Acoustical characterization of flute head joints

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A method is described for characterizing the acoustical properties of flute head joints by measurement of the quantities $F_n = f_n/(2n-1)$, where f_n is the frequency of the nth impedance maximum of the head joint as viewed from the end of a short cylindrical tube simulating a part of the flute body and with the embouchure hole completely unobscured. The tube length is chosen to make $f_1 \simeq 170$ Hz. It is shown that the pattern of this "resonance signature" curve F_n in the frequency range f_n up to 5 kHz is sensitive to the precise geometry of the head joint so that it may serve as a convenient correlate for playing behavior, even though its shape is only indirectly related to conditions in a blown flute. A simple and inexpensive apparatus for determining this resonance signature is described.

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INTRODUCTION

The head-joint section of a flute is generally agreed to be of critical importance in determining the musical responsiveness and tone quality of the instrument, while the keywork and exact hole geometry on the body section influence mechanical reliability and tuning throughout the scale. This statement is an oversimplification, but many fine players use flutes with bodies and heads by different makers and some fine craftsmen concentrate almost exclusively on head joints.

The overall design of the flute has changed very little since it was modernized by Boehm[1] more than a century ago. He replaced the conical-bodied cylindrical-headed baroque flute developed by the Hotteterre family in the 17th century by a cylindrical-bodied tapered-headed design with ingenious keywork to allow eight fingers and one thumb to control the 12 venting holes (and several extra optional keys) necessary to produce a full chromatic scale.

Boehm's head joint, the form of which is shown in Fig. 1, was empirically developed and, rather than having a conical taper, followed a form which he described as "parabolic." The exact tapering law, the height of the chimneylike riser to the embouchure hole, and the exact shape and amount of undercutting of that embouchure hole provide the major variables that distinguish one head joint from another. The material of the tube wall (usually silver alloy, gold alloy, or even platinum) and particularly the wall thickness, may have some influence (there is still debate about this) but it is certainly minor compared with geometrical variables.

As a result of a classic paper by Benade and French[2] we now understand the basic rationale behind the success of Boehm's head-joint design, more recent work having only slightly modified their original conclusions while extending our knowledge of the sound production mechanism.[3-7] Basically the function of the head-joint design is to produce, with the player's lips in position, a set of corrections to the simple cylindrical open tube resonance frequencies that cause the upper passive resonances to agree closely with the harmonics of the sounded lowest mode, this sounded mode frequency normally lying rather above the corresponding passive mode frequency.[3,6]

The optimal form of correction to be produced by the head joint across the compass of the flute will clearly depend upon the lip shape and playing technique adopted by each individual player. It would therefore be useful to have a technique for determining this quantity, or at least something directly related to it, in the simplest manner possible. It is towards this end that the present paper is directed.

I. MEASUREMENT PRINCIPLES

The acoustical properties of a flute head joint are, in principle, completely determined by its geometry, the only adjustable parameter of which is the stopper position, this usually being set close to 17 mm from the center of the embouchure hole. A much simpler, though less precise characterization can be obtained by measuring the acoustical parameters directly, and for such a measurement it is more important to have a simple reproducible acoustic "signature" for each head joint than to measure with difficulty a more complex quantity that is more directly related to conditions during playing. This consideration suggests that the head joint should be measured with the embouchure hole completely unobscured, rather than being partly covered by some arbitrarily defined approximation to the player's lips.

If the head joint is extended by a piece of cylindrical tubing 19 mm in inside diameter, to represent the instrument body, then the passive resonances are the admittance maxima at the embouchure hole with the other end of this tube open. These need to be sampled for a wide range of tube lengths because the taper has a more significant effect on mode frequencies for short tubes than for long tubes.

From a practical point of view, however, it is generally easier to locate impedance maxima rather than admittance maxima, since spurious noise signals then have less effect. However, a configuration with an admittance maximum at the embouchure has an imped-

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ance maximum effectively an odd number of quarter wavelengths down the tube, so that we can meet this problem by shifting the measuring point.

The experimental arrangement is therefore as shown in Fig. 1. The head joint is fitted to a piece of 19-mm inside diameter tubing of sufficient length to bring the fundamental resonance with the remote end closed to a frequency somewhere in the range 100 to 500 Hz. As discussed below, the frequency giving best resolution is about 170 Hz.

