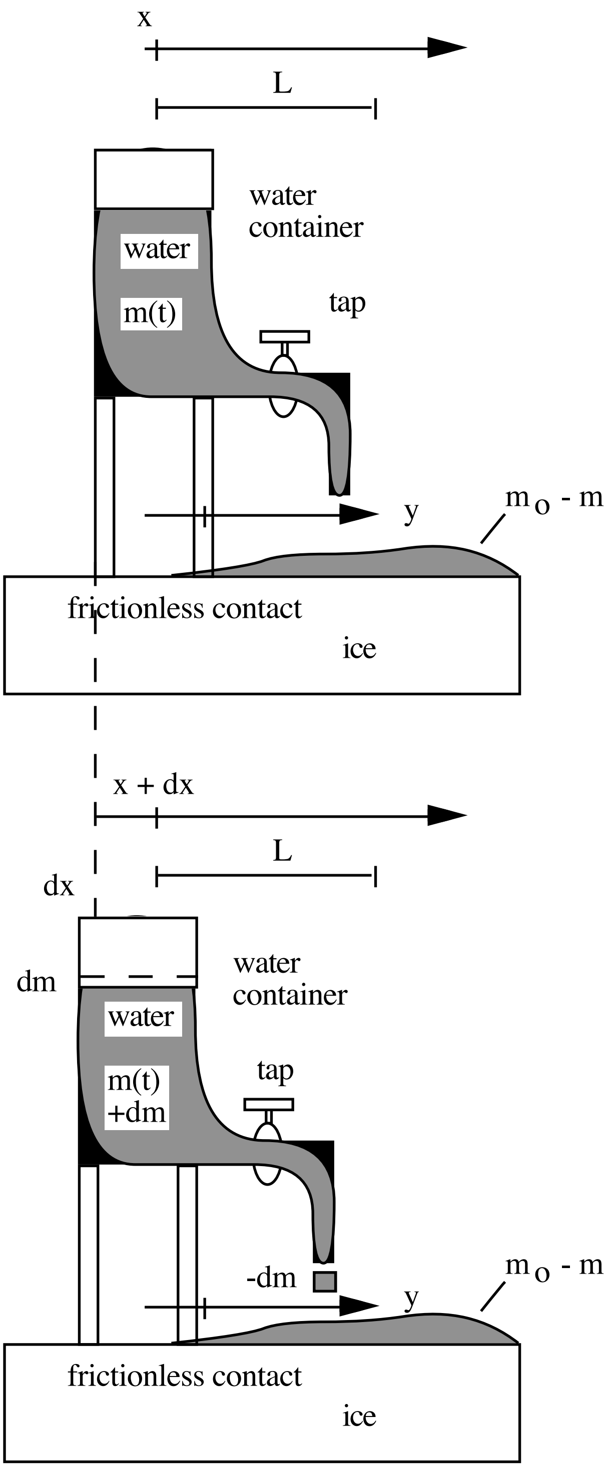

Where does that $%&@ tank go?

Suppose that dm is the increase in m(t), the mass in the tank. (Note dm < 0). A mass −dm leaves the spout. So we have effectively moved (−dm) a distance L to the right. To keep the centre of mass in the same place, the container and the water in it must be displaced by dx (dx < 0) where

$$(m+M)dx + -Ldm = 0$$

$$\frac{dm}{dx} = \frac{1}{L}(m + M)$$

This DE is simpler if we set $\mu = m + M$, which gives

$$\frac{d\mu}{dx} = \frac{\mu}{L} \quad \text{so} \quad \mu = \mu_o e^{x/L}, \quad \text{so}$$

$$m + M = (m_o + M)e^{x/L}$$

So the answer is surprisingly simple:

$$x = L \, ln\left(\frac{m + M}{m_o + M}\right)$$

Dimensions are obviously okay. Lets check the limits: $m = m_o$ gives $x = 0$. Good. $m < m_o$ gives $x < 0$. Good.

As $m \to 0$, $x \to L \, ln\left(\frac{M}{m_o + M}\right)$

x_{final} is *not* a function of t. This last observation is what I found odd at first and what made me set this to puzzle you: the tank actually stops moving at the end. Where does the force to stop it come from? While we're at it, where does the force to start it moving come from?

But let's leave that for the Physics Questions bulletin board, for anyone interested.