   i) Calculate the tension required to tune it so that its fundamental frequency of vibration is 330 Hz.
   ii) State some of the other frequencies can it vibrate, and illustrate the modes of vibration with a sketch.

Question 1 (marks)

a) several examples are possible, of course, including:

$y_1 = A \sin(kx - \omega t)$ $\qquad$ $y_2 = A \sin(kx + \omega t + \phi)$

(ϕ may have any value including zero)

wavelength $\lambda = 2\pi/k$ $\qquad$ frequency $f = \omega/2\pi$ $\qquad$ amplitude = 2A

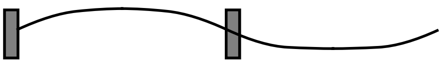
b) i) For an ideal stretched string, fixed at the ends, the longest wavelength standing wave (the fundamental) has

$\lambda = 2L.$

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \therefore\ T = (2Lf_1)^2\mu$$

$$\mu = \frac{m}{L} = \frac{\pi r^2 L.\rho}{L} = \rho\pi(d/2)^2.$$

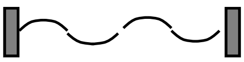
$$T = (2Lf_1)^2\rho\pi(d/2)^2 = \rho\pi(dLf_1)^2 = 88\ \text{N}$$

ii) 

$f_1 = 330$ Hz
(fundamental)

$f_2 = 2^* f_1 = 660$ Hz

$f_3 = 3^* f_1 = 990$ Hz



$f_4 = 4^* f_1 = 1320$ Hz

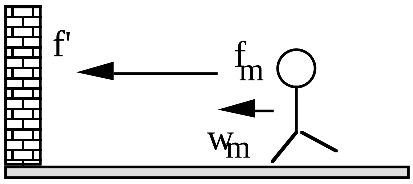
(*a couple of harmonics will suffice to get the marks*)

Question 2 (marks)

a) A man is whistling with a steady frequency f_m of 1000 Hz. He is walking directly towards a wall at speed v_m = 5 km.hr^{-1}. At what frequency would he expect to hear beats, between the sound he is whistling and the echo from the wall? (The speed of sound is 345 m.s^{-1})

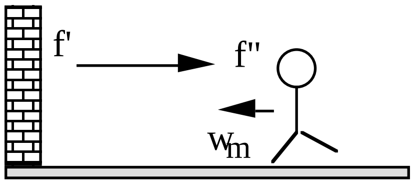
b) A particular ear-plug reduces the sound level at the ear by 23 dB. Determine
- i) By what factor does it reduce the sound power transmitted to the ear?
- ii) By what factor does it reduce the sound pressure at the ear?

c) The speed of sound in water is faster than the speed of sound in air. In one or two sentences, and with one equation, explain why this is so.



First consider the man as the source and the wall as the receiver:

$$f' = f_m \frac{v + v_o}{v - v_s} = f_m \frac{v}{v - v_m}$$



For the echo, consider the wall as source and the man as receiver:

$$f'' = f' \frac{v + v_o}{v - v_s} = f' \frac{v + v_m}{v} = f_m \frac{v}{v - v_m} . \frac{v + v_m}{v} = f_m . \frac{1 + R}{1 - R} \quad \text{where } R = \frac{v_m}{v}$$

f'' = 1008 Hz, therefore he would expect to hear beats at f'' - f_m = 8 Hz

b) $L_2 - L_1 \equiv 10 \log_{10} \frac{I_2}{I_1} \equiv 20 \log_{10} \frac{p_2}{p_1}$

i) $23 \text{ dB} \equiv 10 \log_{10} \frac{I_2}{I_1} \quad \therefore \quad \frac{I_2}{I_1} = 10^{23/10} = 200$

OR: 3 dB is a doubling of power, each 10 dB is a decade increase, so the total increase is 2*10*10 = 200

ii) $\frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = 14$

c) $v = \sqrt{\frac{\text{elastic modulus}}{\text{density}}}$. Although the density of water is much higher than that of air (about 1000 kg.m^{-3}/ 1.3 kg.m^{-3}), the modulus of elasticity of water exceeds that of air by an even larger factor (about 2 GPa/140 kPa). (values not required in answer)

