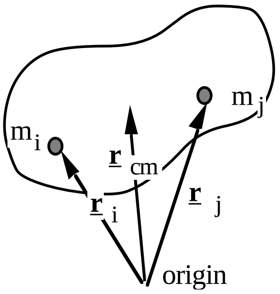
**PHYS 1121-1131. Systems of particles and extended bodies, Centre of mass, Momentum, Collisions** Joe Wolfe, UNSW

## Centre of mass

*In a finite body, not all parts have the same acceleration. Not even if it is rigid. How to apply* $\underline{\mathbf{F}} = m\,\underline{\mathbf{a}}$*?*

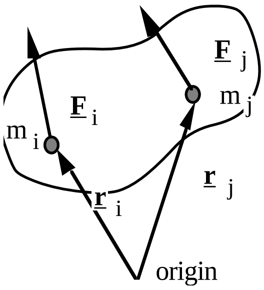



Total mass $M = \Sigma\, m_i$

Define the **centre of mass** as the point with displacement

$$\underline{\mathbf{r}}_{cm} = \frac{\Sigma\, m_i \underline{\mathbf{r}}_i}{M}$$

*Why?* n particles, $m_i$ at positions $\underline{\mathbf{r}}_i$, $\underline{\mathbf{F}}_i$ acts on each. Total force acting on all particles:




$$\Sigma\, \underline{\mathbf{F}}_i = \Sigma\, m_i \underline{\mathbf{a}}_i$$

$$= \Sigma\, m_i \frac{d^2}{dt^2} \underline{\mathbf{r}}_i$$

if masses constant:

$$= \Sigma\, \frac{d^2}{dt^2} m_i \underline{\mathbf{r}}_i$$

$$= \frac{d^2}{dt^2} \Sigma\, m_i \underline{\mathbf{r}}_i$$

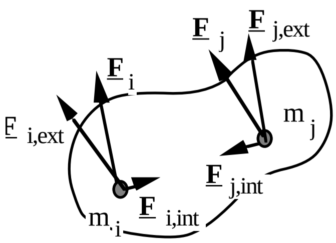
$$= M\frac{d^2}{dt^2}\left(\Sigma\, \frac{m_i \underline{\mathbf{r}}_i}{M}\right)$$

But we defined $\underline{\mathbf{r}}_{cm} = \dfrac{\Sigma\, m_i \underline{\mathbf{r}}_i}{M}$

Then $\Sigma\, \underline{\mathbf{F}}_i = M\dfrac{d^2}{dt^2}\underline{\mathbf{r}}_{cm} = M\, \underline{\mathbf{a}}_{cm}$

(total force) = (total mass)*(acceleration of centre of mass)

*Look at forces in detail:*




Each $\underline{\mathbf{F}}_i$ is the sum of internal forces (from other particles in the body/ system) and external forces (from outside the system)

$$\Sigma\, \underline{\mathbf{F}}_i = \Sigma\, \underline{\mathbf{F}}_{i,\text{ internal}} + \Sigma\, \underline{\mathbf{F}}_{i,\text{ external}}$$

**Newton 3**: All internal forces $\underline{\mathbf{F}}_{ij}$ between $i^{th}$ and $j^{th}$ particles are reaction pairs $\underline{\mathbf{F}}_{ji} = -\underline{\mathbf{F}}_{ij}$

$\therefore\quad \Sigma$ internal forces = 0

$\therefore\quad \Sigma\, \underline{\mathbf{F}}_i = \Sigma\, \underline{\mathbf{F}}_{i,\text{ external}} = \underline{\mathbf{F}}_{\text{external}}$

$\therefore\quad \underline{\mathbf{F}}_{\text{external}} = M\, \underline{\mathbf{a}}_{cm}$

$$\left(\begin{matrix}\text{total}\\ \textbf{external}\text{ force}\end{matrix}\right) = \left(\begin{matrix}\text{total}\\ \text{mass}\end{matrix}\right) * \left(\begin{matrix}\text{acceleration of}\\ \text{centre of mass}\end{matrix}\right)$$

For n discrete particles, **centre of mass** at

$$\underline{\mathbf{r}}_{cm} = \frac{\Sigma\, m_i\, \underline{\mathbf{r}}_i}{\Sigma\, m_i} = \frac{\Sigma\, m_i\, \underline{\mathbf{r}}_i}{M} \qquad \text{(i)}$$

For a continuous body, elements of mass dm at $\underline{\mathbf{r}}$

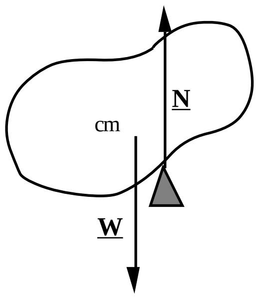
$$\underline{\mathbf{r}}_{cm} = \frac{\int_{body} \underline{\mathbf{r}}\, dm}{\int_{body} dm} = \frac{\int_{body} \underline{\mathbf{r}}\, dm}{M} \qquad \text{(ii)}$$

