PHYS 1121 and 1131. Particle dynamics

www.phys.unsw.edu.au/~jw/1131.html www.physclips.unsw.edu.au

Newton's laws:
force, mass, acceleration also weight
Friction - coefficients of friction
Hooke's Law
Dynamics of circular motion

Aristotle: **v** = 0 is "natural" state
Galileo & Newton: **a** = 0 is "natural" state

Newton's Laws
First "zero (total) force ⇒ zero acceleration"
more formally:

If $\Sigma \mathbf{F} = 0$, ∃ reference frames in which **a** = 0

∃ ≡ "there exists"

called **Inertial frames**

observation: w.r.t. these frames, distant stars don't accelerate

In inertial frames:

Second $\Sigma \mathbf{F} = m \mathbf{a}$ *Σ is important*

($\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$ $\Sigma F_z = ma_z$) *3D → 3 equations*

1st law is special case of 2nd

What does the 2nd law mean?

$\Sigma \mathbf{F} = m_i \mathbf{a}$ and $\mathbf{W} = m_g \mathbf{g}$

are m_i and m_g necessarily the same?
called inertial and gravitational masses

$\mathbf{F} = m \mathbf{a}$
a is already defined.

i) Does this equation define m?
ii) Does this law define **F** ?
iii) Is it a physical law?
iv) All of the above?
v) How?

Newton 1: "Every body persists in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."

postulate

An **inertial frame** of reference is one in which Newton's 1st law is true. *definition*

Such frames exist (and w.r.t. these frames, distant stars don't accelerate) *observation*:

$\therefore$ if $\Sigma \underline{\mathbf{F}} = 0$, $\underline{\mathbf{a}} = 0$ w.r.t. distant stars.

Force causes acceleration. $\underline{\mathbf{F}} \mathbin{//} \underline{\mathbf{a}}$, $\underline{\mathbf{F}} \propto \underline{\mathbf{a}}$

definition

Newton 2: To any body may be ascribed a scalar constant, mass, such that the acceleration produced in two bodies by a given force is inversely proportional to their masses,

i.e. for same F, $\frac{m_2}{m_1} = \frac{a_1}{a_2}$

Already have metre, second, choose a standard body for kg, then choose units of F (Newtons) such that

$$\underline{\mathbf{F}} = m\,\underline{\mathbf{a}}$$

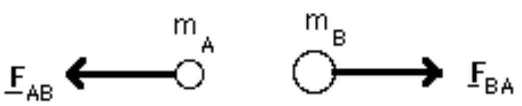
(*this eqn. is laws 1&2, definition of mass and units of force*)

Newton 3: "To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

Or

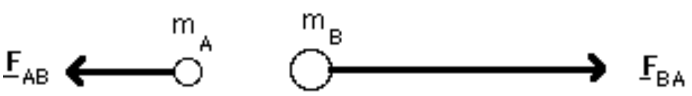
Forces always occur in pairs, $\underline{\mathbf{F}}$ and $-\underline{\mathbf{F}}$, one acting on each of a pair of interacting bodies.

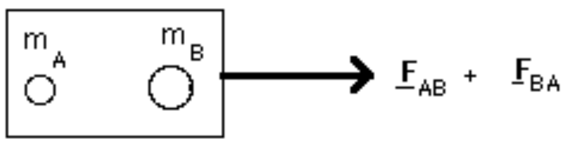
Important conclusion: internal forces in a system add to zero.



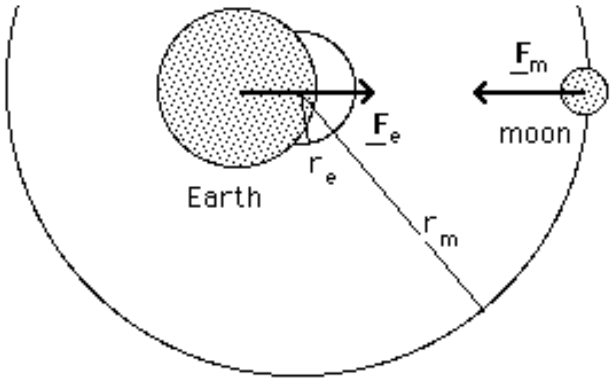
Third $\underline{\mathbf{F}}_{AB} = -\underline{\mathbf{F}}_{BA}$

Why so?





Example Where is centre of earth-moon orbit?



