QUESTION 1 [Marks 8]

Two vectors are given by

$$\mathbf{a} = 2\,\mathbf{i} - \mathbf{j}$$
$$\mathbf{b} = \mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$$

(a) Calculate $\mathbf{a} + 3\,\mathbf{b}$

(b) Calculate $\mathbf{a} \cdot \mathbf{b}$

What do you deduce about the angle between **a** and **b**?

(c) Calculate $\mathbf{a} \times \mathbf{b}$

a) $\underset{\sim}{a} + 3\underset{\sim}{b} = (2\underset{\sim}{i} - \underset{\sim}{j}) + 3(\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$

$= 5\underset{\sim}{i} + 5\underset{\sim}{j} + 3\underset{\sim}{k}$ (Answer)

b) $\underset{\sim}{a} \cdot \underset{\sim}{b} = (2\underset{\sim}{i} - \underset{\sim}{j}) \cdot (\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$

$= 2 \cdot 1 - 1 \cdot 2 = \underline{0}$ (Answer)

We deduce that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular (at 90° to each other), using

$\underset{\sim}{a} \cdot \underset{\sim}{b} = ab\cos\theta$

c) $\underset{\sim}{a} \times \underset{\sim}{b} = \underset{\sim}{i}\,(a_y b_z - a_z b_y)$

$+ \underset{\sim}{j}\,(a_z b_x - a_x b_z)$

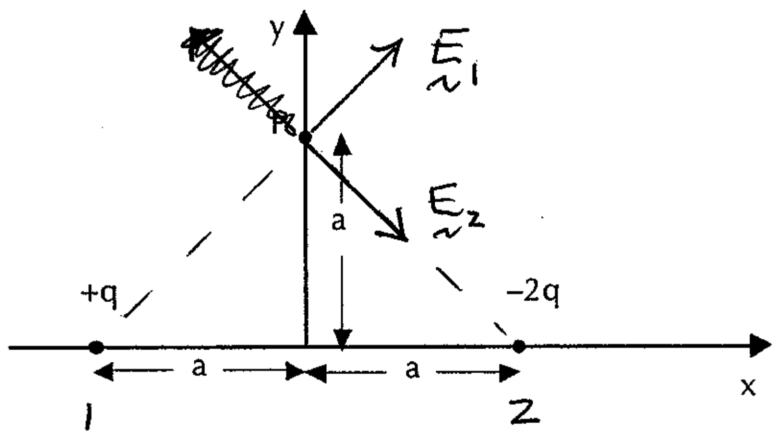
$+ \underset{\sim}{k}\,(a_x b_y - a_y b_x)$

$= \underset{\sim}{i}\,((-1)\cdot 1 - 0\cdot 2) + \underset{\sim}{j}\,(0\cdot 1 - 2\cdot 1) + \underset{\sim}{k}\,(2\cdot 2 - (-1)\cdot 1)$

$= -\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k}$ (Answer)

QUESTION 2 [Marks 10]

Two point charges, one of charge +q and the other of charge –2q, are situated on the x axis as shown.



(a) Derive an expression for vector components of the electric field intensity at P, in rectangular co-ordinates;

(b) Determine the direction of the electric field at P relative to the x axis;

(c) Find an expression for the electric potential at point P.

The fields $\underset{\sim}{E}_1$ and $\underset{\sim}{E}_2$ due to the two charges (labelled 1 and 2 above) are:

$$\begin{cases} E_{1x} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{(2a^2)} \cos 45^\circ \\ E_{1y} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{(2a^2)} \sin 45^\circ \end{cases}$$

and

$$\begin{cases} E_{2x} = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{2q}{(2a^2)} \cos 45^\circ \\ E_{2y} = -\dfrac{1}{4\pi\epsilon_0} \dfrac{2q}{(2a^2)} \sin 45^\circ \end{cases}$$

using $\underset{\sim}{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{r^2} \hat{\underset{\sim}{r}}$ for a point charge.