Can rearrange (i):

$$0 = \Sigma \frac{m_i\, \underline{\mathbf{r}}_i - m_i\, \underline{\mathbf{r}}_{cm}}{M} \quad \rightarrow \quad \Sigma\, m_i\, (\underline{\mathbf{r}}_i - \underline{\mathbf{r}}_{cm}) = 0$$

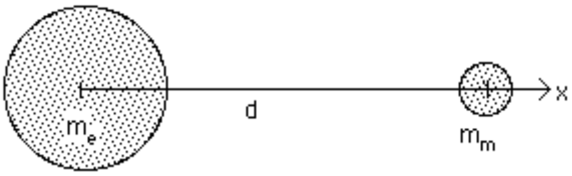
$$\text{(ii)} \rightarrow \quad \int_{body} (\underline{\mathbf{r}}_i - \underline{\mathbf{r}}_{cm})\,dm = 0$$

Later, when doing rotation, we'll consider




which is a useful way to find c.m. experimentally.

**Example**. Where is the c.m. of the earth moon system?




$$\underline{\mathbf{r}}_{cm} = \frac{\Sigma\, m_i\, \underline{\mathbf{r}}_i}{\Sigma\, m_i}$$

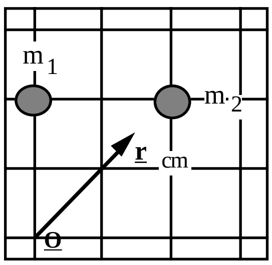
Take origin at centre of earth.

$$x_{cm} = \frac{m_e x_e + m_m x_m}{m_e + m_m} = \frac{m_m d}{m_e + m_m}$$

$$= 4{,}600 \text{ km} \qquad \text{i.e. inside the earth.}$$

*recall: we derived the centre of rotation of the system*

**Example**



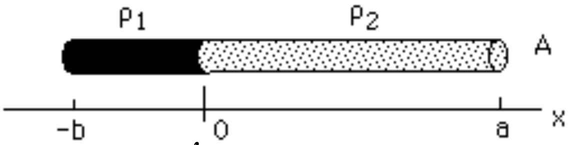

On a square plate (mass $m_p$), we place $m_1$ and $m_2$ as indicated.
$m_p$ = 135 g, $m_1$ = 100 g and $m_2$ = 50 g
Where is the cm of the system?

$$\underline{\mathbf{r}}_{cm} = \frac{\Sigma\, m_i\, \underline{\mathbf{r}}_i}{\Sigma\, m_i} = \frac{m_p(1.5\underline{\mathbf{i}} + 1.5\underline{\mathbf{j}}) + m_1(2.0\underline{\mathbf{j}}) + m_2(2.0\underline{\mathbf{i}} + 2.0\underline{\mathbf{j}})}{m_p + m_1 + m_2}$$

$$= \frac{(303\text{g})\underline{\mathbf{i}} + (503\text{g})\underline{\mathbf{j}}}{285\text{g}} = 1.1\,\underline{\mathbf{i}} + 1.8\underline{\mathbf{j}}$$

*check that $\Sigma\, m_i\, (\underline{\mathbf{r}}_i - \underline{\mathbf{r}}_{cm}) = 0$*

**Example**. Rod, cross-section A, made of length a of material with density $\rho_1$ and length b of material with density $\rho_2$. Where is c.m.? If $\rho_1/ = 2\rho_2$, and a = 2b, where is cm?




$$\underline{\mathbf{r}}_{cm} = \frac{\int \underline{\mathbf{r}}_i \, dm}{\int dm}$$

$$dm = \rho dV = \rho A dx$$

$$x_{cm} = \frac{\int x \, dm}{\int dm}$$

$$= \frac{\int_{-b}^{0} \rho_1 A x \, dx + \int_{0}^{a} \rho_2 A x \, dx}{\int_{-b}^{0} \rho_1 A \, dx + \int_{0}^{a} \rho_2 A \, dx}$$

$$= \frac{-\frac{1}{2}\rho_1 b^2 + \frac{1}{2}\rho_2 a^2}{\rho_1 b + \rho_2 a}$$

$$= \frac{a^2 - rb^2}{2(a + rb)} \qquad \text{where } r = \frac{\rho_1}{\rho_2}$$

**Momentum**

Definition: $\underline{\mathbf{p}} \equiv m\underline{\mathbf{v}}$

*Later we'll see that this is a low velocity approximation to* $\left(\underline{\boldsymbol{p}} = \dfrac{m\underline{\boldsymbol{v}}}{\sqrt{1 - v^2/c^2}}\right)$

