

PHYS1121 HIGHER PHYSICS 1A
SOLUTIONS – Homework Problem Set 2

Q1. Applies N2 in the x direction to the total ($m_1 + m_2$):

(a) $F = (m_1 + m_2)a$

$\therefore a = F / (m_1 + m_2)$

Internal force: $F_{\text{int}} = m_2 a = \frac{m_2}{m_1 + m_2} F = 1.0 \text{ N}$

(b) Here $F'_{\text{int}} = \frac{m_1}{m_1 + m_2} F$.

Same acceleration, but $m_1 > m_2$ so a larger force required to accelerate m_1 .

Q2.

In all cases, the force on the man is $N - mg = ma$ in the up direction.

(a) $a = 0$, so $N = mg = 980 \text{ N}$ up

(b) $a = 0$, so $N = mg = 980 \text{ N}$ up

(c) $N - mg = ma \Rightarrow N = m(a + g) = 100 \times 11.8 = 1180 \text{ N}$ up

(d) $N - mg = ma \Rightarrow N = m(g - a) = 680 \text{ N}$ up

(e) $N - mg = ma \Rightarrow N = 580 \text{ N}$ up

(f) $N - mg = ma \Rightarrow N = m(g + a) = 1480 \text{ N}$ up

Q3. Take a positive if m_2 accelerates upwards.

(a) Newton 2 for m_1 in direction of string:

$$m_1 g \sin \theta - T = m_1 a$$

Newton 2 for m_2 : $T - m_2 g = m_2 a$

Eliminate T : add:

$$m_1 g \sin \theta - m_2 g = (m_1 + m_2) a$$

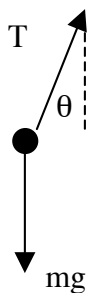
$$a = (m_1 g \sin \theta - m_2 g) / (m_1 + m_2) = -0.10 \text{ g}$$

($a < 0$ means m_2 accelerates downwards)

(b) $m_1 g \sin \theta - T = m_1 a$

$$T = m_1 g \sin \theta + m_1 a = 18 \text{ N}$$

Q4.

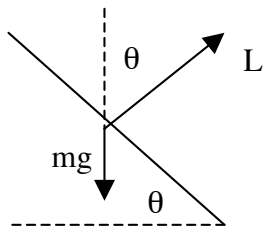


(a) (1) $T \cos \theta = mg$ (N2 in vertical)
 (2) $T \sin \theta = ma$ (N2 in horizontal)
 $\tan \theta = a / g$ divide (1) by (2)
 $a = g \tan \theta$

(b) substitute $\theta = 20^\circ$, find $a = 3.6 \text{ ms}^{-2}$

$$a = 2 \text{ ms}^{-2}, \text{ find } \tan \theta = a/g, \text{ so } \theta = 12^\circ$$

Q5.



$$L \cos \theta = mg \quad (N2 \text{ in vertical})$$

$$L \sin \theta = \frac{mv^2}{r} \quad (N2 \text{ in horizontal})$$

$$\therefore \tan \theta = \frac{v^2}{rg} \quad (\text{divide eqns})$$

$$r = \frac{v^2}{g \tan \theta} = (480 / 3.6)^2 / (9.8 \tan 40^\circ) = 2.2 \text{ km}$$

Q6.

Maximum friction force = $\mu_s N = \mu_s F = 36 \text{ N}$.

Weight = $mg = 29 \text{ N}$.

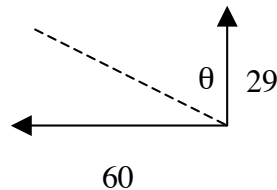
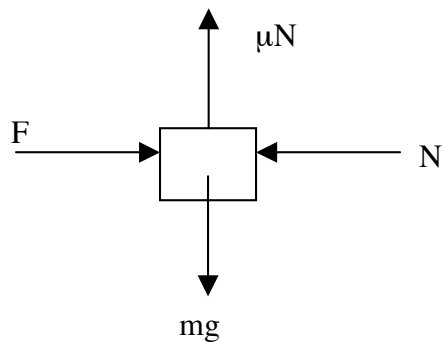
The block will not move.