ANSWER QUESTION 2 ONLY

∴ Total field $\underset{\sim}{E}$ has components

$$E_x = E_{1x} + E_{2x} = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}\left(\frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{3q}{8\sqrt{2}\pi\epsilon_0 a^2}, \quad \text{(Answer)}$$

$$E_y = E_{1y} + E_{2y} = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -\frac{q}{8\sqrt{2}\pi\epsilon_0 a^2}. \quad \text{(Answer)}.$$

b) Direction relative to x axis:

$$\tan\theta = \frac{E_y}{E_x} = -\frac{1}{3}. \quad \therefore \theta = -18.4^\circ$$

∴ direction is 18.4° below the x axis.

c) Electric potential at P, using
$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ for a point charge:

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0 a}\left(\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right)$$

$$= -\frac{q}{4\sqrt{2}\pi\epsilon_0 a}. \quad \text{(Answer)}$$

If there is not enough space here, continue answer on supplementary pages at the end of this booklet.

❑ **Tick this box if you use supplementary pages.**

QUESTION 3 [Marks 10]

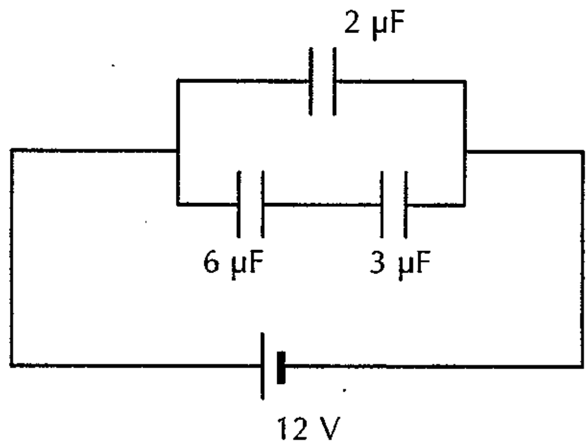
For the circuit shown, showing all your reasoning and working, determine:

(a) the total equivalent capacitance of this combination of capacitors

(b) the total charge drawn from the battery

(c) the charge on the 3 μF capacitor

(d) the potential difference across the 3 μF capacitor

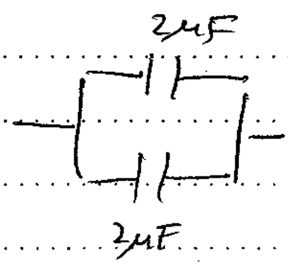


a) The combination 6μF and 3μF in series has equivalent capacitance:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

i.e. $C' = 2\mu F$.

Then



in parallel have equivalent capacitance $C = C' + C_3$
$= 4\mu F$ (Answer).

b) Total charge $Q = CV = 4 \times 10^{-6} \times 12$
$= 4{\cdot}8 \times 10^{-6}$ C. (Answer).

c) Potential difference 12V across the branch with $C' = 2\mu F$ produces charge
$Q' = C'V = 2{\cdot}4 \times 10^{-5}$ C (Answer)

(This same charge will appear on both the 6μF and 3μF capacitors, being in parallel).

ANSWER QUESTION 3 ONLY

d) The potential difference across the 3μF capacitor is now given by

$$Q' = C_2 V_2$$

$$\text{i.e. } V_2 = \frac{2.4 \times 10^{-5}\,C}{3\mu F}$$

$$= \underline{8\ V} \qquad \text{(Answer).}$$

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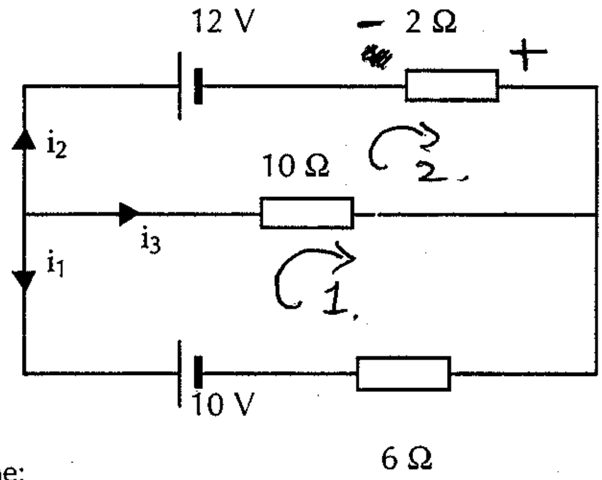
QUESTION 4 [Marks 12]

(a) State Kirchhoff's rules for direct current circuits.