$F_e = F_m = F_g$ equal and opposite *NB sign conventions*

each makes a circle about common centre of mass

$F_g = m_m a_m = m_m \omega^2 r_m$

$F_g = m_e a_e = m_e \omega^2 r_e$

$\therefore \frac{r_m}{r_e} = \frac{m_e}{m_m} = \frac{5.98\ 10^{24}\ \text{kg}}{7.36\ 10^{22}\ \text{kg}} = 81.3$ (i)

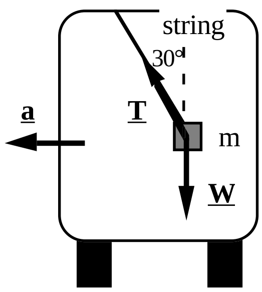
earth-moon distance $r_e + r_m = 3.85\ 10^8$ m (ii)

solve $\rightarrow r_m = 3.80\ 10^8$ m, $r_e = 4.7\ 10^6$ m

$\therefore$ centre of both orbits is inside earth *(later: c. of mass)*

Example. As the bus takes a steady turn with radius 8 m at constant speed, you notice that a mass on a string hangs at 30° to the vertical. How fast is the bus going?

Draw a diagram with physics



We know:
tension in direction of string,
weight down,
acceleration horizontal
circular motion

mass in circular motion with bus, so force on m:

$$F_{horiz} = ma = m\frac{v^2}{r}$$

Only the tension has a horizontal component, so

$$T \sin 30° = m\frac{v^2}{r}$$

Need one more eqn: mass is not falling down, ie

vertical acceleration = 0, so

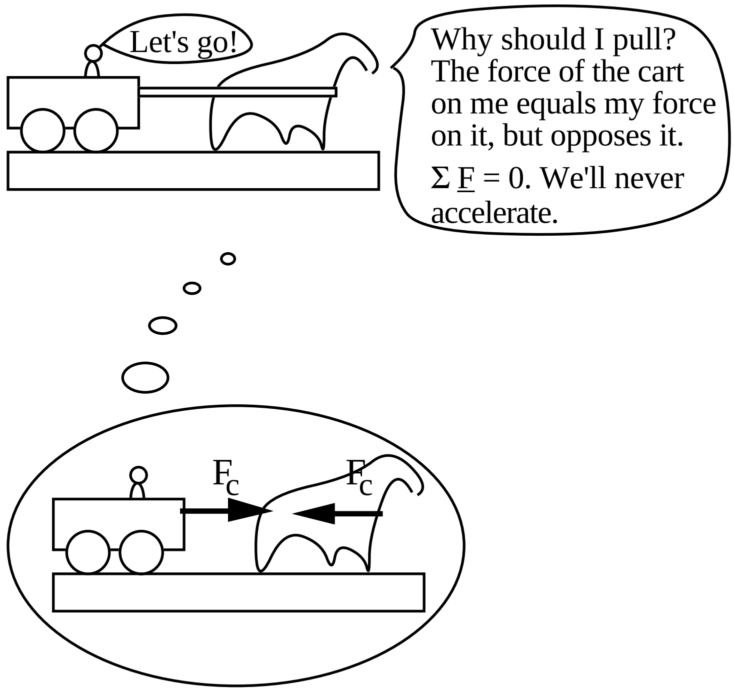
$$T \cos 30° = mg$$

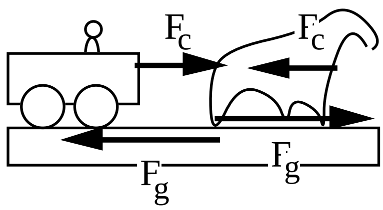
Eliminate T: $\tan 30° = m\frac{v^2}{r}\frac{1}{mg}$

$\rightarrow v = \sqrt{gr \tan 30°} = 6.7$ m/s $= 24$ kph. $\rightarrow$ 20 kph to 1 sig fig

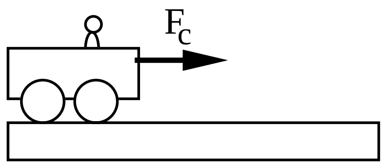
Check dimensions. Reasonable? And so...?

Problem. Horse and cart. Wheels roll freely.