Generalised form of

**Newton 2:** $\Sigma\, \underline{\mathbf{F}} = \dfrac{d}{dt}\underline{\mathbf{p}}$

$$\Sigma\, \underline{\mathbf{F}} = m\frac{d}{dt}\underline{\mathbf{v}} + \underline{\mathbf{v}}\frac{d}{dt}m$$

If m constant, $\Sigma\, \underline{\mathbf{F}} = m\,\underline{\mathbf{a}}$

**System of particles:** *What is system? - you choose.*

$\underline{\mathbf{P}} = \Sigma\, \underline{\mathbf{p}}_i$ and $M = \Sigma\, m_i$

$$\underline{\mathbf{P}} = \Sigma\, m_i \underline{\mathbf{v}}_i = \Sigma\, m_i \frac{d}{dt}\underline{\mathbf{r}}_i$$

$$= \frac{d}{dt}\Sigma\, m_i \underline{\mathbf{r}}_i = M\frac{d}{dt}\left(\frac{\Sigma\, m_i \underline{\mathbf{r}}_i}{M}\right)$$

$$\underline{\mathbf{P}} = M\underline{\mathbf{v}}_{cm}$$

If M constant: $\dfrac{d}{dt}\underline{\mathbf{P}} = M\underline{\mathbf{a}}_{cm}$

$$\Sigma\, \underline{\mathbf{F}}_i = \Sigma\frac{d}{dt}\underline{\mathbf{p}}_i = \frac{d}{dt}\underline{\mathbf{P}}$$

All internal forces are in pairs $\underline{\mathbf{F}}_{ji} = -\underline{\mathbf{F}}_{ij}$

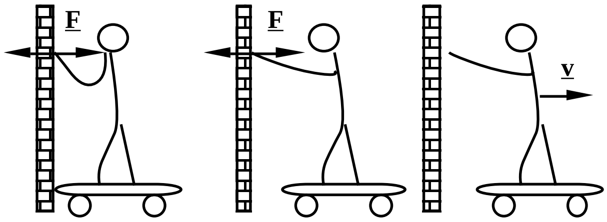
$\therefore$ $\underline{\mathbf{F}}_{ext} = \dfrac{d}{dt}\underline{\mathbf{P}}$ *conclude:*

i) Motion of cm is like that of particle mass M at $\underline{\mathbf{r}}_{cm}$ subjected to $\underline{\mathbf{F}}_{ext}$.

ii) If $\underline{\mathbf{F}}_{ext} = 0$, **momentum is conserved**

**Internal *vs* external work.**

**Problem.** Skateboarder pushes away from a wall




Point of application of $\underline{\mathbf{F}}$ does not move, $\therefore$ normal force does no work, but K changes. Where does energy come from?

$$F_{ext} = Ma_{cm}$$

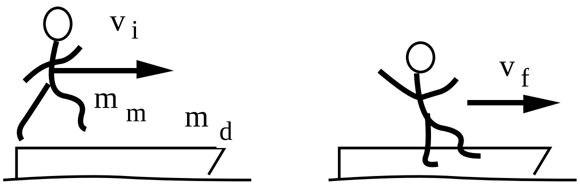
$$F_{ext}\,dx = Ma_{cm}\,dx_{cm} = M\frac{dv_{cm}\,dx_{cm}}{dt} = Mv_{cm}dv_{cm}$$

"Centre of mass work"

$$W_{cm} = \int_i^f F_{ext}\,dx = \left(\tfrac{1}{2}Mv_{cm}^2\right)_f - \left(\tfrac{1}{2}Mv_{cm}^2\right)_i$$

*Work done = that which would have been done if $F_{ext}$ had acted on cm.*

**Example** 90 kg man jumps ($v_j = 5$ ms$^{-1}$) into a (stationary) 30 kg dinghy. What is their final speed? (Neglect friction.)

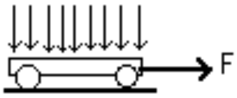



No external forces act in horizontal direction so $P_x$ is conserved.

$$P_i = P_f$$

$$\underset{man}{m_m v_j} \; \underset{dinghy}{+\,0} \;=\; (\underset{man}{m_m} + \underset{dinghy}{m_d})v_f$$

$$v_f = \frac{m_m}{m_m + m_d} v_j$$




**Example** Rain falls into an open trailer (area 10 m$^2$) at 10 litres.min$^{-1}$.m$^{-2}$.

Neglecting friction, what F required to maintain constant speed of 10 ms$^{-1}$?