The far end of this tube is closed with a tightly fitting plug into which are inserted a small electret microphone and an annular capillary tube, of the type described by Backus,[8] connected to an earphone to be used as a sound source. This annular capillary consists of a metal tube 50 mm in length and 6 mm in inside diameter, inside which is a 50 mm length of metal rod 5.8 mm in diameter kept in coaxial location by three wires each 0.1 mm in diameter. Such a tube provides a nearly frequency-independent high acoustic resistance and thus gives an excitation of constant acoustic volume flow if the earphone produces a constant acoustic pressure. In fact such constancy of earphone output is required over only very small frequency ranges since we require only to measure the frequency of the impedance maxima not their magnitudes, but the high acoustic impedance of this capillary prevents it from disturbing these peak frequencies.

When the earphone is fed with a nominally flat sinusoidal signal from an amplifier and the frequency is swept from about 100 Hz to 5 kHz, the passive resonances for this particular configuration are clearly detected as output maxima from the microphone, and their frequencies can be determined with an uncertainty of no more than ±0.1%. With no special precautions other than a quiet environment, the peaks below 5 kHz are all clearly resolved and unambiguous. The situation changes above 5 kHz, but this region is outside our present concern.

By applying this measurement technique to a variety of head joints in succession we can characterize each by the pattern of its resonance frequencies.

II. THE RESONANCE SIGNATURE

Because the head joint, as here mounted, behaves roughly as a cylindrical pipe open at the embouchure end, we expect a series of impedance peaks at the measuring end with frequencies f_n given approximately by

$$f_n \approx (2n-1)f_1 \tag{1}$$

and it is the deviation of the measured resonances from this simple relation that constitutes what we might call the "acoustic resonance signature" of the head under study. It turns out that the most useful quantity to plot is

$$F_n = f_n/(2n-1)\,, \tag{2}$$

for this gives a direct indication of the fundamental frequency f_1 that has been used. F_n is closely related to

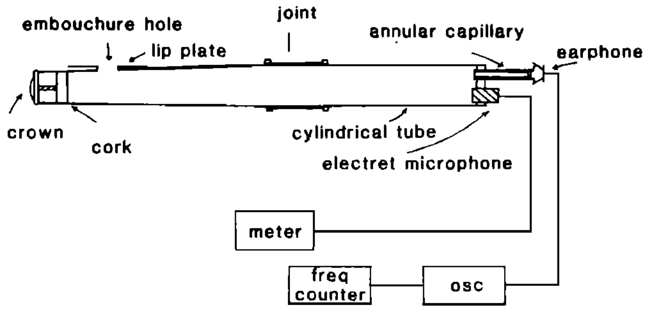

FIG. 1. Schematic section of a typical flute head joint with the impedance head and measuring equipment in place. An alternative simplified electronic system is shown in Fig. 7.

the effective total end correction for the head joint as calculated by Benade and French.[2]

As a preliminary check of the apparatus we can measure the F_n signature of a simple cylindrical tube replacing the flute head joint. This is shown in Fig. 2. As expected, F_n is nearly constant with n but rises slightly at frequencies approaching the radiation cutoff for reflection from the open end[9] (about 5 kHz for a 19-mm tube) and falls at low frequencies because of the effects of viscous and thermal losses to the walls.[10] For a tube with 19-mm inside diameter and length about 50 cm, giving a fundamental resonance of about 170 Hz, the approximate form of the signature curve can be shown from this theory to be

$$F_n^{(170)}(f) \approx 170(1 - 0.17 f_n^{-1/2} + 1.2\times 10^{-10} f_n^2)\,, \tag{3}$$

where f_n is the frequency of the nth resonance. This curve, with appropriate minor adjustment to the factor 170, is shown superposed on the data. The fit is clearly good and could be improved by a small increase in the numerical factor accompanying the $f^{-1/2}$ term, which depends upon the smoothness of the walls and may not have the "ideal" value assumed in Eq. (3).