Actual friction force = 29 N .

$$F = \sqrt{60^2 + 29^2} = 67 \text{ N}$$

$$\theta = \tan^{-1}(60/29)$$

$$\theta = 64^\circ$$



Past exam question

- a) i) Newton's 2nd law in vertical direction: $N = mg$
 Newton's 2nd law in horizontal direction: $F_f = lma$
 greatest frictional force when $F_f = \mu_s N$

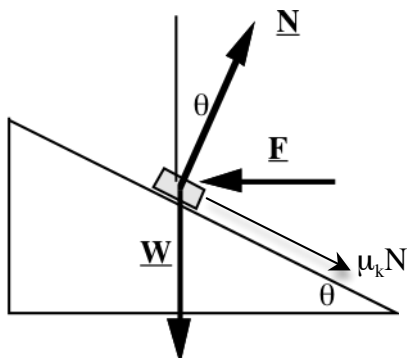
$$l a = \mu_s N / m = \mu_s g$$

Decelerating with constant acceleration $0^2 - v^2 = 2as$, where $a < 0$.

$$s_b = -v^2 / 2a = v^2 / 2a = v^2 / 2\mu_s g.$$

- ii) For 50 kph, $s_b = 12 \text{ m}$
 For 80 kph, $s_b = 30 \text{ m}$

b)



i) No acceleration, so $\Sigma \text{forces} = 0$.

Resolve in the plane and perpendicular to it:

// to plane: $F \cos \theta = \mu_k N + W \sin \theta$

perpendicular to plane $N = F \sin \theta + W \cos \theta$

Hence $F \cos \theta = \mu_k F \sin \theta + \mu_k W \cos \theta + W \sin \theta$

$$\therefore F[\cos \theta - \mu_k \sin \theta] = W[\sin \theta + \mu_k \cos \theta]$$

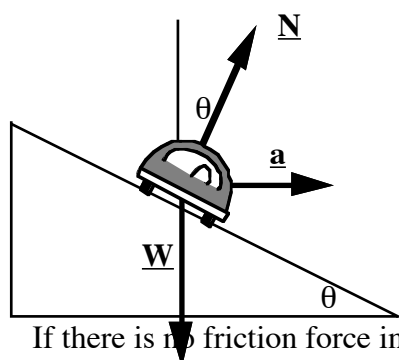
$$\therefore F = W \frac{\sin \theta + \mu_k \cos \theta}{\cos \theta - \mu_k \sin \theta}$$

check: in the limit of no friction, $\mu_k = 0$,

so $F = W \sin \theta / \cos \theta = W \tan \theta$.

also check when $\theta = 0$, $F = \mu_k W$

Exercise for the student: demonstrate this is true.



ii) Let $\underline{ds}$ be the displacement up the plane:

$$\begin{aligned} \text{Power} &= \frac{d(\text{work})}{dt} = \frac{\mathbf{F} \cdot \mathbf{ds}}{dt} \\ &= \frac{F \cos \theta ds}{dt} = F v \cos \theta \end{aligned}$$

iii) If there is no friction force in the plane of the diagram, then the horizontal acceleration a satisfies

$$ma = N \sin \theta$$

There is no vertical acceleration so

$$W = N \cos \theta$$

and the centripetal acceleration required is v^2/R , so

$$\begin{aligned} m \frac{v^2}{R} &= N \sin \theta = \frac{W}{\cos \theta} \sin \theta = mg \tan \theta \\ \theta &= \tan^{-1} \left(\frac{v^2}{Rg} \right) \end{aligned}$$

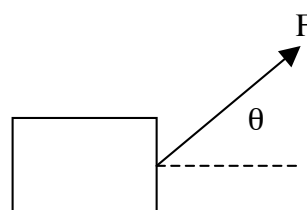
iv) In the normal direction – the same direction as N in the diagram. The driver of the car is also travelling in circular motion with the same speed and radius, so exactly the same equations apply, but with a different value of m.

Q7.