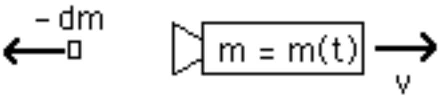
*10 litres has mass 10 kg*

$$F_x = \frac{d}{dt}(mv_x) = m\frac{d}{dt}v_x + v_x\frac{d}{dt}m$$

$$= 10 \text{ ms}^{-1} \times \left(\frac{10 \text{ kg.m}^{-2}}{60 \text{ s}} 10 \text{ m}^2\right)$$

$$= 17 \text{ N.}$$

**Example.** Rocket has mass m = m(t), which decreases as it ejects exhaust at rate $r = -\frac{dm}{dt}$ and at relative velocity u. What is the acceleration of the rocket? $\left(\frac{dm}{dt} = \begin{matrix}\textit{rate of increase of}\\ \textit{mass of rocket}\end{matrix} < 0\right)$




No external forces act so momentum conserved. In the frame of the rocket, forwards direction:

$$dp_{rocket} + dp_{exhaust} = 0$$

$$m.dv + (-dm).(-u) = 0$$

$$dv = -u\frac{dm}{m}$$

$$a = \frac{dv}{dt} = -\frac{u}{m}\cdot\frac{dm}{dt}$$

$$a = \frac{ur}{m} \qquad \textit{1st rocket equation}$$

$$dv = -u\frac{dm}{m} = \quad dv = -u\, d(\ln m)$$

$$\int_i^f dv = v_f - v_i = u \ln\frac{m_i}{m_f} \qquad \textit{2nd rocket equation}$$

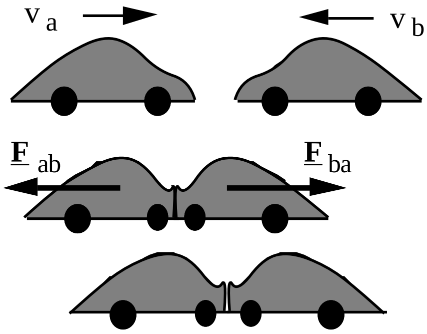
*need high exhaust velocity u (c?), else require $m_i >> m_f$*

## Collisions

**Collisions** Definition: in a collision, "large" forces act between bodies over a "short" time.

*In comparison, we shall often neglect the momentum change due to external forces.*

**Example 1:**




$F_{ab} = F_{ba} = 0$

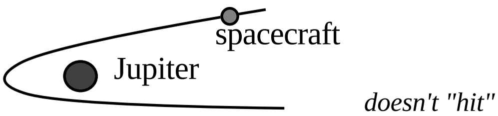
$F_{ab} = F_{ba} =$ large!

$F_{ab} = F_{ba} = 0$

*forces that crumple cars during (brief) collision are much larger than friction force (tires - road), ∴ neglect $F_{ext}$.*

In previous example: car decelerates from 30 kph to rest in a 60 cm 'crumple zone'. Average a = 58 ms$^{-2}$, so force on car during collision ~ ma ~ 58 kN, compared with friction at ~ 10 kN

**Example 2**




Here, start and finish of collision not well defined

At large separation before and after, $F_{ab} = F_{ba} \cong 0$

During collision (fly-by), forces are considerable.

However, $F_{grav} \propto 1/r^2$, so much smaller at large distances.

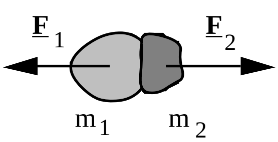
**Impulse ($\underline{\mathbf{I}}$) and momentum**

Newton 2 ⇒ $\quad d\underline{\mathbf{p}} = \underline{\mathbf{F}}\,dt$

$$\therefore \quad \int_i^f d\underline{\mathbf{p}} = \int_i^f \underline{\mathbf{F}}\,dt \qquad \textit{so}$$

Definition: $$\underline{\mathbf{I}} \equiv \underline{\mathbf{p}}_f - \underline{\mathbf{p}}_i = \int_i^f \underline{\mathbf{F}}\,dt$$

Usual case: external forces small, act for small time, therefore $\int_i^f \underline{\mathbf{F}}_{ext}\,dt$ is small.



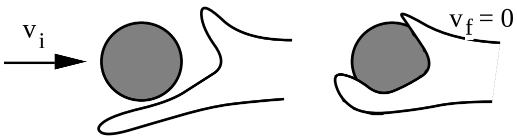

$$\Delta \underline{\mathbf{p}}_1 = \int_i^f \underline{\mathbf{F}}_1\,dt \equiv \overline{\underline{\mathbf{F}}}_1 \Delta t \qquad \Delta \underline{\mathbf{p}}_2 = \int_i^f \underline{\mathbf{F}}_2\,dt = -\int_i^f \underline{\mathbf{F}}_1\,dt$$

$$\therefore \quad \Delta \underline{\mathbf{p}}_1 = -\Delta \underline{\mathbf{p}}_2$$

$$\therefore \quad \Delta \underline{\mathbf{P}} = \Delta \underline{\mathbf{p}}_1 - \Delta \underline{\mathbf{p}}_2 = 0$$

***If external forces are negligible*** (in any direction), then the momentum of the system is conserved (in that direction).

**Example**. Cricket ball, m = 156 g, travels at 70 $ms^{-1}$. What impulse is required to catch it? If the force applied were constant, what average force would be required to stop it in 1 ms? in 10 ms? What stopping distances in these cases?




m = 0.156 kg, $v_i$ = 70 $m.s^{-1}$ $v_f = 0$.