In measuring an actual head joint it is important to decide upon the length of cylindrical tube to which it is to be joined, or equivalently on the fundamental frequency on which the F_n series is to be based. If this tube is short, then the head-joint taper will have maximal effect upon the shape of the signature curve F_n but only a few points will be measured below the 5-kHz limit. If, on the other hand, the tube is long, then there will be a large number of points on the measured curve but, because the tapered head is only a small fraction of the total tube length, F_n will be relatively flat and

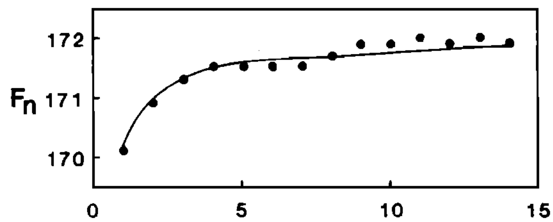

FIG. 2. Measured resonance signature F_n for a simple cylindrical tube 19 mm in diameter and about 50 cm long. The curve is calculated from theory for a smooth tube.

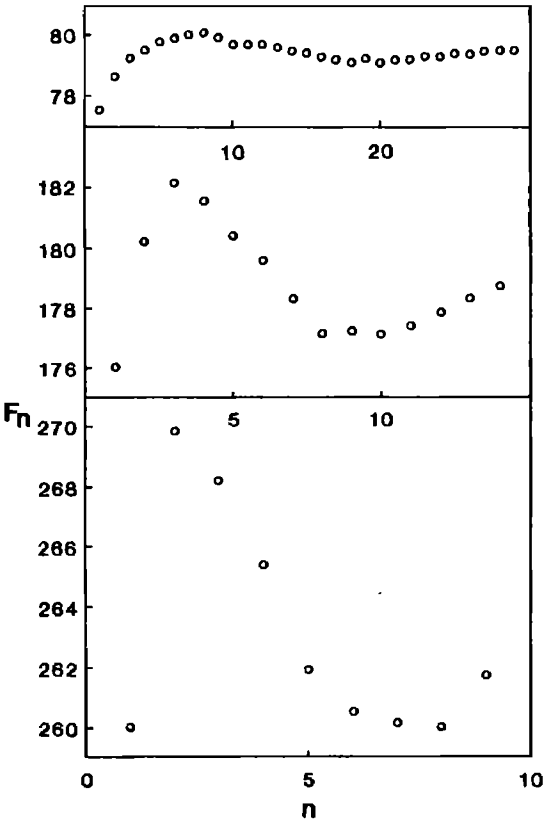

FIG. 3. Measured resonance signatures F_n for a conically tapered head joint attached to a length of cylindrical tube so as to give a fundamental frequency of about (a) 78, (b) 176, and (c) 260 Hz.

frequency resolution may become a problem. These points are illustrated in Fig. 3 for three different tube lengths giving fundamental resonances f_1 near 80, 170, and 260 Hz, respectively. The intermediate choice with $f_1 \approx 170$ Hz appears to be the best compromise and was therefore adopted for all our subsequent measurements. Minor variations in f_1 of up to ±5 Hz, caused

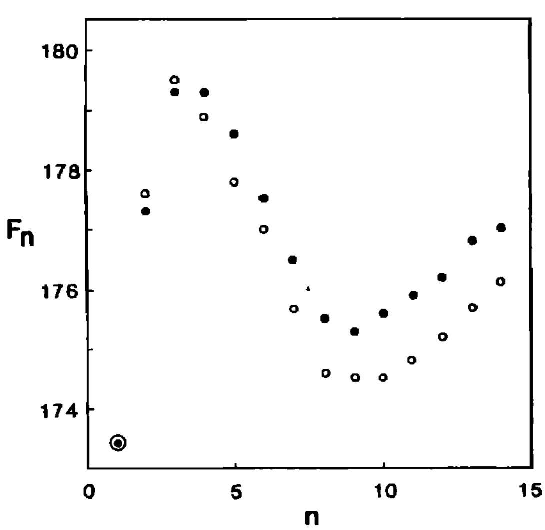

FIG. 4. Measured resonance signatures F_n of a student flute head joint (filled circles) and of a simple conical head joint (open circles). Each was fitted to a cylindrical tube to give a fundamental resonance near 170 Hz and the curves have then been displaced vertically to coincide at that point ($n = 1$).