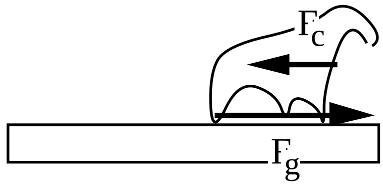


Horizontal forces on cart (mass m_c)



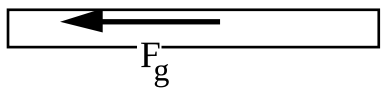
$F_c = m_c a_c = m_c a$

Horizontal forces on horse (mass m_h)



$F_g - F_c = m_h a$

Horizontal forces on Earth (mass m_E)

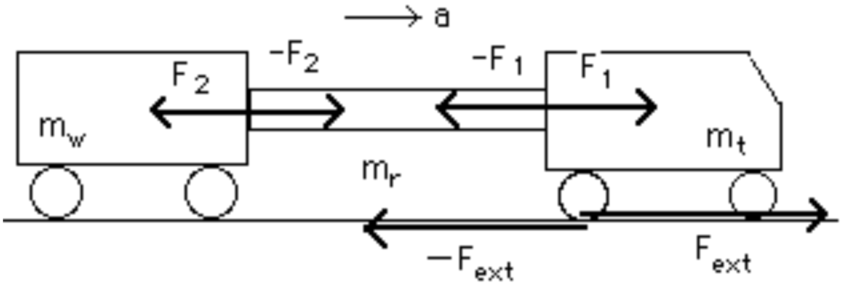


$m_E >> m_h + m_c$

"light" ropes etc.

Here, light means m << other masses

Truck (m_t) pulls wagon (m_w) with rope (m_r).
All have same **a**.



i) wagon: $-F_2 = m_w a$.

ii) rope: $F_1 - F_2 = m_r a$

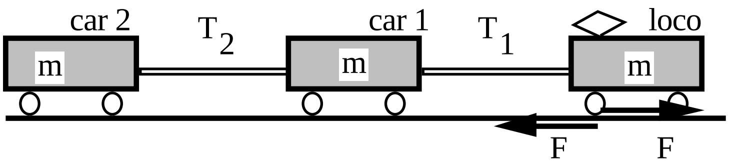
iii) truck: $-F_1 + F_{ext} = m_t a$

(ii)/(i) $\rightarrow$ $\frac{F_1 - F_2}{-F_2} = \frac{m_r a}{m_w a}$

$\therefore$ if $m_r << m_w$, $F_1 = F_2$.

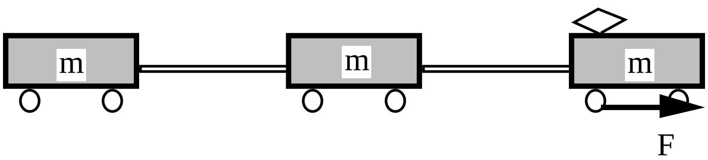
Forces at opposite ends of light ropes etc are equal and opposite.

Example. Train. Wheels roll freely. Loco exerts horizontal force F on the track. What are the tensions T_1 and T_2 in the two couplings?



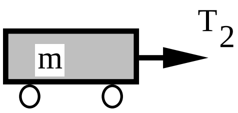
Whole train accelerates together with a.

Look at the external forces acting on the train (horiz. only).



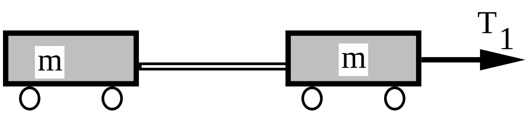
$F = (m+m+m)a \rightarrow a = F/3m$

Look at horiz forces on car 2:



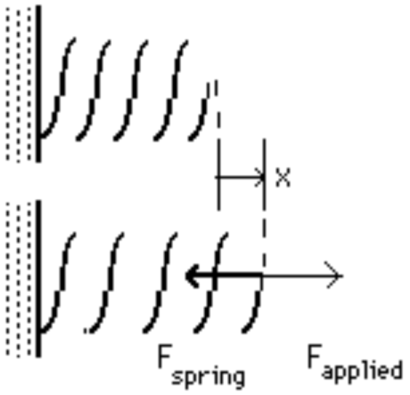
$T_2 = ma = F/3$

and on cars 2 and 1 together



$T_1 = 2ma$

Hooke's Law.