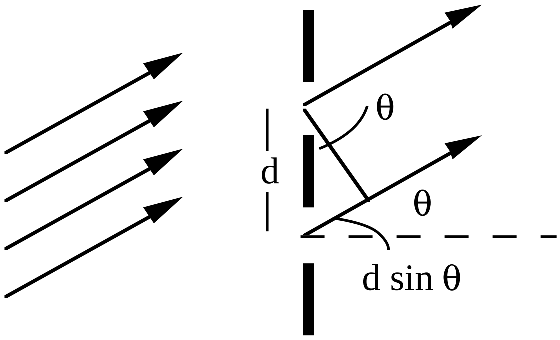
Question 3 (marks)

a) i) What is a diffraction grating? (One clear sentence will do.)
ii) Light with a particular wavelength λ strikes a diffraction grating at normal incidence. Draw a sketch of the intensity I as function of sin θ, the sin of the angle between the normal and the light leaving the grating
iii) With the aid of a sketch, derive an expression for the angle θ_1, the angle of the first maximum in I on either side of the central maximum at $\theta = 0$.

b) i) With the aid of a diagram, explain how an anti-reflective coating on a lens works.
ii) Estimate the thickness of the coating if the coating material has a refractive index of 1.4 and the antireflective property is optimised for light of wavelength λ = 550 nm in air.

Question 3 (marks)

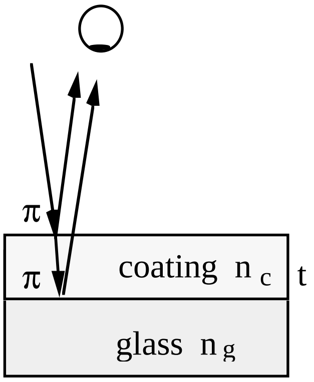
a) i) A diffraction grating has a series of equally spaced lines cut or etched so as to make a series of parallel slit sources of light. (*Not needed: It is used to produce optical spectra and to measure wavelengths.*)
ii) (*Sketch showing a series of narrow maxima.)*



iii)
the first order maximum will occur when the pathlengths of rays from adjacent slits differ by λ, ie when $d \sin\theta = \lambda$

$$\theta = \sin^{-1} \lambda/d$$

b)



If the film has a value of refractive index intermediate between that of the glass and that of the external medium, then there will be a phase change of π at the two reflections shown. For normal incidence, these two rays will have a phase difference of π if

optical pathlength difference = $\lambda/2$

$2tn = \lambda/2$ ∴ $t = \lambda/4n = 100$ nm 5

Question 4 (marks)

a) A tank has a volume V_0 at a reference temperature T_o, and is made of a material with linear coefficient of thermal expansion α_m. At temperature T_o, it is completely filled by a liquid with volumetric coefficient of thermal expansion β_l. Derive an expression for the volume ΔV that overflows from the tank at a temperature T.

b) Using the equation of state for an ideal gas, derive an expression for the density ρ of an ideal gas in terms of the pressure P, temperature T, molecular mass m and constants.

Question 4 (marks)

a) At $T = T_o$, $V_{tank} = V_{liquid} = V_o$,

At $T \neq T_o$, $V_{tank} = V_o(1 + \beta_m(T - T_o))$

$= V_o(1 + 3\alpha_m(T - T_o))$

$V_l = V_o(1 + \beta_l(T - T_o))$

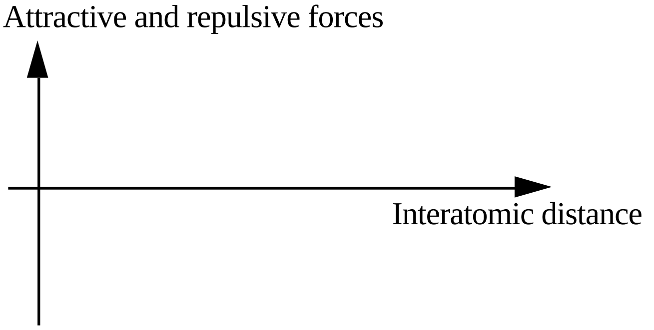
Overflow $= V_l - V_{tank} = V_o(\beta_l - 3\alpha_m)(T - T_o)$ ( )