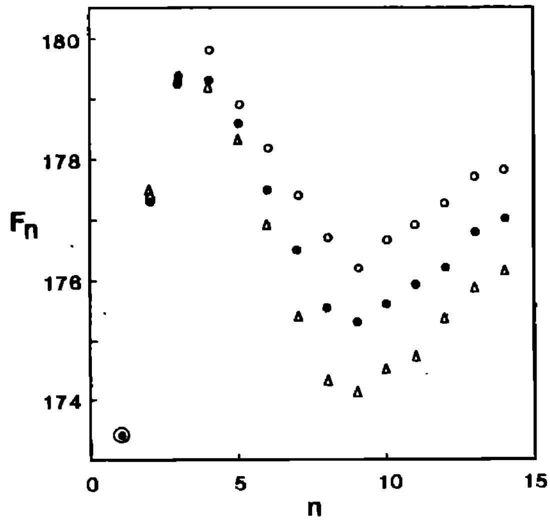

FIG. 5. Signature curves F_n for a typical student flute head joint with cork position set at 15 mm$^{\circ}$, 17 mm$^{\bullet}$, and 19 mm$^{\Delta}$ from the center of the embouchure hole.

by temperature changes or small differences in tube length are not significant within the resolution of F_n and may be compensated for by simple displacement of the frequency scale.

To illustrate the sensitivity of the signature to minor changes in geometry, we show in Fig. 4 the measured signatures of the head joint from a typical student flute (Armstrong model 105) and of a head joint with simple conical taper as made in our workshop.

Even more striking is the sensitivity of F_n to quite small displacements of the stopper in the top of the head joint. The normal position for this stopper is close to 17 mm from the center of the embouchure hole and in Fig. 5 we show the considerable change in F_n produced by variation of this distance between 15 and 19 mm.

III. SIGNIFICANCE OF THE RESONANCE SIGNATURE

As was remarked before, the resonance signature is not directly related to the playing properties of the flute, if only because of the influence of the player's lips in partly covering the embouchure hole. It does, however, provide a much simpler means of comparing the acoustic properties of two head joints than would a detailed dimensional measuring procedure and, if signature curves were available for head joints of different design, this would allow a player to express his musical preferences in terms of an objective physical measurement.

More than this, since the signature is an objective and indeed a calculable quantity if precise dimensions are known, any systematic preference by players for head joints with particular signature features could lead to the design of models to emphasize those features. The formal basis for such perturbation calculations of the effects of small changes in geometry has been set out by Benade and French,[2] and has been discussed less formally in relation to other woodwind instrument bores by Benade.[11] Detailed calculations from first principles are also possible, as demonstrated for the oboe by Plitnik and Strong.[12]

It is important to emphasize, however, that the signature, as we have defined it, is by no means the only determinant of the playing quality of a flute head joint. Indeed one might argue that the exact shape of the embouchure hole, the sharpness and undercut angle of its edge, and even the shaping of the lip plate are at least as important as is the resonance signature in determining subtleties of tone and response. These are, however, matters for argument, and there does not seem to be any way of characterizing these variables at the present time other than by detailed dimensional measurement. At least the resonance signature is well defined, easily measurable, and clearly though indirectly related to the musical behavior of the instrument.

IV. SOME THEORY

A detailed discussion of the flute head joint was given by Benade and French[2] and it is unnecessary to repeat this here. It is useful, however, to see how the various features of the signature curve arise and how they might be modified.

We have already remarked that wall effects reduce the speed of sound in a tube at low frequencies, and this droop in the signature is shown for a simple cylindrical tube in Fig. 2. When the tube is tapered so that it is narrowed towards the open (embouchure) end, then this droop is exaggerated—by wave propagation effects rather than by added wall losses—to produce the sharp drop at low frequencies (small n) characteristic of the signature.