No applied force
(x = 0)

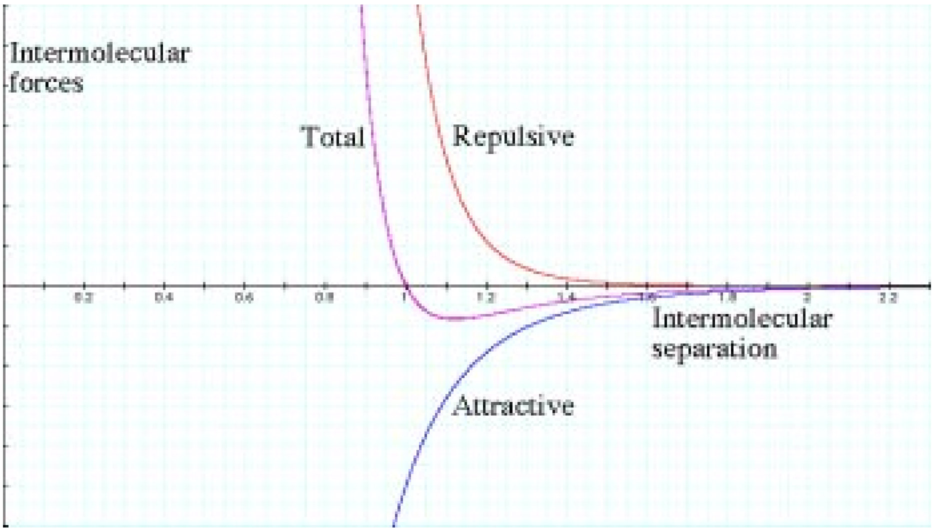
Under tension
x > 0

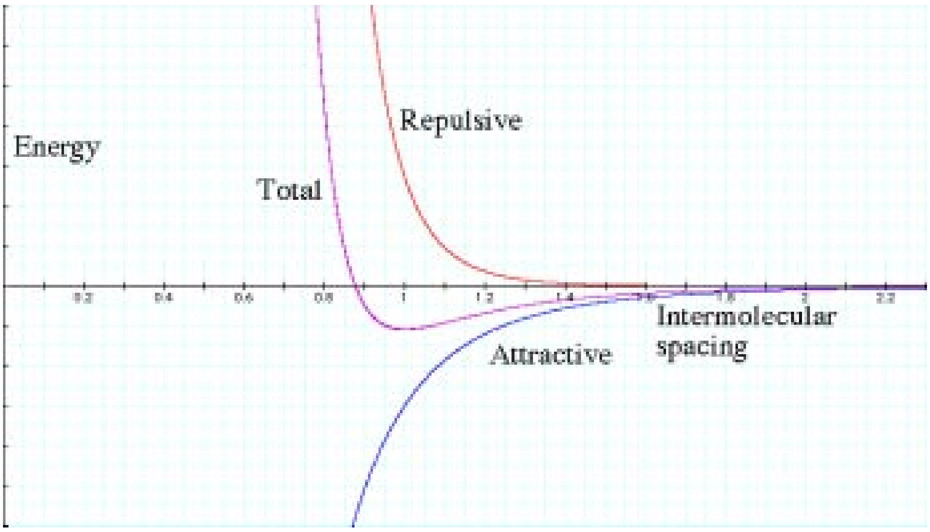
$\underline{\mathbf{F}}_{spring}$ in opposite direction to x.
Experimentally, $|\underline{\mathbf{F}}_s| \propto |x|$ over small range of x

$$F = -kx$$ **Hooke's Law.**

linear elastic behaviour - more in S2.

Why linear elasticity?
Intermolecular forces F and energies U:





See tut problem on interatomic forces

Mass and weight
(inertial) mass m defined by F = ma
observation:
near earth's surface and without air,
all (?) bodies fall with same a (= –g)
weight W = – mg

Warning: do not confuse mass and weight, or their units
kg → mass N → force ($kg.m.s^{-2}$)
kg wt = weight of 1 kg = mg = 9.8 N

What is your weight?

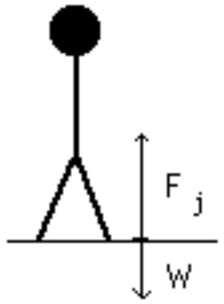
Why is $W \propto m$? *Why is $m_g \propto m_i$?*
ma = F = W = (Grav field).(grav. property of body)

- Mach's Principle
- Principle of General Relativity
- Interactions with vacuum field

Example Grav. field on moon $g_m = 1.7\ ms^{-2}$. An astronaut weighs 800 N on Earth, and, while jumping, exerts 2kN while body moves 0.3 m. What is his weight on moon? How high does he jump on earth and on moon?