(b) For the circuit shown, use Kirchhoff's rules to write down three equations from which the currents i_1, i_2 and i_3 could be determined,
DO NOT GO ON TO SOLVE THOSE EQUATIONS.



i_3 (N.B.!)

(c) Given the current ~~i_2~~ has a value 1A, determine:
(i) the potential difference across the 2Ω resistor
(ii) the value of the current i_1

(d) Is power supplied by, or delivered to the 12 V battery? Give your reasoning.

a) Junction Rule: The algebraic sum of the currents flowing into any junction is zero.
Loop Rule: The algebraic sum of the potential changes around any loop of the circuit is zero.

b) For the circuit shown:

Junction Rule: $i_1 + i_2 + i_3 = 0$ (1)

Loop 1: $-10 i_3 + 6 i_1 + 10 = 0$ (2)

Loop 2: $-12 - 2 i_2 + 10 i_3 = 0$ (3)

ANSWER QUESTION 4 ONLY

~~c) i) The p.d. across the 2Ω resistor~~

~~= $i_2 R$ (Ohm's Law) = 2V (Answer)~~

~~ii) From i), total potential~~

c). i) P.d. across 10Ω is $i_3 \times 10$ = 10V

∴ p.d. across 2Ω resistor must be 2V, in the direction shown

(i.e. p.d. across top branch = 12 − 2 = 10 V also)

ii) the p.d. across the lower branch must also total 10V

∴ $i_1 = 0$! (Answer)

d) By the junction rule, the current i_4

$i_2 = -i_3 - i_1$

$= -1 A.$

∴ current is being drawn from the 12V battery

∴ power is being supplied by the 12V battery.

If there is not enough space here, continue answer on supplementary pages at the end of this booklet.

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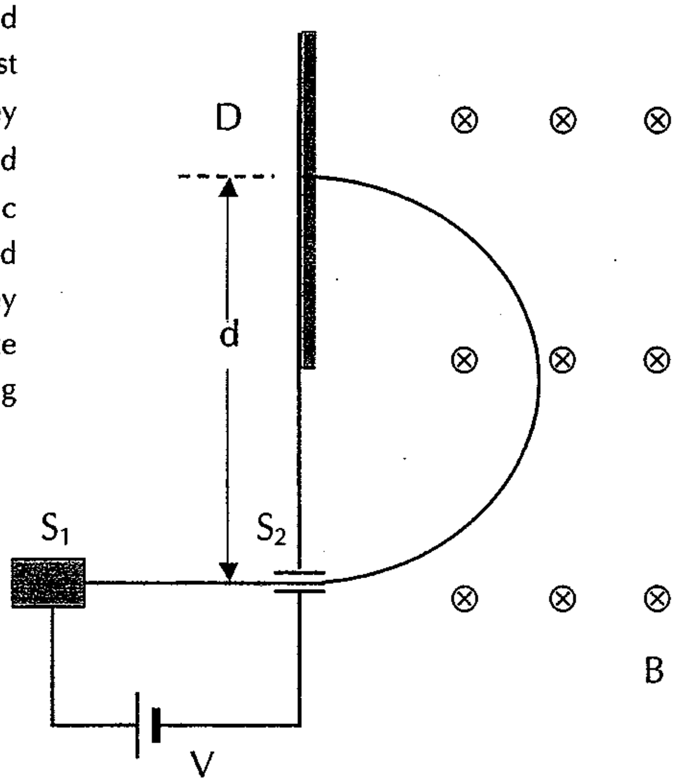
QUESTION 5 [Marks 10]

The diagram shows a plan view of a mass spectrometer. In it ions of mass m and charge +q are accelerated to S_2 from rest at S_1 by a potential difference of V. They emerge through a small hole at S_2 and enter a uniform magnetic field, of magnetic induction B, directed at right angles and into the plane of the paper. In the field they follow a semicircular path until they strike the detector D. Showing all your reasoning and working, derive expressions for



(a) The kinetic energy gained by the ions from S_1 to S_2

(b) The velocity of the ions on entering S_2

(c) The distance d from S_2 at which the ions strike the detector array.

a) By conservation of energy,
Kinetic energy gained = potential energy lost

$\therefore K = qV$. (Answer)

b) $\therefore$ on entering S_2,

$\frac{1}{2} m v^2 = qV$.