The other major feature of the signature curve is a broad dip around 3000 Hz ($n \approx 9$ for $f_1 \approx 170$ Hz). The origin of this can be seen from consideration of a lossless tube of constant diameter for which the electric analog circuit is shown in Fig. 6. The characteristic impedance of the line is $Z_0 = \rho c/S$ where S is the cross section of the tube and ρ is the density of and c the velocity of sound in air. The impedance measurement for determining the signature is made at $x=0$, the open-circuited end at $x=L$ representing the rigid stopper and Z_m the impedance of the embouchure hole at $x=L-l$. If radiation resistance is neglected, then Z_m is purely inductive since the chimney height is very small.

Analysis of this circuit immediately yields as condition for an impedance maximum at $x=0$

$$\tan[k(L-l)] = j(Z_0/Z_m) - \tan kl\,, \qquad (4)$$

where $k = 2\pi f/c$. Since $|Z_m| \ll |Z_0|$ and $kl \ll \pi/2$ in a typical case, these impedance maxima occur near $k(L-l) = (n+\frac{1}{2})\pi$, with terms on the right-hand side providing an "end correction." It is the variation of this end correction with frequency that is responsible for the dip in the signature.

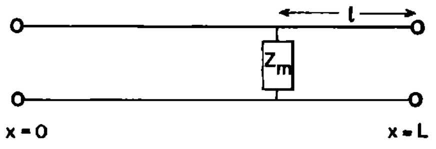

FIG. 6. Electric transmission line analog for a flute head joint. The impedance is measured at $x=0$, the head is closed by a rigid stopper at $x=L$, and the embouchure hole appears as a shunt of impedance Z_m at a distance l from the stopper.

The signature curve measured with the apparatus set up as in Fig. 1 can, in principle, be calculated exactly once the geometry of the head is known, either using the step-by-step numerical procedures that were applied successfully to the oboe by Plitnik and Strong[12] or the variational methods of Benade and French.[2]

An aim of greater interest, however, is to compute the geometry necessary to achieve any particular distribution of resonance frequencies and hence any desired F_n curve. This can be done, in principle, by application of variational techniques to a basic version of a head joint.

This general approach has been applied with some success to the problem of obtaining vocal tract cross-sectional areas from measured resonances by Mermelstein[13] and by Schroeder.[14] The tract area as a function of distance along the tract is expressed in terms of cosine perturbations to a cylindrical tract, spatial resolution being limited by the number of measured resonances available. There is some problem with lack of uniqueness of the solution so obtained but this can be alleviated by imposing physically reasonable constraints on the tract shapes allowed.

Another approach that could be used is the computer sorting routine described by Atal *et al.*[15] for vocal tract analysis. This approach reduces essentially to the computation of signatures for a wide variety of configurations, the ordering of these configurations on the basis of certain selected dimensional parameters, and then a sorting routine to proceed from the specified signature to the stored signature and hence physical configuration giving best fit.

V. A PRACTICAL SIMPLIFIED MEASUREMENT SYSTEM

The measurement system used in this paper was assembled using standard laboratory equipment but, if the

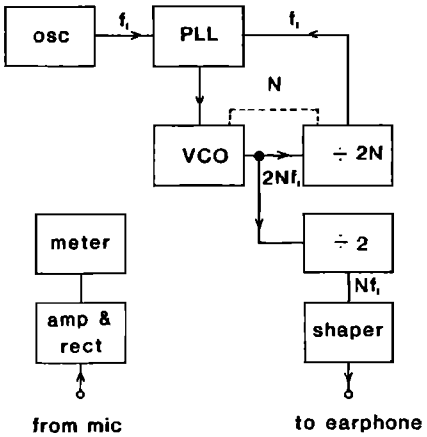

FIG. 7. Schematic circuit for a simple harmonic generator and measurement system. The principle of operation is described in the text.