$mg_E = W_E \rightarrow \quad m = \dfrac{800\ N}{9.8\ ms^{-2}} = 82\ kg$

$W_m = mg_m = 82kg\ 1.7ms^{-2} = 140\ N$

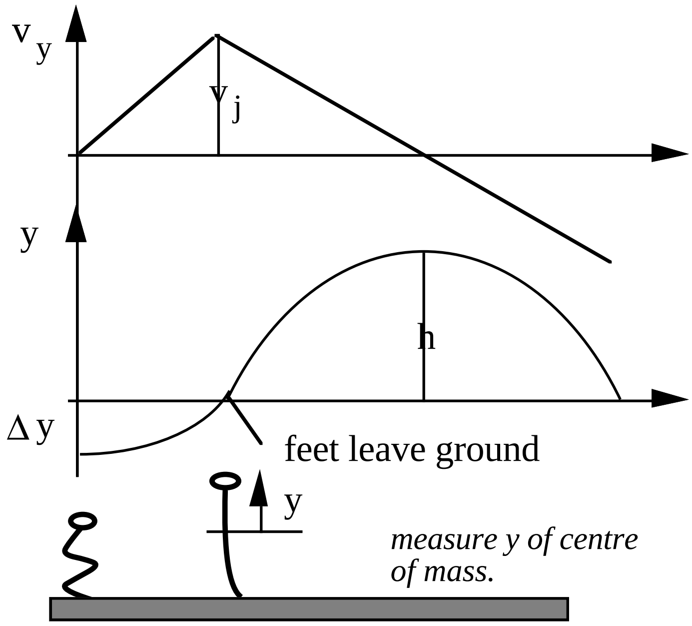


Vertical (y) motion with const accel. While feet are on ground,
$\Sigma F = 2\ kN - W_E$
$= 1.2\ kN$ (Earth)
Moon:
$\Sigma F = 2\ kN - mg_m = 1.9\ kN$

Jump has two parts:

feet on ground $\left(a = \dfrac{\Sigma F}{m}\right)$ $\quad v_i = 0, v_f = v_j$

feet off ground $a = -g$ $\quad v_i = v_j, v_f = 0$



While on ground:

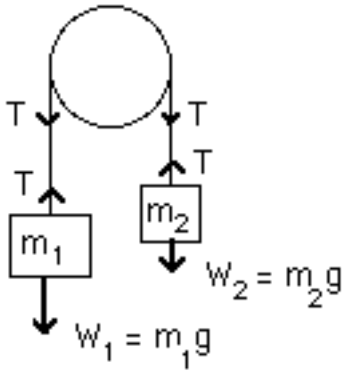
$v_j^2 - v_o^2 = 2a_j\Delta y = 2\dfrac{\Sigma F}{m}\Delta y$

Earth $\rightarrow v_j = 3.0\ ms^{-1}$ Moon $\rightarrow v_j = 3.7\ ms^{-1}$

While above ground:

$v^2 - v_j^2 = -2gh \quad \rightarrow \quad h = \dfrac{v_j^2}{2g}$

$h_E = 0.5$ m. $\quad h_m = 4$ m



Example

Light pulley, light string. What is acceleration of the masses?

Let a be accel (down) of m_1 = accel (up) of m_2.

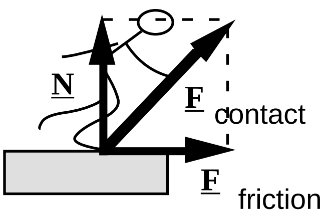
Newton 2 for m_1: $T - m_1 g = -m_1 a$

" " " m_2: $T - m_2 g = +m_2 a$

subtract: $-m_1 g + m_2 g = -m_1 a - m_2 a$

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

(Check: if $m_1 = m_2$, a = 0. If $m_2 = 0$, a = g.)



Contact forces

The normal component of a contact force is called the **normal force <u>N</u>**. The component in the plane of contact is called the **friction force <u>F</u>**$_f$.

Normal force: at right angles to surface, is provided by deformation.

- If ∃ relative motion, **kinetic** friction (whose direction opposes relative motion)
- If ∃ *no* relative motion, **static** friction (whose direction opposes applied force)

Define coefficients of kinetic (k) and static (s) friction:

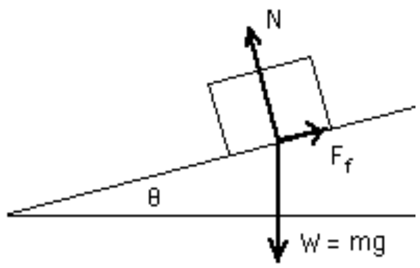
$|F_f| = \mu_k N$ $|F_f| \le \mu_s N$

Friction follows this **approximate** empirical law

μ_s and μ_k are approx. independent of N and of contact area.