$\underline{\mathbf{I}} \equiv \underline{\mathbf{p}}_f - \underline{\mathbf{p}}_i$

$= m(v_f - v_i)$ to right

= .... = 10.9 $kgms^{-1}$ to *left*.

$\underline{\mathbf{I}} = \int_i^f \underline{\mathbf{F}}\, dt$ if $\underline{\mathbf{F}}$ constant, $J = F\Delta t$.

$\therefore$ $F_{av} = J/\Delta t$.

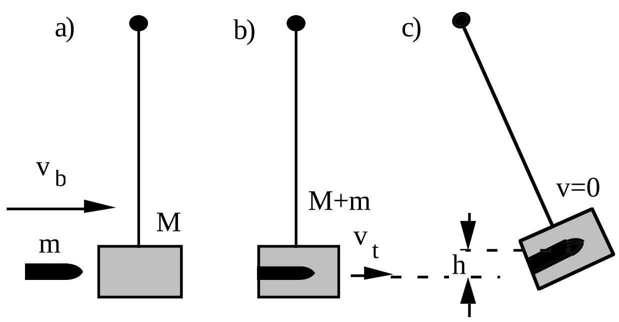
Const F $\Rightarrow$ const a. $s = v_{av}\Delta t$. $v_{av}$ = 35 $m.s^{-1}$

| | | | |
|---|---|---|---|
| $\Delta t$ | 1 ms | 10 ms | |
| F | 11 kN | 1.1 kN | *ouch!* |
| s | 3.5 cm | 35 cm | |

**Example**. (*Common method to measure speed of bullet.*) Bullet (m) with $v_b$ fired into stationary block (M) on string. (i) What is their (combined) velocity after the collision? (ii) What is the kinetic energy of the bullet? (iii) of the combination? (iv) How high does the block then swing?

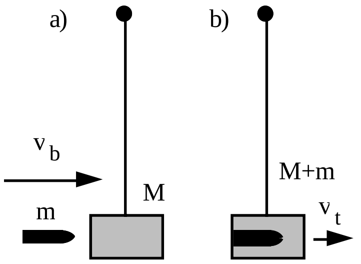
*a-b): collision, no horizontal external forces* $\therefore$ ***momentum conserved****. Friction does work, so* ***mechanical energy is lost****, not conserved*

*b-c): during this phase, external forces* ***do*** *act, so* ***momentum is lost****, not conserved. However, there are no non-conservative forces, so* ***mechanical energy conserved****.*




*Note the different stages:*

Analyse a) to b)




No horizontal ext forces during collision $\therefore$ momentum conserved

i) $P_{xi} = P_{xf}$

$$mv_b = (m + M)v_t$$

$$v_t = \frac{m}{m + M} v_b$$

ii) $K_b = \frac{1}{2} mv_b^2$

iii) $K_t = \frac{1}{2}(m + M)v_t^2 = \frac{1}{2}(m + M)\left(\frac{m}{m + M} v_b\right)^2$

$$= \frac{1}{2} \frac{m^2}{m + M} v_b^2 < K_b.$$

Conclusion: $U_i = U_f$, $K_i \neq K_f$.

Mechanical energy is ***not*** conserved - deformation of block is **not elastic**; heat is produced.

During a collision with negligible external forces,

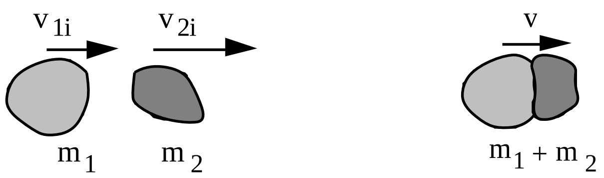
$$\underline{\mathbf{P}} = M\underline{\mathbf{v}}_{cm} \quad \text{is conserved}$$

M constant $\therefore$ $\underline{\mathbf{v}}_{cm}$ is constant $\therefore$ $\frac{1}{2} M\underline{\mathbf{v}}_{cm}^2$ constant

K of c.m. is ***not*** lost. But the K of components with respect to c.m. *can* be lost.

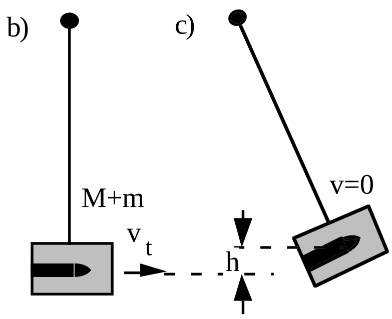
Greatest possible loss of K: if all final velocities = $\underline{\mathbf{v}}_{cm}$, i.e. if all objects stick together after collision.