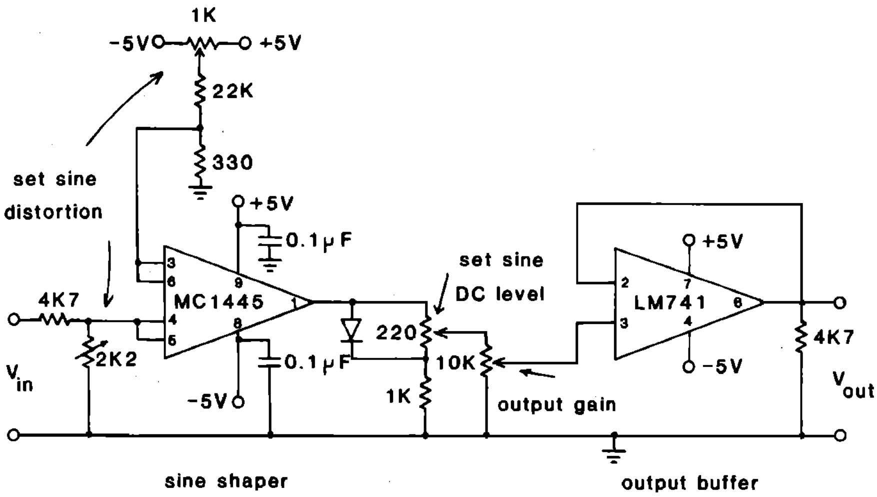

FIG. 8. Details of the shaping circuit used to convert triangular waves to sine waves in the system of Fig. 7. Adjustment of the signal level is critical to good operation. (Original source of circuit is not known.)

analysis of head-joint signatures is to enjoy any practical use for flute makers and purchasers of their instruments, it is desirable that a system be devised which is both simpler in operation and less expensive. In this section we describe such a system.

The impedance head illustrated in Fig. 1 is noncritical in design and construction. It must be a good fit in the tube but otherwise it is made from simple components readily obtainable for a few dollars from any radio hobby shop.

The simplified measurement system developed to determine the signature F_n is shown in schematic form in Fig. 7. Essentially it consists of an oscillator of adjustable frequency f_1 coupled to a harmonic generating system to produce sine waves of frequencies $(2n-1)f_1$. For each value of n selected in turn, the oscillator frequency f_1 is adjusted to produce a maximum in the signal from the microphone, and the resulting set of f_1 values comprise the signature function F_n. Modest stability and incremental calibration accuracy of the basic oscillator is all that is required to produce a reliable F_n series; no counter is needed, and the absolute frequency of the oscillator is not critical. Indeed the oscillator might reasonably have one control to adjust the initial value of f_1 for reasonance and a calibrated Δf_1 control to determine the F_n curve.

The harmonic generating circuit is the heart of the system and operates as follows. The voltage controlled oscillator (VCO) is set, by switching ganged to a dividing circuit ($\div 2N$) used to select the value of $N = 2n-1$, to run freely at a frequency near $2Nf_1$. The divider reduces this frequency back to about f_1 and this frequency is locked exactly to the original oscillator frequency f_1 through the phase-locked loop (PLL). The output frequency of the VCO then has a frequency of exactly $2Nf_1$. This output frequency is then divided by 2 to give the required frequency $Nf_1 = (2n-1)f_1$. In this arrangement the initial divisor is taken as $2N$ rather than N to avoid frequency modulation effects which can arise when N is odd.

This output waveform may be either a square or a triangular wave, depending upon details of the circuit, and must then be shaped to a sinusoid with adequately small distortion components. This shaping requirement is not particularly critical since a maximum response is being sought at the fundamental frequency of the waveform. A biased diode chain could reasonably be used, but the circuit given in Fig. 8, when carefully adjusted in level, produces a reasonably well-shaped sinusoid with very little circuit complexity when operating on a triangular input waveform.

The microphone signal is simply amplified, rectified, and displayed on an uncalibrated meter, a peak in the reading as f_1 is varied indicating the frequency F_n. The whole equipment is thus simple, compact, and inexpensive.

VI. CONCLUSION

We have described the development of a simple system for the partial characterization of the acoustic properties of flute head joints by means of a resonance signature curve. It is hoped that such a curve may serve flute makers and flute purchasers as an objective means of comparison between different head designs, though actual playing tests must be used by each individual to decide the type of signature curve best suited to his playing style. With practical use in mind, we have also developed a simple and inexpensive system for the determination of resonance signatures without special skill or sophisticated laboratory equipment.

While recognizing that the resonance signature is far from being a complete characterization of the playing properties of a flute, we hope that use of this method may help to quantify at least some aspects.

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