Often $\mu_k < \mu_s$.

(It takes less force to keep sliding than to start sliding.)



Example. θ is gradually increased to θ_c when sliding begins. What is θ_c? What is a at θ_c?

Newton 2 in normal dirn:

$$N - mg\cos\theta = 0 \qquad \text{(i)}$$

Newton 2 in dirn down plane:

$$mg\sin\theta - F_f = ma. \qquad \text{(ii)}$$

No sliding: $a = 0$

$\therefore$ (ii) $\Rightarrow$ $mg\sin\theta = F_f$

$\leq \mu_s N$

(i) $\Rightarrow$ $= \mu_s mg\cos\theta$

$mg\sin\theta \leq \mu_s mg\cos\theta$

$\tan\theta \leq \mu_s$, $\theta_c = \tan^{-1}\mu_s$ *useful technique for μ_s*

Sliding at $\theta = \theta_c$: $a > 0$

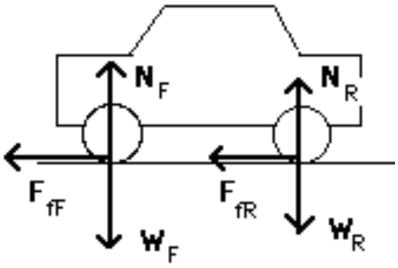
$\therefore$ (ii) $\Rightarrow$ $a = g\sin\theta_c - \frac{F_f}{m}$

$= g\sin\theta_c - \frac{\mu_k N}{m}$

(i) $\Rightarrow$ $= g\sin\theta_c - \mu_k g\cos\theta_c$

$=$

$= g\cos\theta_c(\mu_s - \mu_k)$



Example. Rear wheel drive car, 300 kg wt on **each** front wheel, 200 kg wt on rear. Rubber-road:

$\mu_s = 1.5$, $\mu_k = 1.1$

Neglect rotation of car during accelerations. Assume that brakes produce 1.8 times as much force on front wheels as on back. (i) What is max forward accel without skidding? What is maximum deceleration (ii) not skidding? (iii) 4 wheel skid?

(i) $F_{fRs} \leq \mu_s N_R = \mu_s W_R$

$$= 1.5\text{x}200\text{x}9.8 \text{ N} = 2.9 \text{ kN}$$

$$a_{max} = \frac{F}{m} = \frac{2 \text{ x } 2.9 \text{ kN}}{(2\text{x}300 + 2\text{x}200)\text{kg}} = 5.8 \text{ ms}^{-2}$$

Stopping.

For all wheels, $F_{fs} \leq \mu_s N = \mu_s W$

$F_{fF} = 1.8\, F_{fR}$. $\mu_s W_F = 1.5\, \mu_s W_R$,

$\therefore$ front wheels skid first, when $F_{fF} > \mu_s W_F$.

$$\text{max total friction} = \text{(front + rear)} = \left(2 + \frac{2}{1.8}\right).\mu_s W_F$$

$$= 1400 \text{ kg.wt} = 14 \text{ kN}$$

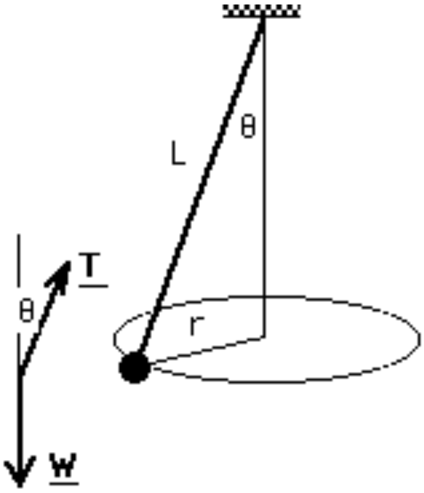
ii) $a_{max} = \dfrac{F_{max}}{m} = \dfrac{14 \text{ kN}}{1000 \text{ kg}} = 14 \text{ ms}^{-2}$

iii) $a = \dfrac{\Sigma F_k}{m} = \dfrac{\Sigma \mu_k W}{m} = \ldots = 11 \text{ ms}^{-2}$

Questions:

Does area of rubber-road contact make a difference?