$\therefore v = \sqrt{\frac{2qV}{m}}$ (Answer) (1)

ANSWER QUESTION 5 ONLY

e) In the magnetic field the ions travel in a semicircle of radius given by

$$F = ma$$

i.e. $qvB = \frac{mv^2}{r}$

$\therefore$ $r = \frac{mv}{qB}$ (Larmor radius)

$\therefore$ $d = 2r = \frac{2mv}{qB} = \frac{2m}{qB}\sqrt{\frac{2qV}{m}}$

$= \frac{2}{B}\sqrt{\frac{2mV}{q}}$ (Answer)

If there is not enough space here, continue answer on supplementary pages at the end of this booklet.

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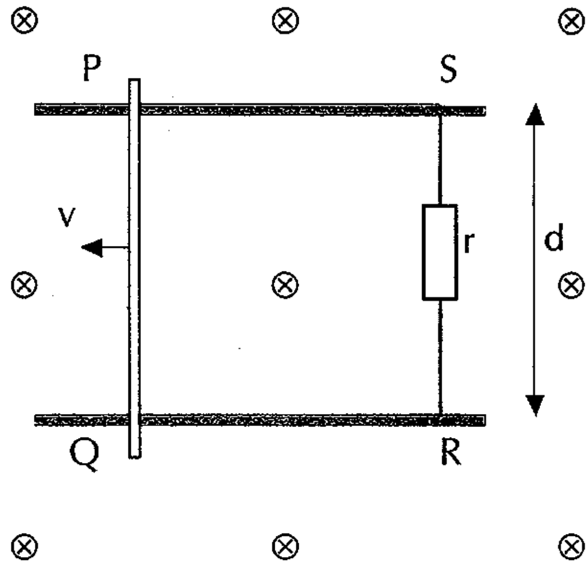
QUESTION 6 [Marks 10]

(a) State Faraday's Law of electromagnetic induction, defining all the symbols used.

(b) The diagram shows a conducting rod PQ making contact with parallel metal rails PS and QR, which are a distance d apart. Both the rails and the rod have negligible resistance.

The rails are connected at S and R by a resistor of resistance r. A uniform magnetic induction B is directed vertically down into the plane of the paper. The rod moves to the left at speed v, as shown in the diagram.



(i) Showing all your reasoning and working, derive an expression for the magnitude of the current in the loop PQRS.

(ii) What is the direction of this current? Justify your answer.

a) Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where $\mathcal{E}$ is the e.m.f. induced in a closed loop due to a magnetic flux Φ_B passing through the loop which is changing with time t, in a direction given by the Right Hand Rule

ANSWER QUESTION 6 ONLY

b) i) Magnetic flux into the paper

$$\Phi_B = \underset{\sim}{B} \cdot \underset{\sim}{A} = BA$$

$$\therefore \frac{d\Phi_B}{dt} = B\frac{dA}{dt} = Bdv$$

$$\therefore \mathcal{E} = -\frac{d\Phi_B}{dt} = -Bdv$$

$\therefore$ magnitude of current $i = \frac{|\mathcal{E}|}{r}$

$$= \frac{Bdv}{r} \quad \text{(Answer)}$$

ii) ~~#~~ A flux into the paper corresponds to a clockwise e.m.f., by the Right Hand Rule. The answer is negative, so the e.m.f. and ~~curr~~ hence the current run anticlockwise.

This will produce an induced $\underset{\sim}{B}$ field out of the paper, counteracting the increasing flux through the loop into the paper, in agreement with Lenz's law.

If there is not enough space here, continue answer on supplementary pages at the end of this booklet.

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