(If $\beta_l > \beta_m$, as is usual, overflow occurs for $T > T_o$. If $\beta_l < \beta_m$ overflow occurs on cooling, but the equation is valid in both cases.)

b) $PV = nRT$ $\rho \equiv \frac{\text{mass}}{\text{volume}} = \frac{nN_A m}{V} = \frac{PN_A m}{RT}$ ( )

Question 5 (12 marks)

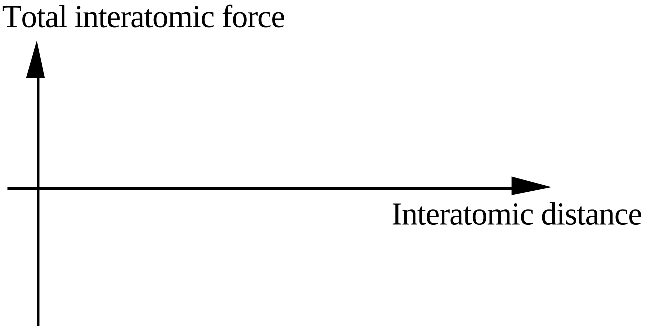
a) Use these axes to sketch your answers to the following questions. If you spoil this copy, Use the copy of these axes on the opposite page.



i) On the upper graph, sketch the interatomic attractive force as a function of interatomic distance

ii) Also on the upper graph, sketch the interatomic repulsive force as a function of interatomic distance.

Indicate clearly which is which.



iii) On the lower graph, and to the same scale, sketch the total interatomic force as a function of interatomic distance.

iv) On the lower graph, sketch the behaviour corresponding to Hooke's law (*ie* behaviour with a constant Young's modulus).

Indicate clearly which is which.

b) i) Define stress

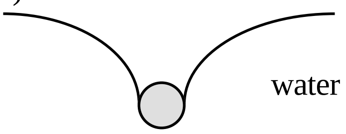
ii) Define longitudinal strain

iii) Sketch a stress-strain relation for a material for which an elastic modulus may be defined. On your sketch, indicate a feature related to the elastic modulus. ***Also*** on your sketch, indicate hysteresis (or history dependent behaviour).

Question 6 (marks)

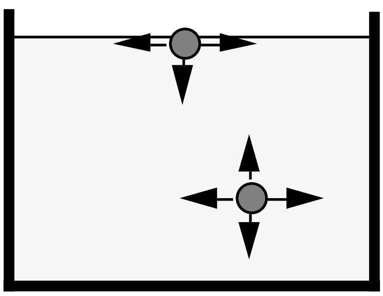
a) With the aid of simple sketch, explain why molecules at the surface of a liquid have a higher energy than those in the interior of the liquid.

b)



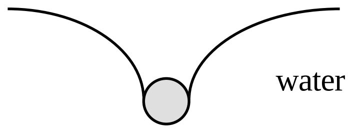
A spherical bead, with density ρ, is supported at the surface of water. What is the diameter of the largest bead that can be thus supported, with the total force due to surface tension equal to the weight of the object? Express you answer in terms of the diameter of the bead and the surface tension γ_W of water. (You need not consider the effects of bouyancy.)

Question 6 (marks)



A molecule in the bulk is uniformly attracted in each direction. Molecule at surface has ~ no attraction to atmosphere, $\therefore$ work done against the nett force in order to make a surface.

b)



The tension exerted by the water surface cannot exceed γ_w, and it is a maximum if it is vertical. Hence

$$F_{surface\ tension} = 2\pi r.\gamma_w$$

$$\text{Weight} = \rho V = \rho \frac{4}{3} \pi r^3$$

$$2\pi r.\gamma_w = \rho \frac{4}{3} \pi r^3$$

$$\frac{3}{2} \frac{\gamma_w}{\rho} = r^2 \qquad r_{max} = \sqrt{\frac{3}{2} \frac{\gamma_w}{\rho}}$$