Does the size of the tire make a difference?



Example
Conical pendulum
(Uniform circular motion.)
What is the frequency?

Apply Newton 2 in two directions:

Vertical: $a_y = 0 \quad \therefore \quad \Sigma F_y = 0$

$\therefore \quad T \cos\theta - W = 0$

$$T = \frac{mg}{\cos\theta}$$

Horizontal:

$$\frac{mv^2}{r} = ma_c = T \sin\theta$$

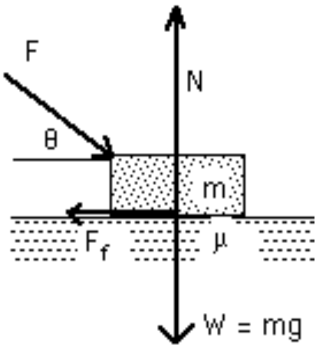
$$= \frac{mg \sin\theta}{\cos\theta}$$

$$\therefore \quad \frac{v^2}{r} = g \tan\theta$$

$$\therefore \quad v = \sqrt{rg \tan\theta}$$

$$\therefore \quad \frac{2\pi r}{T} = \sqrt{rg \tan\theta}$$

$$\therefore \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g \tan\theta}{r}}$$



Example.
Apply force F at θ to horizontal. Mass m on floor, coefficients μ_s and μ_k. For any given θ, what F is required to make the mass move?
Eliminate 2 unknowns N and $F_f \to F(\theta, \mu_s, m, g)$

Stationary if $F_f \le \mu_s N$ (1)

Newton 2 vertical: $N = mg + F \sin\theta$ (2)

Newton 2 horizontal: $F \cos\theta = F_f$ (3)

(1,3) → stationary if $F \cos\theta \le \mu_s N$

$F \cos\theta \le \mu_s(mg + F \sin\theta)$ *(using (2))*

$F(\cos\theta - \mu_s \sin\theta) \le \mu_s mg$ (*)

note importance of sign of (.....)

if $(\cos\theta - \mu_s \sin\theta) = 0$,

$\theta = \theta_{crit} = \tan^{-1}(1/\mu_s)$.

If $\theta < \theta_c$, then $(\cos\theta - \mu_s \sin\theta) > 0$

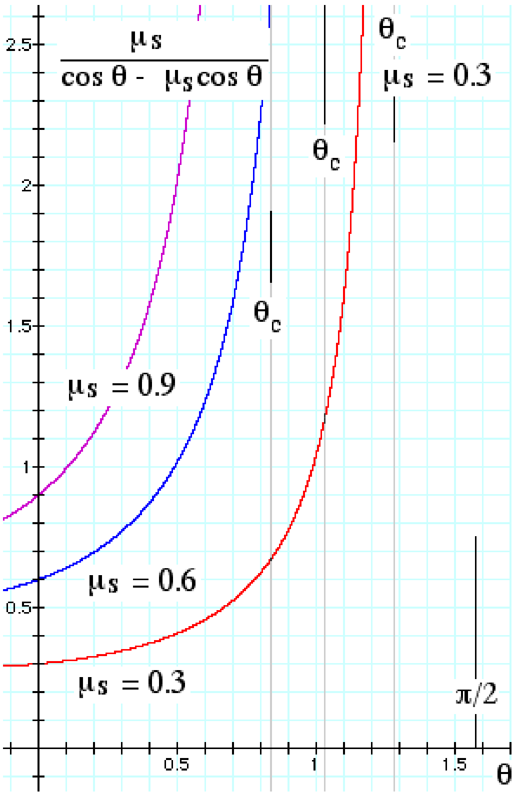
stationary if $F \le \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$

i.e. moves when $F > F_{crit} = \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$

If $\theta > \theta_c$, then (....) > 0

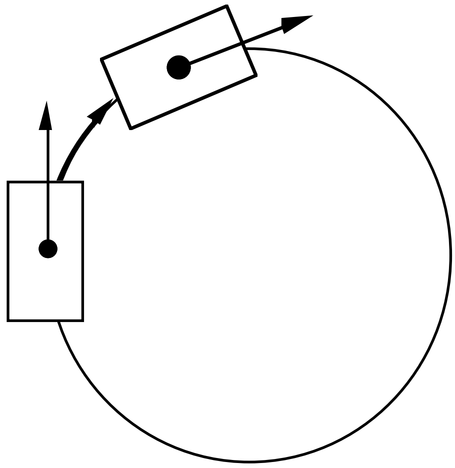
(*) ⇒ stationary if $F \ge \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$

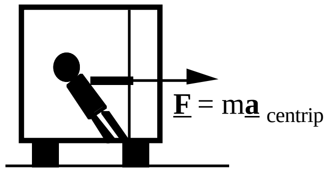
i.e. stationary no matter how large F becomes.