Called **completely inelastic collision.**




**Completely elastic collision** is one in which no non-conservative forces do work, so mechanical energy is conserved.

Part (iv) of previous example (b-c):



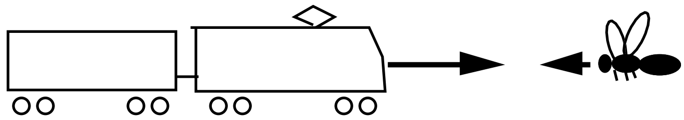

here the external forces (gravity and tension) *do* do work and change momentum. But there is no non-conservative force and so *in this part of the process* conservation of mechanical energy applies:

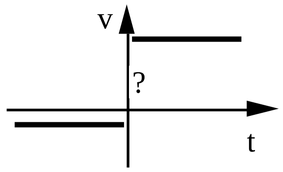
$$\Delta U + \Delta K = 0$$

$$(M + m)g\,(\Delta h - 0) + (K_t - 0) = 0$$

$$\Delta h = \ldots = \frac{1}{2} \frac{m^2}{g(m + M)^2} v_b^2$$

**Puzzle.** Fly travelling West at 5m/s meets train travelling East at 30 m/s.






initally, $v_{fly} = -5\ ms^{-1}$

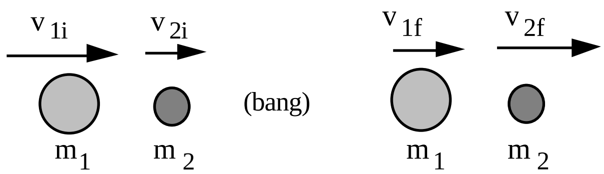
finally, $v_{fly} = +30\ ms^{-1}$

At some time t, the fly travels at 0 $ms^{-1}$

i) does this occur before or after the fly first touches the windscreen.

ii) how fast was the windscreen going when the fly was going 0 $ms^{-1}$?

## Elastic collision in one dimension




**Collision:** neglect external forces $\Rightarrow$

$$p_i = p_f$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad \text{(i)}$$

**elastic** $\Rightarrow$ $K_i = K_f$

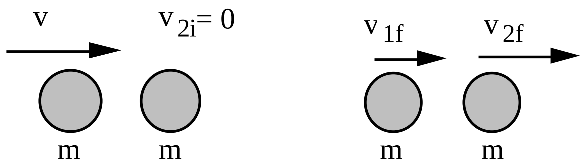
$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \qquad \text{(ii)}$$

usually know $m_1$, $m_2$, $v_{1i}$, $v_{2i}$. Two unknowns ($v_{1f}$, $v_{2f}$), $\therefore$ we can always solve.

Or: transform to frame where (e.g.) $v_1 = 0$

Or: transform to centre of mass frame.

**Example**. Take $m_1 = m_2$, $v_{2i} = 0$, $v_{1i} = v$.




neglect external forces $\Rightarrow$ $p_i = p_f$

$$mv + 0 = mv_{1f} + mv_{2f} \qquad \text{(i)}$$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 \qquad \text{(ii)}$$

(i) $\rightarrow$ $v_{2f} = v - v_{1f}$ (iii)

*substitute in (ii)* $\rightarrow$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}m(v^2 + v_{1f}^2 - 2vv_{1f})$$

$$\therefore \quad 0 = v_{1f}^2 - vv_{1f}$$

$$0 = v_{1f}(v_{1f} - v) \qquad \textit{2 solutions}$$

Either: $v_{1f} = 0$ and (iii)$\rightarrow$ $v_{2f} = v$

i.e. 1st stops dead, all p and K transferred to $m_2$

or: $v_{1f} = v$ and (iii)$\rightarrow$ $v_{2f} = 0$

i.e. missed it.

**Example** Show that, for an elastic collision in one dimension, the relative velocity is unchanged.

*i.e. show* $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

p and K conservation gave:

(i) $m_1(v_{1i} - v_{1f}) = m_2(v_{2i} - v_{2f})$

(ii) $\frac{1}{2}m_1(v_{1i}^2 - v_{1f}^2) = \frac{1}{2}m_2(v_{2i}^2 - v_{2f}^2)$

If they hit, $(v_{1i} - v_{1f}) \neq 0$, $(v_{2i} - v_{2f}) \neq 0$

(ii)/(i) $\Rightarrow$ $v_{1i} + v_{1f} = v_{2i} + v_{2f}$

$$\therefore \quad v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

i.e. relative velocity the same before and after Solve $\rightarrow$ $v_{2f} = \dfrac{2m_1}{m_1 + m_2}v_{1i} + \dfrac{m_2 - m_1}{m_2 + m_1}v_{2i}$

**Example** Two similar objects, mass m, collide completely inelastically.

case 1: $v_{1i} = v, \quad v_{2i} = 0.$

case 2: $v_{1i} = v, \quad v_{2i} = -v.$

What energy is lost in each case?