(a) $v = \text{const.} \therefore W_{\text{total}} = \Delta K = 0$

(b) $W = F \cos \theta = 30.1 \text{ J}$

(c) Gravity does no work, total work is zero,
therefore $W_{\text{friction}} = -30.1 \text{ J}$



(d)

$$F_{\text{friction}} = W_{\text{friction}} / d = 7.42 \text{ N} = \mu_k N$$

$$N2 \text{ vertical } N + F \sin \theta = Mg$$

$$N2 \text{ horizontal } N = Mg - F \sin \theta = 33 \text{ N}$$

$$\mu_k = 7.42 / 33 = 0.23$$

Q8.

$$(a) \quad F = -\frac{dU}{dx} = \frac{12A}{x^{13}} - \frac{6B}{x^7}$$

$$(b) \quad F = 0 \therefore x^6 = 2A/B$$

$$\therefore x = (2A/B)^{1/6}$$

Q9.

$$(a) \quad F = 52.8x + 38.4x^3$$
$$W = \int_a^b F dx = \int_a^b (52.8x + 38.4x^3) dx = \left[26.4x^2 + 9.6x^4 \right]_a^b = 28.8 \text{ J}$$

(b) Non conservative forces do no work, so mechanical energy conserved, so $\Delta K + \Delta U = 0$

$$\Delta K = 1/2 mv^2 = 28.8 \text{ J} \Rightarrow v = \sqrt{\frac{2 \times 28.8}{2.17}} = 5.15 \text{ m/s}$$

(c) We have shown in (a) that $U = U(x)$, which is the definition of a conservative force.

Q10.

(a)

Assume it slides,

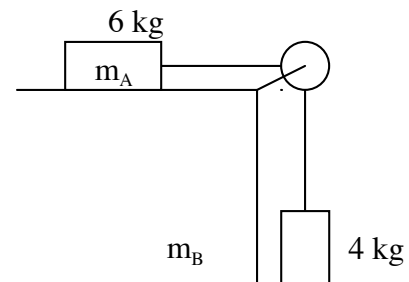
$$m_B g - F_{fr} = (m_A + m_B) a$$

$$F_{fr} = \mu N = \mu m_A g < m_B g$$

$$a = \frac{m_B g - \mu_k m_A g}{m_A + m_B} = 2.16 \text{ ms}^{-2}$$

$$N2 \text{ for } m_B : m_B g - T = m_B a$$

$$\therefore T = m_B (g - a) = 30.6 \text{ N}$$



(b)

$$v^2 = v_0^2 + 2as = 4.32 \times 1.5 = 6.5 \text{ m}^2/\text{s}^2$$

$$K = 1/2 mv^2 = 1/2 \times 10 \times 6.48 = 32 \text{ J}$$

(c)

$$\Delta U = -4.0 \times 1.5 \times 9.8 = -59 \text{ J}$$

$$\Delta E = -58.8 + 32.4 = -26 \text{ J}$$

$$\therefore \text{heat} = 26 \text{ J}$$

Q11. no non-conservative forces do work so

Mechanical energy is conserved :

$$U_i + K_i = U_f + K_f$$

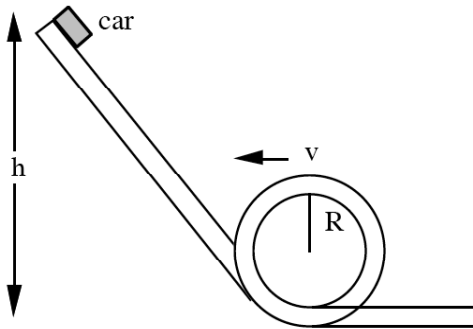
$$U_i = U_f$$

$$(1/2)kx^2 = mgh$$

$$h = \frac{kx^2}{2mg} = 2.0 \text{ m}$$

$$d = h / \sin \theta = 2h = 4.0 \text{ m}$$

Past exam question



i) v must be sufficiently great that the centripetal force at the top of the loop at least equals the weight of the car. (Faster than this, a downwards normal force is required.)

No non-conservative forces do work, so conservation of mechanical energy applies:

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg \cdot 2R + \frac{1}{2}mv^2$$

$$v^2 = 2g(h - 2R)$$

It loses contact when normal force = 0.

$$N + mg = F_{\text{centrip}} = mv^2/R \quad \text{i.e.} \quad \text{falls when } mg = mv^2/R$$

$$v^2 = gR$$

Therefore it just falls off if h satisfies

$$gR = v^2 = 2g(h - 2R)$$

$$5gR = 2gh \quad \therefore \quad h_{\text{min}} = 5R/2$$

ii) No. All forces are proportional to the mass and so scale accordingly.

(iii) For the marble, some of the initial potential energy is converted into rotational kinetic energy, so there is proportionally less translational kinetic energy at the top of the loop, so its speed is less. So the centripetal force required is less than the weight, so it falls off the track.