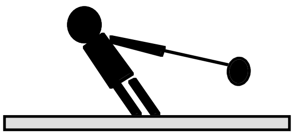
Centripetal acceleration and force

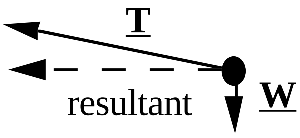
Circular motion with ω = const. and v const. eg bus going round a corner





Or consider hammer thrower

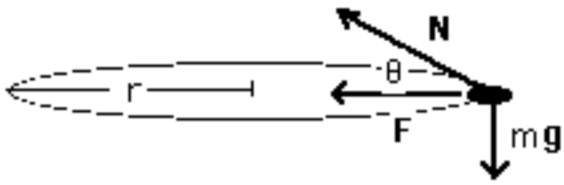




Resultant force produces acceleration in the horizontal direction, towards the centre of the motion

Centripetal force, centripetal acceleration

Example Plane travels in horizontal circle, speed v, radius r. For given v, what is the r for which the normal force exerted by the plane on the pilot = twice her weight? What is the direction of this force?



Centripetal force $F = m\frac{v^2}{r} = N\cos\theta$

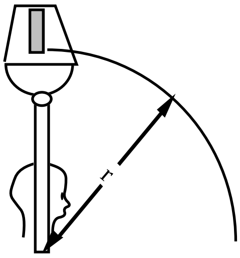
Vertical forces: $N\sin\theta = mg$

eliminate θ: $N^2 = m^2\left(\frac{v^4}{r^2} + g^2\right)$

$$\left(\frac{N^2}{m^2} - g^2\right) = \frac{v^4}{r^2} \rightarrow r = \frac{v^2}{\sqrt{\frac{N^2}{m^2} - g^2}}$$

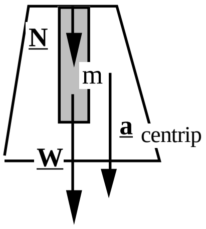
$$\sin\theta = \frac{mg}{N} = \frac{1}{2}$$

$\therefore$ 30° above horizontal, towards axis of rotation



Example. Foolhardy lecturer swings a bucket bricks in a vertical circle. How fast should he swing so that the bricks stay in contact with the bucket at the top of the trajectory?

Draw diagram & identify important variables
pose question mathematically.



W and **N** provide centripetal force.

$mg + N = ma_c$

For contact, we need

$N \geq 0$

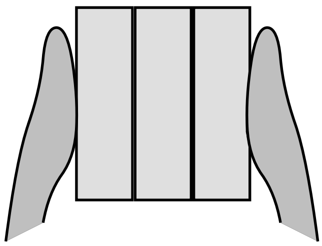
so $ma_c \geq mg$ *how to express a_c?*

$$a_c = \frac{v^2}{r} = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$$ *T is easy to measure*

$$T = 2\pi\sqrt{\frac{r}{a_c}} \leq 2\pi\sqrt{\frac{r}{g}}$$

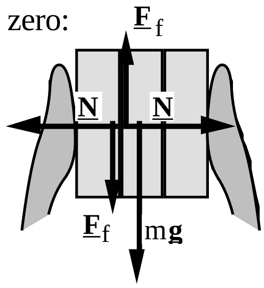
$r \sim 1\text{m} \rightarrow T \leq 2$ s. *please check for errors!*

Question. Three identical bricks. What is the minimum force you must apply to hold them still like this?



Vertical forces on middle brick add to zero:

$$2F_f = mg$$



Define μ_s

$$F_f \leq \mu_s N$$

$$\therefore \quad N \geq \frac{F_f}{\mu_s} = \frac{mg}{2\mu_s}$$

Bricks not accelerating horizontally, so normal force from hands = normal force between bricks.

$\therefore$ (each) hand must provide $\geq \frac{mg}{2\mu_s}$ horizontally.

Vertically, two hands together provide 3mg.