p conserved $\rightarrow \quad mv_{1i} + mv_{2i} = 2mv_f$

$$v_f = \frac{v_{1i} + v_{2i}}{2}$$

$$\Delta K = K_f - K_i = \frac{1}{2}(2m)v_f^2 - \frac{1}{2}mv_{1i}^2 - \frac{1}{2}mv_{2i}^2$$

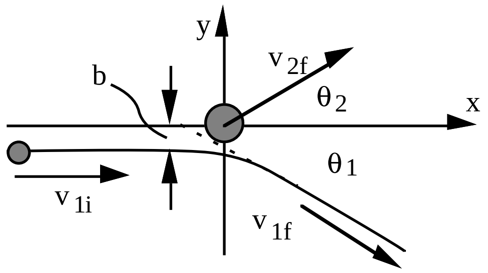
case 1: $\quad \Delta K = \frac{1}{2}(2m)\left(\frac{v+0}{2}\right)^2 - \frac{1}{2}mv^2$

$$= -\frac{1}{4}mv^2$$

case 2: $\quad \Delta K = \frac{1}{2}(2m)\left(\frac{0+0}{2}\right)^2 - \frac{1}{2}mv^2 - \frac{1}{2}mv^2$

$= -mv^2$ 4 times as much energy lost

**Elastic collisions in 2 (& 3) dimensions**




Choose frame in which $m_2$ stationary, $v_{1i}$ in x dir$^n$

b is called impact parameter (distance "off centre")

$p_x$ conserved $\quad m_1v_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$

$p_y$ conserved $\quad 0 = mv_{2f}\sin\theta_2 - mv_{1f}\sin\theta_1$

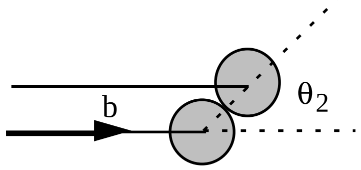
K conserved

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 + \Delta K \qquad \text{(iii)}$$

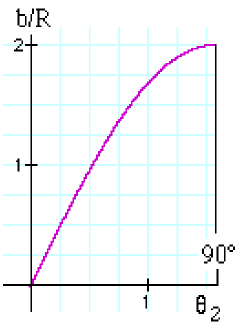
where $\Delta K = 0$ for elastic case

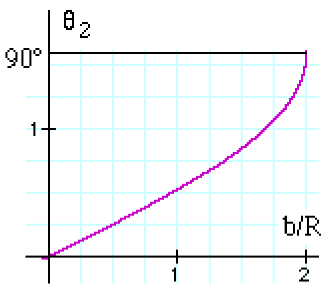
3 equations in $v_{1f}$, $v_{2f}$, $\theta_1$ and $\theta_2$: need more info (often given $\theta_1$ or $\theta_2$)

Incidentally: for billiard balls, neglecting rotation and friction (*reasonable during collision, but not after*)

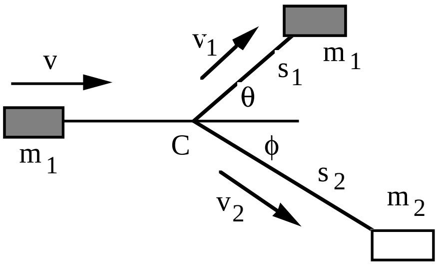



$(R + R)\sin\theta_2 = b \qquad\qquad \theta_2 = \sin^{-1}\frac{b}{2R}$




i) Note that as $\theta \rightarrow 90°$, small error in b gives large error in $\theta_2$. ii) Does $b = R$ give $\theta_2 = 30°$?




**Example**. Police report of road accident. Car 1, mass $m_1$ strikes stationary car $m_2$ at point C. They then slide to rest in positions shown. Given $\mu_k = \mu$ (assumed same for both) find the initial speed v of $m_1$. Can you check assumption? (*real example*)

After collision, a for both $= \frac{F_f}{m} = -\ \mu \frac{W}{m} = -\mu g$

$$v_f^2 - v_i^2 = 2as = -2\mu g s$$

$$0 - v_1^2 = -2\mu g s_1$$

$$v_1 = \sqrt{2\mu g s_1} \qquad\qquad v_2 = \sqrt{2\mu g s_2}$$

Neglect external forces during collision: $\Delta P = 0$

$P_x$: $\quad m_1 v = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi \quad$ (i)

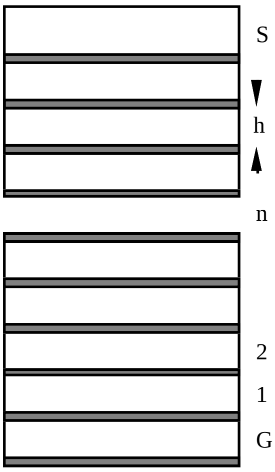
$P_y$: $\quad 0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\phi \quad$ (ii)

(i) $\Rightarrow \quad v = \sqrt{2\mu g s_1}\cos\theta + \frac{m_2}{m_1}\sqrt{2\mu g s_2}\cos\phi$

Note the "spare" equation—we can use it to check the model or assumptions:
(The $\mu$ may not be the same for the two: surfaces different etc)

(ii) $\Rightarrow \quad m_1\sqrt{2\mu_1 g s_1}\sin\theta = m_2\sqrt{2\mu_2 g s_2}\sin\phi$

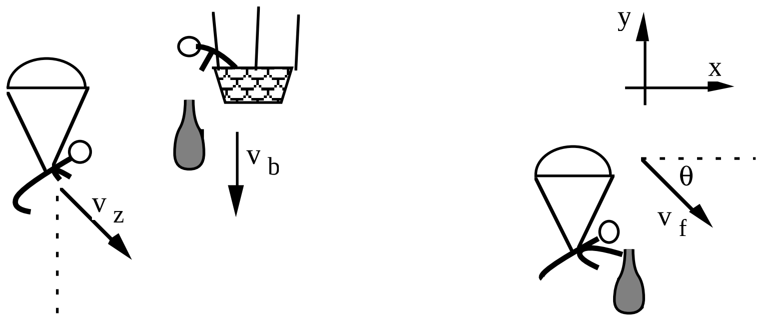
$$\frac{\mu_2}{\mu_1} = \frac{s_1 m_1^2 \sin^2\theta}{s_2 m_2^2 \sin^2\phi}$$




**Example** A building has S stories, each of height h. An explosion destroys strength of $n^{th}$ floor. How long before $(n+1)^{th}$ floor hits ground, falling vertically?

Assume inelastic collisions between floors. To obtain a lower estimate, assume negligible strength between floors.

**Example**




Balloonist Albert writes message on a bottle (1 kg) and drops it over the side. It is falling vertically at 40 m.s$^{-1}$ when caught by parachutist Zelda (m = 50 kg), travelling at 1 ms$^{-1}$ at 45° to vertical. Collision (bottle—Zelda's hand) lasts 10 ms.

i) If only gravity acted, what is $\Delta p$ for Zelda over 10 ms ?

ii) Neglecting ext forces during collision, what is the velocity of (Zelda+bottle) after collision?

iii) What impulse is applied to bottle during collision?

iv) What is the impulse applied to Zelda?

v) What is the average force during collision?

i) due to $\underline{\mathbf{W}}$, $\Delta\underline{\mathbf{p}} = \underline{\mathbf{W}}\,\Delta t = .. = 5$ kgm.s$^{-1}$ down

ii) Neglect ext forces $\Rightarrow$ momentum conserved.

$$m_b \underline{\mathbf{v}}_{bi} + m_Z \underline{\mathbf{v}}_{Zi} = m_{(Z+b)} \underline{\mathbf{v}}_{(Z+b)f}$$

$$1(-40\,\underline{\mathbf{j}}) + 50(1\cos 45°\,\underline{\mathbf{i}} - 1\cos 45°\,\underline{\mathbf{j}}) = 51(v_x\,\underline{\mathbf{i}} + v_y\,\underline{\mathbf{j}})$$

$\underline{\mathbf{i}}$ dir$^n$: $\quad v_x = \cos 45° \cdot \frac{50}{51} \text{x} 1 = 0.7 \text{ ms}^{-1}$

$\underline{\mathbf{j}}$ dir$^n$: $\quad v_y = \frac{-1\text{x}40 - 50\cos 45°}{51} = -1.5 \text{ ms}^{-1}$

$\therefore \quad |v_f| = \sqrt{v_x^2 + v_y^2} = 1.6 \text{ ms}^{-1}$

$\theta_f = \tan^{-1}\frac{v_y}{v_x} \Rightarrow 67°$ to horizontal

iii) $\underline{\mathbf{I}} \equiv \underline{\mathbf{p}}_{bf} - \underline{\mathbf{p}}_{bi} = 1\text{x}(v_x\,\underline{\mathbf{i}} + v_y\,\underline{\mathbf{j}}) - 1(-40\,\underline{\mathbf{j}})$

$= (1.6\,\underline{\mathbf{i}} + 38\,\underline{\mathbf{j}})$ kgm.s$^{-1}$

iv) $\underline{\mathbf{I}}_Z = -\underline{\mathbf{J}}_b = -(1.6\,\underline{\mathbf{i}} + 38\,\underline{\mathbf{j}})$ kgm.s$^{-1}$

$|\underline{\mathbf{I}}| = \sqrt{1.6^2 + 38^2} = 38$ kgm.s$^{-1}$

v) $\quad \bar{\underline{\mathbf{F}}}_Z = \frac{\Delta\,\underline{\mathbf{p}}_Z}{\Delta t} = \frac{|\underline{\mathbf{I}}|}{\Delta t} = .. = 380$ N